Surveying stereotomy: Investigations in arches, vaults and digital stone masonry

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ABSTRACT: The art of fabricating, making, crafting and the traditional sense of the stone mason has been diminishing due to dramatic changes in technology. The path of increasing complexity in formal stereotomy is causing the increased use of structural analysis influencing current design. This paper will look into past methods of stereotomic stone masonry construction techniques, and how new practitioners are utilizing stereotomy. Through case studies and historical precedents, methods in stereotomic analysis will be investigated, focusing on the machine generation of the arch.

The arch in its simplest form is the fundamental structure for spanning in unreinforced masonry and has the potential to be combined into many variations. The arch produces a line of thrust which is a theoretical line that represents the path of compressive forces through the structure; which can be used to design and analyze vaulted structures in masonry. The paper traces the change in the meaning of stereotomy, from geometric form finding to advanced static, construction analysis and production. It will survey past, present and emerging practitioners and investigate stereotomic methodology in the creation of vaulted spaces.

KEYWORDS: Stereotomy, Voussoir geometry, Stone Arches, Vaulting systems, Computational Craft

INTRODUCTION

The renewed interest in stereotomy relating to the construction of freeform and vaulted spaces is due to emerging Computer Aided Design and manufacturing technologies, as well as the revival of interest in stone as a construction material due to its ecological properties. As José Carlos Palacios states, after the peak of stereotomy research in the nineteenth century, the advancement of the industrial revolution brought the rise of new building techniques and materials which were primarily structural steel construction. The modern aesthetic of the time further diminished the place of traditional stonework and ornamentation. Across these decades,

> stereotomy collapses and falls into oblivion; the massive masonries of traditional architecture succumbs to give way to a new architecture based on a structural set up never imagined before. Nevertheless, there is still some hope. New geometrical skills gave way to new developments in stereotomy. Digital monitoring allows expansion of the limits of geometry. (Palacios 2008)

The return of stereotomy is largely due to the improvement of computational tools, since these tools offer a pathway to continue the stalled tradition of stone architecture. ‘Computer graphics enable architects to explore sophisticated forms, while subjecting them to static analysis for safety’ (Etlin, Fallacara and Tamborero 2008) [1].

The increased ability to analyze the line of thrust in an arch or vault is contributing to the renewed interest. This paper will cross examine case studies of two practitioners. The work of Professor Mark Burry in the restoration and extension of the Basílica i Temple Expiatori de la Sagrada Família in Barcelona, Spain, involves various “stereotomic techniques in the construction of doubly curved surfaces for the stone work” (Burry 2006). Another practitioner Philippe Block from the Block research group at the ETH in Zurich is investigating digital stereotomy in terms of voussoir geometry for freeform masonry vaults informed by fabrication and structural constraints (Block 2011).

1.0 STEREOTOMY DEFINED

The term stereotomy can be simply described as the art of cutting solids. In relation to architectural construction, it is a set of geometric instructions and techniques of drawing and cutting blocks of stone for their assembly into complex structures. It has its origins as a French term combining two Greek words meaning 'stereos'(solid) and 'tomy'(cut).[2] The earliest and most influential texts on cutting geometrical solids are Euclid's 3rd Century BC texts[3]. Although there is no unambiguous literary evidence for theorizations of stereotomy between the Classical eras and seventeenth century France, Euclid's works were probably the core texts for the practice of stone masonry. Giuseppe Fallacara suggests that the French term has it origin in Philibert de L'Orme's 1567 treatise on architecture.[4] Although Fallacara does not suggest "a direct geometrical connection between Euclid and de L'Orme", rather, that L'Orme had an interest in "Euclid's work, considering it a very high digest of methodological rigor." (Fallacara, Digital Stereotomy and Topological Transformations: Reasoning about Shape Building 2006.) Fallacara hypothesizes that the birth and evolution of projective geometry came from Euclidean geometry speculated by de L'Orme; describing it as a 'methodically setting' according to Euclidean logic. It is interesting to find that Robin Evans categorizes the two geometries as "Platonic and projective rather than Euclidean and Vitruvian" (Evans 2000).

According to Danila Aita, stereotomy, through geometrical principles "allows one to visualise a tridimensional object by means of a bidimensional reproduction… [and] each of the voussoirs." (Aita 2003) Sakarovitch in his paper, 'Stereotomy, a multifaceted technique' describes stereotomy as "the art of drawing the shapes to be given to stones (and bricks) for future assembly" (Sakarovitch 2003) which echoes the French expression 'art du tait', or art of line drawing. The basis for stone cutting was an orthographic projection called the 'tait'.[5] (Evans 2000) A tait can be defined as layout drawings used to enable for precise cutting of masonry blocks into complex architectural forms. There is an intriguing inconsistency here, several versions of the phrase appear, other than Evan's "tait", there is 'traint' (Block 2009) and 'trait' (Fallacara 2006).

Another approach to stereotomy is the one of Girard Desargues in 1640 who published a unique rule to try to solve the problems of stereotomy. This method presents an important graphical development where Desargues presented a global image through a central perspective of the architectonic model that he wanted to study.

> In this sense, Desargues approach was in some way didactical and epistemological, focused on describing a methodology, clearly in contrast to the previous treaties, which described stone cutting work from a geometrical point of view. (Rodriguez, Andres and Alvarez 2011)

Both Sakarovitch and Evans state that stereotomy is a combination of relationships to the history of architecture, masonic geometry, the geometry of the mathematician, studies in the field of mechanics, and the history of crafts and their emergence. (Sakarovitch 2003, Evans 2000)

Whilst Sakarovitch argues that 'statics' is responsible as the fundamental design principle for building stone arches; Aita is quite pertinent on the geometrical problems in finding solutions to the design of arches.

> In antiquity, the arch was considered as a pre-eminent example of geometrical perfection, containing in itself a principle of static perfection: the common conviction was that geometry, not statics, could provide the safest proportions for designing arches. (Aita 2003)

According to Giuseppe Fallacara stereotomy is where "specific geometrical rules and correspondences set the relationship between system and part."(Fallacara 2006.) This implies a parametric approach to design where sets of relationships are defined between components and their assemblage. These parameters are employed by the practitioner based on their requirements. Sakarovitch subsequently explains his preference for Claude Perrault's definition of "the art of using the weight of stone against itself so as to hold it up thanks to the very weight that pulls it down" (Sakarovitch 2003).

This sequence of definitions implies the arch as the generator of stereotomy which gives a "historical depth of 23 or 24 centuries" (Sakarovitch 2003). Stereotomy began as a methodology and descriptive geometry, over centuries it acquired meanings associated with, statics and mechanics. Its continued evolution due to time, technological innovations and context depicts the mutable nature of the term itself.

1.1. Increasing complexity of stereotomy

There is a correlation between the development of stereotomy and advancement of construction tools. Constructing particular shapes and vaults is partly reliant to stone cutting techniques such as half squaring, repointing, squaring and cutting with a template. The cutting method is also dependent on the stone material and size, in relation to finding stability compared to that obtained using larger stones to the smaller ones. Another issue is the fact that stone is characterized by a high resistance to compression and low resistance to traction and bending. As such ancient temples used a maximum distance of 4-5 metres between the axes of columns.

Aita explains the 3 archaic cutting methods which require no preparatory trace. These include cutting *par ravalement* where the stones are cut when they are placed in the vault; "cutting *a la demande* where each stone is hewn for subsequent retouching" (Aita 2003). And the 3rd method cutting *par equarrissement* (also known as *derobement*), "consists of cutting the stone without the help of the panneaux, using the height and depths delimiting the voussior to be made." (Aita 2003)

A construction technique called the *encorbellement* method was developed to get over larger inter axis spaces and coverings. (Figure 1). This method consists in using overhanging(corbelled) stones, with the beds always being horizontal. This was prevalent in works dating back to the 13th century of Mycenean architecture, as well as from the Etruscans of the 7th to 2nd Century BC, "who frequently used corbelling to cover funeral chambers and to make arches." (Aita 2003) The encorbelled face could then be cut in-situ, using the technique Aita named, *par ravalement.*

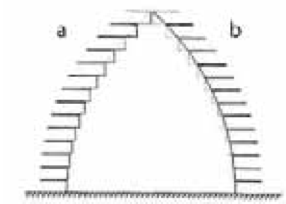

Figure 1: a) The *Encorbellement* method. b) Method of cutting *par ravalement* Source: (Aita 2003)

1.2. Stereotomy and Voussoir Geometry

In the historical record the segmented arch and the tunnel vault (also known as a barrel vault) first appear as brick constructions in the 3rd millennium BC in regions where there was a shortage of wood, such as Mesopotamia and the valley of the Nile. Stereotomy. *a la demande,* stone cutting developed with the introduction of the *voussoir*. A voussoir is defined as a "wedge shaped stone with two oblique faces by means of which it rests on the adjacent voussoirs, laterally transferring the vertical forces due to its own weight and any other loads." (Aita 2003) Figure 2 reveals a segmented arch composed of four voussoirs either side of a central key stone.[6]

Figure 2: Segmented arch with defined terms: (Purchase 1904)

The problem faced by medieval builders in the realization of vaults was how to cut the voussoirs constituting a structure. Aita implies that the medieval builders answered the structural questions, coincidently by solving the geometrical problems: symmetry, stability, material resistance and equilibrium of forces. Jacques Heymen, in 'The Stone Skeleton' describing the design requirements for masonry arches, "failure of a masonry structure will occur when the line of thrust can no longer be contained within the stonework." (Heyman 1966) He then describes in terms of design principles certain assumptions which can be made about the material. These include:
(1) Stone has no tensile strength (assuming that the arch voussoirs are laid dry or with very little mortar).
(2) The compressive strength of stone is effectively infinite.
(3) Sliding of one stone upon another cannot occur. It implies that wherever there is a weak plane, for example between voussoirs, the line of thrust should not depart too far from normality to that plane.

What shapes can a stable arch take? The arch produces a line of thrust which can be used to design and analyze vaulted structures in masonry.

> The structural action of complex vaults and domes can be understood by analyzing them as a series of two-dimensional arches contained within these shapes. When analyzed this way, the additional structural integrity resulting from the three-dimensional aspect of vaults and domes provides a further margin of safety. (J. A. Ochsendorf 2012)

Figure 3 reveals a simple thrust line analysis of the arch of Taq-I Kisra (540AD Ctesiphon, Iraq) that was used to estimate the magnitude of the internal compressive forces and to demonstrate its safety.

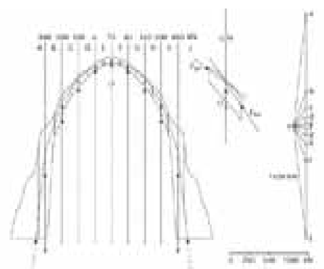

Figure 3: Thrust lines for Taq-I Kisra arch (image: Joseph Dahmen 2012).

1.3. Gaudi's caternary chain models

Spanish Catalan architect Antoni Gaudi(1852-1926) created hanging chain models as extraordinary formal and structural design models based on catenary curvature. The word catenary is derived from the Latin word catena, which means "chain". It is based on the delicate balance of opposing forces that gives rise to a certain amount of structural stability. A chain suspended from two points will always try to form a catenary. This happens because the chain is hung in a state known as "pure tension," so it will always adjust itself to find this balanced state. Only tension forces can exist in the hanging chain; inverting the shape into an arch reverses those into pure compression forces. All that compression force acts along the curve and never at right angles to it. The connecting faces of voussoirs in catenary arches are approximately, but never precisely, normal to the line of thrust. This makes the inverted catenary very stable, particularly for spanning a horizontal distance. For example Gaudi made a famous scale model of the loads and thrusts involved in the structure of his building, as shown in Figure 4. He hung cords in loops to correspond upside down to the placement and shapes of piers and arches of the vault. The catenary curves of the hanging cords were then distorted into funicular polygons by attaching weights shown in the drawing by Gaudi's assistant Juan Rubio. Using hanging chain models as a graphic analysis tool to investigate structural integrity of arched and vaulted spaces was an effective communicative tool in form finding.

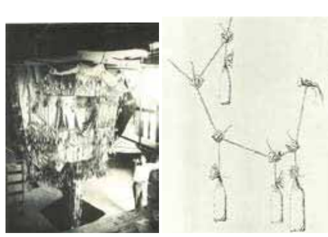

Figure 4: Left: Colonia Giuell church, exterior view of Gaudi's funicular model of the church as it hung in the workshed Source: (George R. Collins 1983)Right: Drawing of Funicular model by Gaudi's assistant in structural matters, Juan Rubio Source: (George R. Collins 1983)

2.0. DIGITAL STEREOTOMY

The renewed interest in the construction of freeform and vaulted spaces is due to emerging Computer Aided Design and manufacturing technologies, as well as the revival of interest in stone as a construction material. Although there are other renowned practitioners[7] which can be studied such as Guiseppe Fallacara's work on "digital stereotomy showing computational modeling techniques when applied on classical stereotomy studies" (Fallacara,2006); two current practitioners Professor Mark Burry and the Block Research group will be examined in this paper for their use of stereotomic principles.

2.1. Burry's reworking of Gaudi's models for the church of Sagrada familia

The Sagrada Familia in Barcelona, Spain, began as a gothic cathedral in 1882 and taken over by Antoni Gaudi in 1883 who worked on the project until his death in 1926. He was perhaps one of the most enigmatic architects in history. Born into a family of metal workers he grew up with a passionate curiosity about nature. Despite the fact that he was a Catalan at a time of great resistance to absorption into Spanish culture, he was regarded as a master craftsman, reminiscent of a medieval artisan. Gaudi's extraordinary working methods have kept the Sagrada Familia not only an architectural icon but also a continuous piece of scholarship[8]. At the time of the Spanish civil war during the 1930s, "the church was raided, the drawings burnt and plaster models smashed." (Burry 2006) Gaudi primarily relied on models rather than drawings to convey his design and construction intent; therefore the models were carefully restored in subsequent decades.


Figure 5: Left: Sagrada Familia Church in context from South East. Source: (Burry 2006) Right: North East View 2009 (Weir 2009)

While classical and medieval arches were based mainly on circular arcs, Gaudi based his geometry on hyperboloids, paraboloids, and hyperbolic paraboloids which closely resembles catenary curvature tracing the arch's line of thrust. The geometrical problems of working with hyperboloids, paraboloids, and hyperbolic paraboloids are not covered in Euclid's work. This means that most masons were probably unprepared to produce these shapes and that the graphical technique for the tait was insufficient for the task which explains why Gaudi worked with models. As the work undertaken today is the effective reverse engineering of Gaudi's models, a graphic technique was developed by his successors. Burry states,

> Such graphic techniques were the tools of the stereotomers of the past but the graphic technique for the intersection, for example between a sphere and cone, could not be adapted for the intersection between a circular and elliptical hyperboloid inclined differently with respect to the datum plane. (Burry 1993)

Burry further explained that the benefits of computational models are the ready application of such tools to diverse aspects of the construction process: automated manufacture, calculation of volume and mass of the irregular shaped pieces, and the calculation of the centre of gravity to facilitate the correctly orientated hoisting of the heavy masonry pieces. It is a combination of a surface and solid modelling facility which is now making a significant impact on the working method. As software solid modelling is conceptually distinct from any other drafting packages is more likely to be used by architects. It replicates the plaster model-makers methodology almost exactly. Computational modeling also enables an opportunity to work in a 'parametric' rather than 'explicit' environment. This is where the object can be described as a series of relationships to which dimensional values can be given later and changed at will. "This facility elevates the tool from working slavishly to a known intention to working more intuitively with the designer's considerations."(Burry 1993) The models produced on the screen from these software programs can then be used to drive saws or mills and fashion either a model, a prototype, mold or finally the finished article.

2.2. Block research group - Stone Vault Pavilion

Philippe Block with the Block Research group has been researching masonry modelling and fabrication techniques for many years. The Stone Vault Pavilion is part of a research project in the advancement of freeform masonry construction. This project was a collaboration with the Texas based company Escobedo Construction and Prof. John Ochsendorf, MIT. By combining the ETH / MIT team's expertise in three dimensional equilibrium of complex structures in unreinforced masonry and Escobedo Construction's expertise in fabrication and construction of stone buildings, a prototype project of a state of the art stone vaulted pavilion was developed.

> The vault was designed using Thrust Network Analysis, a new graphical form finding tool for exploring three dimensional compression only shapes. This vault prototype serves to learn what is needed to construct new stone forms out of complex stone cut pieces.(Block 2014)

This is shown in Figure 6 with the 3D model showing the load paths and capacity of structural loads, thus revealing the model used as a structural analysis tool. Figure 5 reveals the finished scaled model of the stone vault pavilion.[9] This project highlights the impact in which current and emerging trends in digital design and fabrication can produce complex geometries using natural materials.

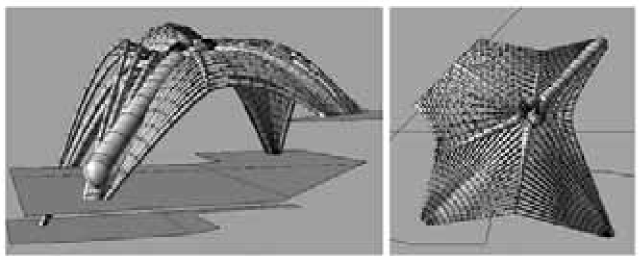

Figure 6: Stone Vault Pavilion 3D model. Source (Block 2009)

Figure 7: Stone Vault Pavilion. Source (Block 2009)

Block's research highlights the trend towards new types of stone vaults with new form finding methods that integrates formal geometry with structural analysis. The CNC process also accelerates the precise procedure of cutting complex forms.

> Except for the new challenges related to 'geometrical' complexity, the building industry is also facing environmental constraints. 'Freeform' is no longer accepted at any cost; efficiency of material use is a key consideration to embrace today's economical and ecological demands. (Block 2011)

Analysis tools such as Thrust Network Analysis was developed by Philippe Block as a methodology which finds possible funicular solutions under gravitational loading within a defined envelope. Adopting similar advantages of techniques such as graphic statics in a three dimensional environment, "using projective geometry, duality theory and linear optimization." (Block 2009) These computational techniques not only increase the complexity in graphic analysis but also increases the possibility of more complex stereotomic techniques adopted for more varied vault designs.

CONCLUSION

This paper presents a summary of stereotomy from its origins in the ancient writings of Euclid to the current work Burry and Block. We observed a progressive increase in the complexity of stereotomy from formal, geometric description to increasingly accurate methods of structural analysis. After the demise of stereotomy in the nineteenth century, the next significant step was in the stereotomic innovations of Gaudi and later by his successors, Burry and Block. This approach sought more efficient structural solutions requiring more complex geometries than those found in Euclid, aimed at more closely aligning form with static forces.

Most recently, stereotomy's applicability in new fabrication methods is fuelling a revival of stereotomy. Increasing economic and ecological constraints present a positive argument to reimagine the value of the ancient craft in stone masonry. As demonstrated in the studies of Gaudi, Burry and Block, developments in stereotomy afforded advances in architectural form. As new construction materials and methods for producing vaulted spaces emerge, stereotomy will be continuously redeployed to respond to these new conditions.

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ENDNOTES

[1] This text is partially referenced from:(Fallacara, Toward a Stereotomic Design: Experimental Constructions and Didactic Experiences 2009)
[2] Greek: στερεός (*stereós*) "solid" and τομή (*tomē*) "cut "
[3] Euclid: Greek (Εὐκλείδης) 300BC (of the Elements and Optics), not to Vitruvius (Fallacara, Digital Stereotomy and Topological Transformations: Reasoning about Shape Building 2006)
[4] Philibert de '-Orme is attributed with the origin of the stereotomic discipline. (Fallacara, Digital Stereotomy and Topological Transformations: Reasoning about Shape Building 2006)
[5] Robin Evans in his Essay 'Drawn Stone' describes stereotomy as a 16th century technique developed through taits.
[6] Defined terms of voussoir segmented arch: (Purchase 1904)

- The under-surface/soffit is called *intrudos*, the outer surface the *extrudos*
- The *voussoirs* are the separate stone blocks composing the arch, the *keystone* is the central voussoir
- The *springers* are the first stones on either side commencing the curve of the arch
- The *span* of the arch is the extreme width between the piers or opening; and springing line is that which connects the two points where the *intrudos* meets the inposts on either side.
- The *radius* is the distance between the centre and curve of the arch
- The highest point in the intrudos is called the *crown* and the height of this point above the springer is termed the *rise* of the arch.
- The *centre* is a point where the arch is struck, and lined drawn from the centre to the arch are radiating joints, and are also called *normals*. Variations in the normal and produce variations in the shape of the arch.
- All joints in arches should be radii of the circle/s or ellipses forming the curve of the arch, and will therefore converge to the centre or centres from which these are struck.

[7] Other digital stereotomy practitioners include Richard Etlin and Luc Tambeorero as well as the work of MIT researchers J.A. Ochsendorf
[8] Descharnes, R., & Prévost, C. *Gaudi - The Visionary.* New York: Viking Press, 1971;Collins, George R. *The designs and drawings of Antonio Gaudí* George R. Collins, Juan Bassegoda Nonell. Princeton, N.J. : Princeton University Press, c1983; Burry, Mark. *Antoni Gaudi, Expiatory Church of the Sagrada Familia Barcelona 1882-.* London: Phaidon Press, 1993.
[9] The entire process of the Stone vault pavilion can be found on this blog page: https://equilibriumstone.wordpress.com/page/3/