A parametric metamorphosis of Islamic geometric patterns: The extraction of new from traditional

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ABSTRACT: A method for the exploration of design space in traditional Islamic geometrical patterns is presented. The method uses traditional Islamic geometry as a starting point and performs a metamorphosis operation for geometrical creation and exploration of possible variations.

KEYWORDS: Parametric Design, Metamorphosis, Patterns Generation, Islamic Geometric Pattern

INTRODUCTION

A considerable amount of research has been conducted to answer the question of how to create both traditional and new Islamic geometrical patterns. Researchers have attempted to answer this question manually and digitally. Some scholars focus on how to simulate traditional patterns, while others aim to create new patterns from scratch. These studies enrich the literature by simulating the methods used by the original artists, architects, and craftsman; by presenting new design methods; and by improving existing methods. This study takes a new approach toward answering the initial question. Our work focuses on how to extract new patterns out of existing ones.

According to Syed Abas and Amer Salman (Abas et al. 1995) Islamic geometric patterns are defined as patterns that contain:
1.Arabic calligraphy
2.Where created between 900 AD and 1500 AD by Muslims or non-Muslims in a society where the common practiced religion was Islam
3.Patterns derived from Arabic calligraphy or traditional patterns
In this paper, we build upon this third definition to explore Islamic geometric patterns by deriving them from existing geometries.

By starting with decomposing and parameterization existing traditional Islamic geometry, we can derive new, unique geometries from them. In other words, this research tries to move beyond the creation of the geometry to the exploration of the geometry. In that sense, this method can be used to conduct an analytical study of the geometric patterns to improve the understanding of the formal qualities of the geometric pattern. For instance, by exploring all possible combinations of the geometric components within a pattern, it will be possible to identify the desirable systems of proportion.

Because large number of geometrical derivations can be reached, which makes counting designs relatively impossible; this paper will express the geometric metamorphoses that provide a convenient way to explore possibilities. By adding the fourth dimension of time to the process, it becomes possible for the designer to express the keyshape of the geometry, i.e., the state of the geometry at a particular point in time (Kolarevic 2004). The question that needs to be asked at this point is it possible for a keyshape of one geometry to be qualitatively and quantitatively equal to an existed Islamic geometry?

We have built a computer representation model to construct the entities of the geometrical patterns with modifiable attributes. This is known as a parametric model, and it allows us to explore all design variations with ease (Barrios Hernandez 2006). By defining certain rules that govern the parameter values, the designer can to explore the patterns in a manual or metamorphic manner. The variation performed on the fundamental unit level, which is a specifically defined portion of the repeat unit, will populate the level to the repeat unit using symmetry and either translation, rotation, or a combination of the two. Thus, the parameterization of the fundamental unit will allow designers to manipulate the whole pattern.

1.0 PRECEDENCE

Scholars have done magnificent work on how to define the fundamental unit and populate it on the geometry. Haresh Lalvani presents a method of coding that generates classes of various patterns from different cultures (Lalvani 1989). His method involves defining the fundamental unit and using symmetry to populate the resulted geometries of the pattern.

Ahmad Aljamali takes an approach similar to Lalvani's, defining a fundamental unit to control the shape of the pattern by the radius of the repeated unit and the angle of rotation (Aljamali and Banissi 2003). Craig Kaplan creates a program that allows him to explore the design space of Islamic stars by manipulating the sub-motifs (Kaplan and Salesin 2004). Ali Izadi creates a code that explores different geometries by controlling of the main points in the motif (Izadi, Rezaei, and Bastanfard 2010).

Most of them have worked to provide a method for simulating traditional patterns and for exploring new patterns made from scratch. The method we are proposing does not simulate traditional geometries, neither does it generate new geometry from scratch. Rather, it performs post-design analysis of an existing geometry to produce new, unique design patterns.

2.0 THE PROPOSED METHOD

The proposed method works on existing geometry. It does not create new geometries from scratch but rather performs spatial transformations on the provided Islamic geometry based on specific rules. But before presenting the proposed exploration method, it is necessary to introduce two approaches to geometrical exploration of Islamic geometric patterns. The first approach enables the exploration without preserving the qualitative properties of the geometry (i.e. performing topological transformation), and the other preserves the geometry's qualitative properties (i.e., the produced geometry will have the same number of points, lines, and faces) (Kolarevic 2004).

Both approaches share the method of finding of the fundamental unit and building the parametric model, i.e., defining the fixed and modifiable attributes; however, different rules govern their special transformations.

2.1. The fundamental unit

The method acts on the repeated unit level and then populates the results globally to the pattern. Both Issam El-Said (El-Said et al. 1993) and Rima Al Ajlouni (Al Ajlouni 2012) have clearly distinguished between the repeat unit (the seed), which is the basic geometrical composition, and the pattern (structure), which is the product of systematically repeating the geometry.
To find the fundamental unit, the repeat unit is decomposed to its constructional components. This operation will produce a fundamental unit, which is defined as the minimum motif that cannot be reached with symmetry.

To make the process of finding the fundamental unit easier, we first need to determine the cell unit. According to Abas and Salman (Abas et al. 1995), the cell unit is the region with the minimum motif that may be repeated to create the whole geometry. In this paper we differentiate between the cell unit and the fundamental unit. Because the method we are defining does need a completed geometry to begin, it is always a good idea to break down the steps of finding the fundamental unit by analyzing the type of the geometry.

Cell units can be created by dividing the polygon that contains the repeat unit into triangles. The first point of each triangle is located at the center of the polygon, and the other two points are located at the constructional points of one of the sides (Figure 1). Cell units can hold more than one of a fundamental unit, a whole fundamental unit, or less than one of a fundamental unit. There are no specific rules that govern the relationship between the fundamental unit and the cell unit; in fact, it depends on who originally designed the geometry. Aljamali (Aljamali and Banissi 2004) proved that point by breaking up the steps of creating Islamic geometric patterns into four stages: the planer surface stage, the divisional stage, the artistic stage, and the extension stage. The artistic stage is an important factor to consider in determining the fundamental unit. The combined cell units that contains the fundamental unit are defined as the fundamental region.

2.2. Fundamental unit parameterization

In this step, a point should be assigned at each segment intersection in the fundamental unit. An intersection can occur between one segment and another or between the intersection of a segment and the boundaries of the fundamental region.

Having constructed the points, the next step is to build the parametric model. This step involves deconstructing each defined point from the previous step into x and y coordinates and then adding or subtracting a number from the coordinates to relocate a point with the new coordinates. The line segments should correspond to the changes that occur in the point, thus creating a new geometry.

2.3. Rules of spatial transformation

In general, points can be categorized into: (1) constrained points, or points that can travel toward and against the center of the polygon; (2) linked constrained points, which occur when two constrained points are symmetrical and located on the sides of the fundamental region, those two points must always be moving in accordance with each other by taking the same value of change; and (3) anchored points, or points that cannot move at all cases (moving an anchored point can break the continuity of the pattern as well as create additional intersections). All the points located on the outer edge of the repeated polygon are considered anchored points, and the rest are either constrained or linked constrained.

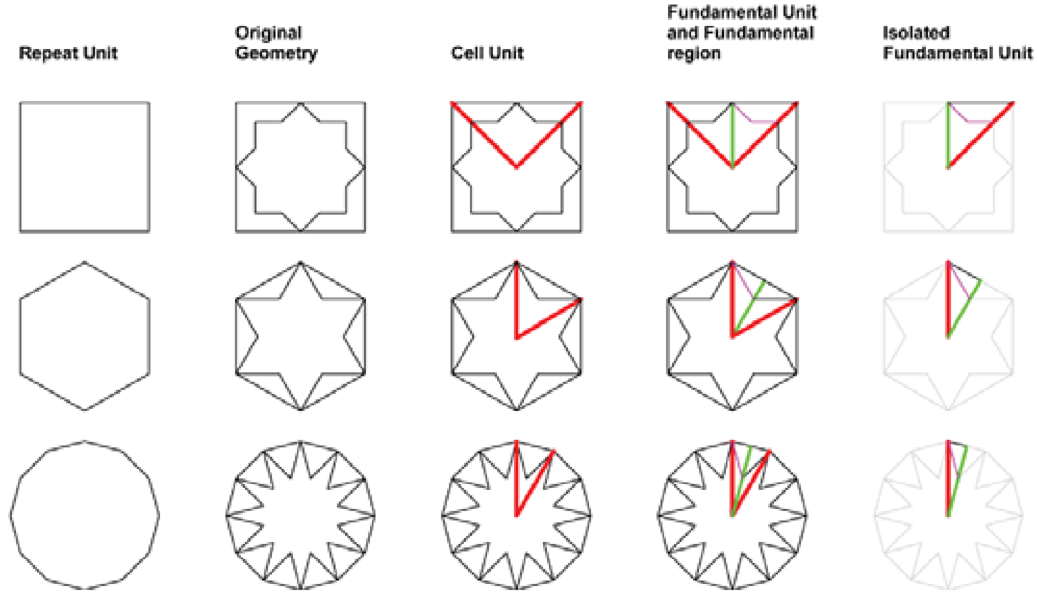

Figure 1: Identifying the fundamental unit. The cell unit is in red, the fundamental region limits are in green, and the fundamental unit is shown in pink.

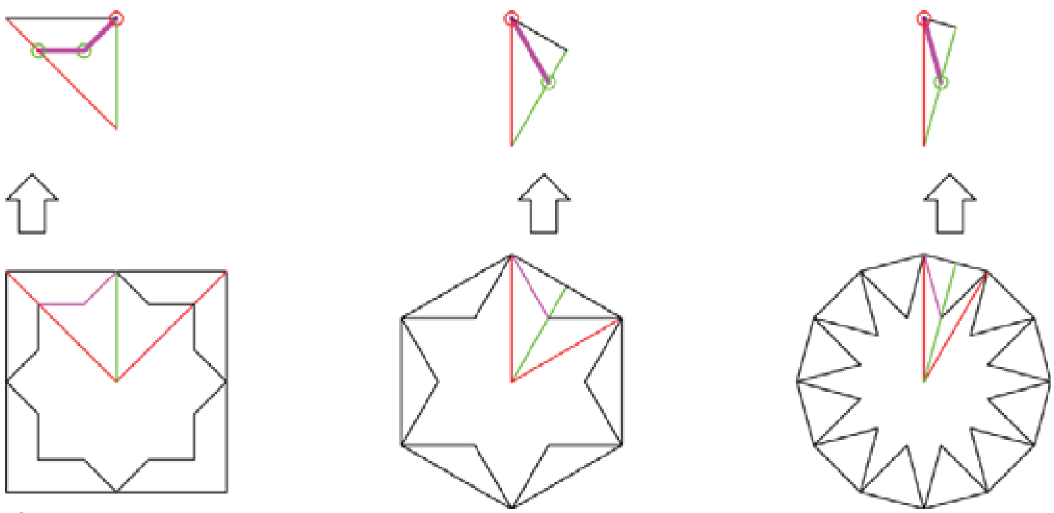

Figure 2: Building the parametric model of the fundamental unit.

2.3.1. The first approach

Qualitative Variant. In this section we discuss the first method in which the manipulation of the geometry should result in a new geometry qualitatively different from the original geometry.

Rules for the First Approach: New geometries can be generated by following set rules that should apply to all the points within the fundamental region. First, at least one point should overlap. Second, lines not allowed to overlap. Third, no additional intersections are allowed. Finally, points should not leave the fundamental region.

Results of the First Method: In this section, we apply the first approach on the six-point star and the eight-point star. For the six-point star, there is only one parameter to control because it contains only one constrained point. However, the eight-point star has two points to control, and more geometries are therefore possible. See the figure 3.

Each geometry in the previous figure has different qualitative properties. The number of points, line segments, and faces is different in the new geometry. In fact, if we apply the same rules from the first method again, we will not be able to reach the original geometry, which is caused by the overlapping that engenders a reduction in the geometry's components.

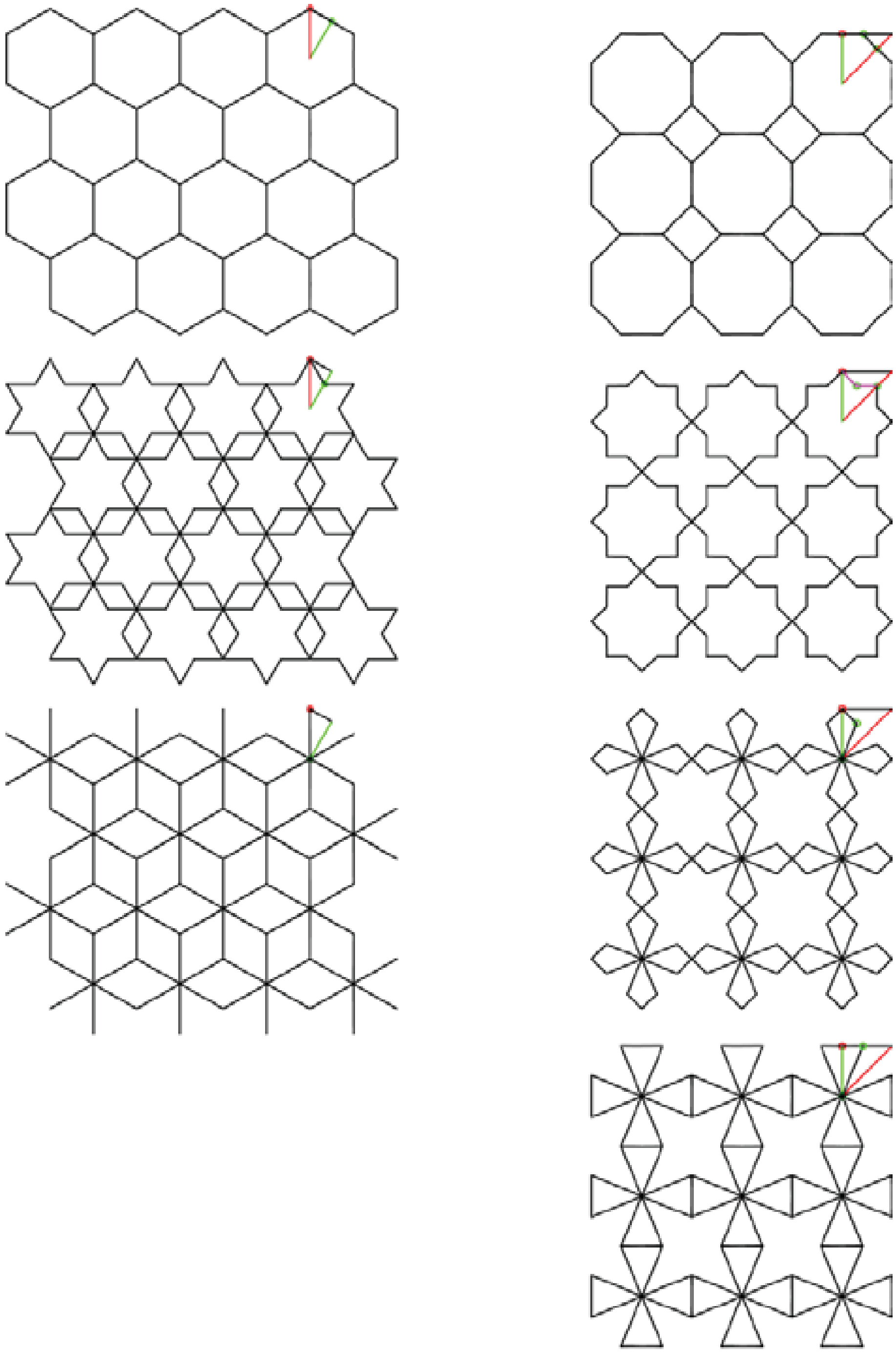

Figure 3: Series of qualitative transformation of two different patterns. Left, six-point star geometrical pattern. Right, eight-point star geometrical pattern.

2.3.2. The second approach

Qualitative Invariant. In this section we discuss the second approach, in which manipulation of the geometry should not result in a topological transformation; the new geometry should be always qualitatively same as the original geometry. The number of points, edges, and faces in the resulted geometry should be always equal to that of the original geometry.

Rules for the Second Approach: New geometries can be generated by adhering to the following rules, for all the points within the fundamental region: First, no point overlap is allowed. Second, line overlapping is not allowed. Third, no additional intersections are allowed. And finally, points should not leave the fundamental region.

Results of the Second Method: In this section we apply the second method on the six-point star and eight-point star. It should be noted that the original geometry and the new geometry are qualitatively invariant. See figure 4.

Reapplying the rules will always return the new geometry to the form of the original Islamic geometry because this method does not cause any reduction in the geometrical components.

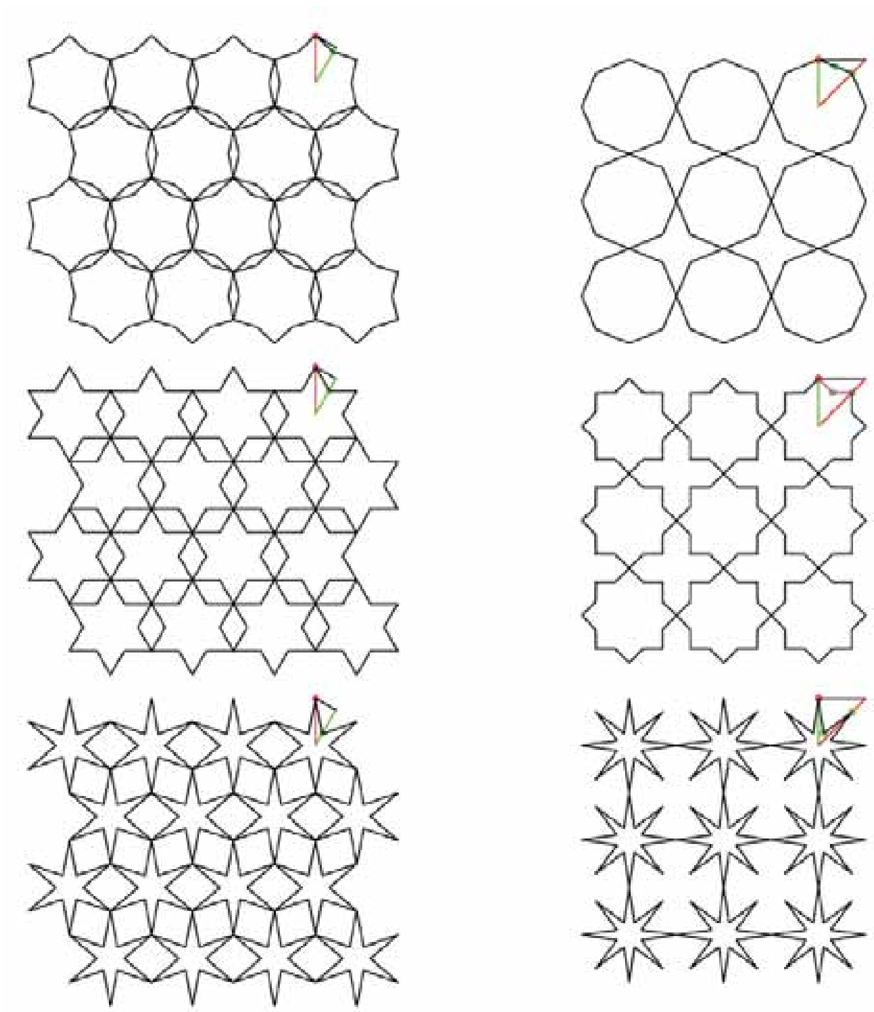

Figure 4: Qualitatively invariant patterns. Left, six-point star geometry. Right, eight-point star geometry.

2.3.3. Metamorphosis approach

To morph the geometry in relation to time, one needs to apply the rules from approach one and approach two and assign different values to the parameters at different points in time. In other words, this approach combines both approaches and adds a time factor, thus broadening the search domain, guiding and easing the exploration process.

Rules of the Metamorphosis Approach:

1. All constrained and linked constrained points of the geometry should be relocated to the center of the polygon. Consider the geometry marked "A" in figure 5. If we relocate all the constrained points to the center

of the geometry, we will have the geometry marked “B” in figure 5. As it is shown, all green circles are on the center.

2. The next step is to release one point at a time (Figure 5 C) until the point reaches the limits of the fundamental unit (Figure 5 D). Then, we release the other point one step only (Figure 5 E) and move the first point back toward the center of the polygon (Figure 5 F), the point stops if it intersect line, overlap another point, or the point leave the fundamental region (Figure 5 G). This procedure will allow us to explore the design domain for both previous approaches. A point can travel a specific distance within a specific amount of time. The time and distance are variables.

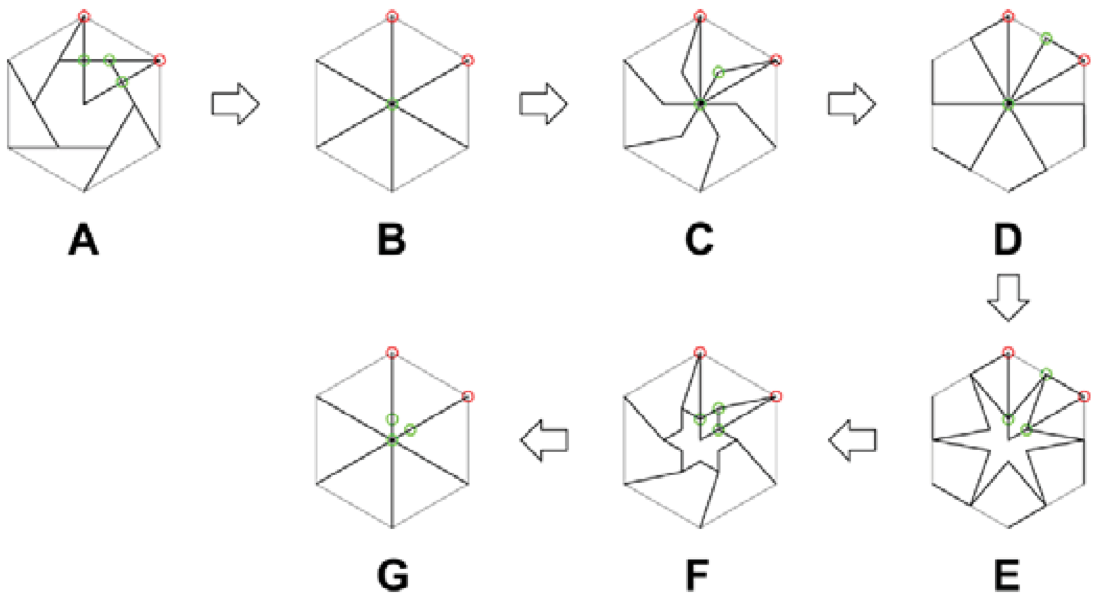

Figure 5: Metamorphoses of Islamic geometry (the seed unit).

All of these geometries in figure 5, except F, have fewer points, and so they are topologically different from A. If the other points were released and all the rules from the second approach were applied, we would be able to get back to the topology of the original geometry. Figure 6 illustrate the rules of the third approach. In this example, there are only two parameters to control. Different geometries may have more parameters and that will make the exploration process longer and more complex, but by using the specified procedure, it is indeed achievable.

Results of the Metamorphosis Approach:The results of the third approach, in the previous example, revealed that the original geometry we used to initiate the process, circled in blue in figure 6, can be transformed into many geometries. Three of them were originally existing geometries, circled in green, and two of them were partially existing patterns, circled in yellow. In other words, different geometries are existed with different status of one geometry in different points in time. To confirm that, we developed a shape code of geometry, which is a numerical description that represents each status/geometric pattern (Lalvani 2003). Consider the code below:

This code describes a six-point star. It uses the Cartesian coordinate system to position points and lines that construct the geometry. Number six represents the number of cells within the geometry. The second and third lines represent the two lines that exist within a single cell unit. Each line has two constructing points. Each point has two coordinates, i.e. X and Y coordinates. It will be read as (x1, y1), (x2, y2). The lines are positioned for one cell unit. The rest of the cells are filled out by continuously populating the same lines. An expanded version of the code might include/add methods to analyze the geometry extensively and document the results of the exploration process.

DISCUSSION

It is possible for two distinct geometries that exist in traditional Islamic patterns to have same geometry topologically, but each one represents a different point in time. Consider this, El-said, in his book Islamic Art and Architecture, demonstrates how to generate an eight-point star. Later in the same chapter, he explains how to generate the octagonal pattern. In other words, he shows two different geometries with two different set of rules to generate them. El-said expresses the eight-point star as A:B:A, which represents the proportions of the constructional grid, while he expresses the octagon as A+B:B. However, using the method presented in this paper, it will be possible for a designer to manipulate these proportions to reach the octagon from the eight-point star and vice versa (El-Said et al. 1993).

So back to the original question: what else can we derive? The answer is that a seemingly unlimited number of geometries can be derived by considering fractions of distance in relation to time. Now, to differentiate geometries, we need a new system that can classify based on when they occur. This predicament implies the second question as to considering the new patterns to be Islamic. If we consider the third proviso that prescribes Islamic patterns as ones derived from other Islamic patterns, then we can say with confidence that the new patterns are in fact Islamic. Furthermore, some of the new patterns seem to fit the visual imagery of the traditional Islamic counterparts, but this perhaps will require further studies to determine the degree of likeness to the traditional patterns. “Just like nature, there is a universal code, there must be one like this for architecture,” Lalvani (Lalvani 2010) said in a TEDx Brooklyn talk. This method is a step toward in finding the code of the original Islamic geometries, and in generating new geometries through a guided exploration of Islamic geometry.

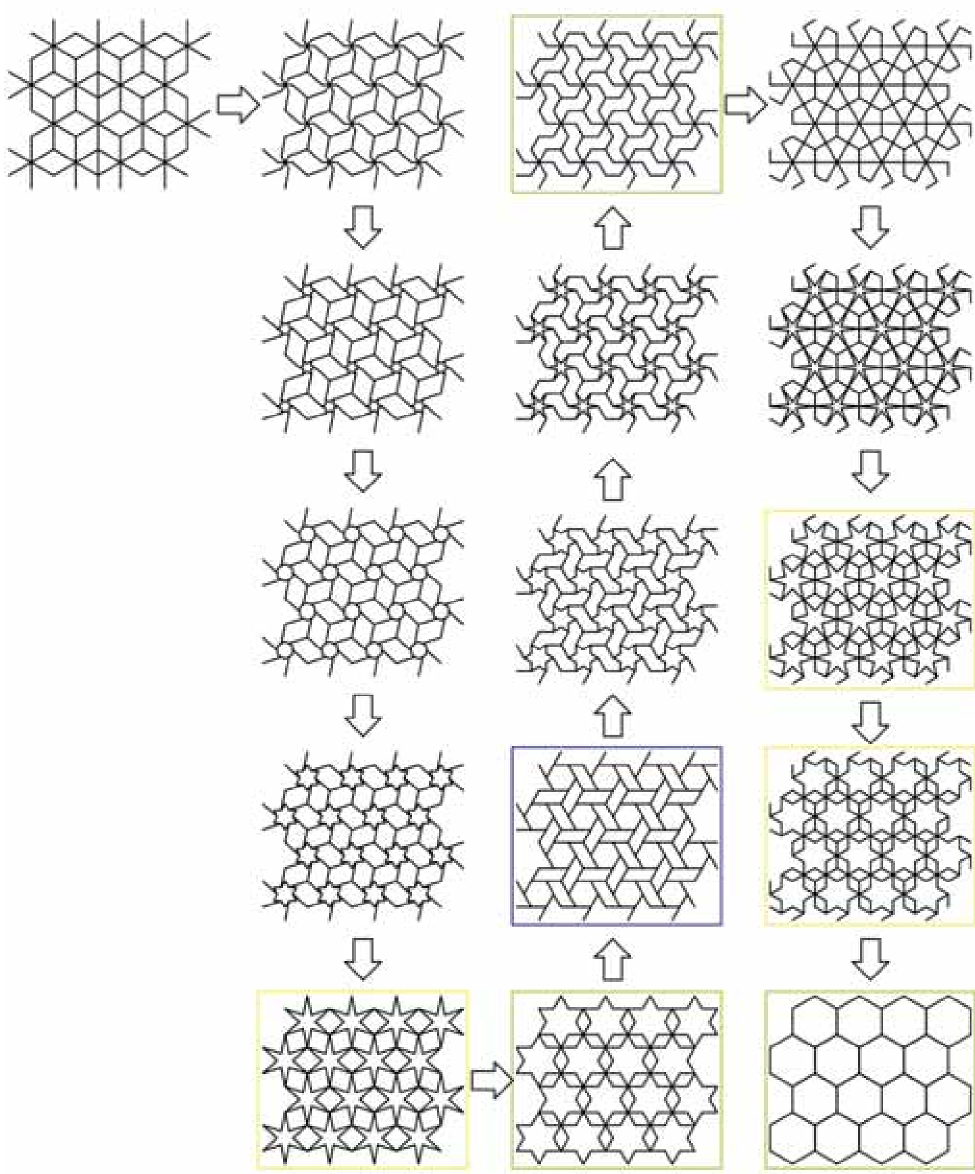

Figure 6: Metamorphoses of Islamic pattern.

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