AUTOMATED GENERATION OF MINIMUM ENERGY COST BUILDING DESIGNS: A RESPONSE OF COMPUTER-AIDED DESIGN

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Rather than attempting a broad survey of the applications of computer-aided design techniques in responding to social change, this paper focuses on one specific case: the use of the computer in responding to the growing need for energy-conserving design.

1. The problem

Energy cost considerations are clearly becoming an increasingly major determinant of the organization and form of the built environment. However traditional methods of architectural design, based upon manipulation of drawings and scale models of buildings, do not directly allow a designer to assess the energy cost implications of design decisions or to systematically attempt to minimize energy cost. Reliance must generally be placed upon rough rules of thumb, such as the avoidance of western exposures for large glass areas to reduce cooling loads, the use of forms with a low surface/volume ratio to minimize winter heat losses, and so on. Here I consider an emergent alternative approach: the representation of built forms by means of appropriate mathematical models, and the manipulation of these models by computer to generate minimum energy cost designs. To keep the topic within reasonable bounds, the emphasis is on one particular aspect of minimum energy cost design which is of particular concern to architects and urban designers: the discovery of building geometries which minimize heat losses.

2. Models which consider only overall form of the building envelope

An initial approach to the problem can be made by utilizing a highly simplified model of a building and some fairly sweeping assumptions. Consider, for example, a building in the form of a rectangular parallelepiped. A very simplified description of this building can be produced by giving values for the variables x, y, z which represent length, width, and height, respectively. Assuming that total building volume remains constant, that the average thermal transmittances of the walls, floor and roof are equal to some fixed value U, that there is negligible heat loss through the floor, and that there are no losses by air exchange, the total heat loss occuring in a specified period of time is given by the following formula

$$\text{Heat loss} = (2(x + y)z + xy)U$$

By performing some simple algebraic manipulations it can be shown (March 1972) that, in this case, heat loss is minimized when

$$x = y = 2z$$

In other words, the building form which minimizes heat loss is a half cube.

Atkinson and Phillips (1964) utilized a slightly more elaborate form of this type of model to conduct a rather extensive theoretical investigation of the effects of hospital building geometry on heat loss. They assumed a rectangular plan, and calculated heat losses according to the following formula

$$\text{Heat loss} = 2h(a + b)\sqrt{\frac{An}{ab}}\, U_1 + \frac{A}{n}\, U_2$$

Where A = total floor area

U_1 = average thermal transmittance for walls

U_2 = composite average thermal transmittance for roof and floor

h = floor to floor height

a = length of one side

b = length of other side

n = number of floors

Results were presented in the form of graphs illustrating the interrelations between heat loss, floor plan proportions, and numbers of floors (figure 1). In the example shown it can be seen that under the given assumptions, for a total floor area of 200,000 sq. ft., the optimum configuration is a 223 ft. square plan of 4 floors.

A further refinement can be introduced into the model by allowing for different average thermal transmittances on different walls. This condition could arise due to differing constructions, differing proportions of fenestration, or the effects of differing exposures. March (1972) investigated this situation and showed that in general a simple rectangular building of given volume (or given floor area and constant floor to ceiling height) loses the least amount of heat if the dimension of each edge is proportional to the mean thermal transmittance value of the faces defined by the other two edges.

3. Introduction of internal planning constraints

Of course these simplified models which I have discussed so far are of limited practical use, since they ignore internal planning constraints. There is no guarantee that it is feasible to plan the building in such a way that it fits within the "optimum" envelope. In this section I will introduce an approach to modeling the problem which allows internal planning constraints to be realistically incorporated.

The basis of the modeling method is the concept of <u>minimum representation</u> of geometric forms. The notion of a minimum representation is perhaps best explained by means of an example. Consider the series of rectilinear forms shown in figure 2(a). Although they are different in size and proportion, they clearly have certain shape properties in common; we can describe them all as "U-shaped." The reason for this apparent commonality becomes obvious if we surround each form by a rectangular frame, and superimpose a <u>minimum rectangular grating</u> (Newman 1964, March 1972, Mitchell 1974) as shown in figure 2(b). This minimum rectangular grating is formed by drawing horizontal and vertical lines across the frame in such a way that all vertices of the original form lie on an intersection, and each grating line intersects at least one vertex. It can be seen that the minimum grating for each form, in the example shown, consists of six rectangular cells. If we now adjust the dimensions of the minimum gratings so that each cell in the grating becomes square, as shown in figure 2(c), we find that each of the differently-dimensioned shapes reduces to the same U-shaped figure. Such a representation of a rectinlinear form within a uniform square grating may be termed its minimum representation.

Rectilinear building floor plans may be reduced to their minimum representations as shown in figure 3(a). If each cell constituting a particular space is labeled with an integer, as shown, then the minimum representation may be symbolically described by means of a rectangular integer array (figure 3(b)). This integer array form of the minimum representation is particularly convenient for storage and manipulation in a computer.

The dimensional properties of a given floor plan may be described by means of x and y dimensioning vectors applied to its minimum representation, as shown in figure 4. Applying different dimensioning vectors to a minimum representation will produce a family of floor plans in which the spaces have different dimensions and areas, but the adjacency relations between the spaces remain constant.

This suggests that the problem of finding a built form which minimizes heat losses, while complying with internal planning constraints, can be approached in two steps. Firstly, an appropriate plan arrangement, which allows required functional relationships between spaces to be satisfied, should be selected. Secondly, dimensioning vectors which result in minimum heat loss, subject to meeting constraints on areas, dimensions, and proportions of spaces should be found. The next part of the paper describes a systematic method for finding geometric arrangements which comply with specified requirements for relations between spaces, and this is followed by a discussion of methods for discovering dimensioning vectors which result in minimum heat loss designs.

4. Generating an appropriate plan arrangement

For the purposes of this discussion I will retain the assumption that the envelope of the building is to be rectangular. I will further assume for the sake of simplicity, that the building is single storied and that all the interior spaces are themselves rectangular. In other words, the plan is a dissection of a rectangle into smaller rectangles.

Now the number of distinct such dissections of a rectangle into a given number of smaller rectangles is finite, and it is quite feasible to exhaustively enumerate all plan possibilities if the number of interior spaces is relatively small. The known numbers of different dissections into n rectangles are as shown on next page.

Figure 5 illustrates minimum representations of all the possible dissections into four or fewer rectangles. Steadman (1973) has published an exhaustive catalogue of all possible dissections into six or fewer rectangles.

Computer programs have been developed to exhaustively enumerate rectangular dissections, and their output can be stored on magnetic tape (in the integer array form discussed previously). The task of finding an appropriate geometric arrangement for a small building, then, becomes a task of searching through such a tape to find those configurations which can potentially satisfy required relationships between rooms (e.g., that the dining area of a house should be adjacent to the kitchen). Since for a small building the number of rooms will be small, the number of different dissections which must be considered in such a search is at most a few hundred thousand.

It is possible to program a computer to perform this search very rapidly, and a program for this purpose has been experimentally implemented at UCLA. Input to the program consists of a matrix specifying required adjacencies between spaces plus the tape of rectangular dissections. For each dissection, the program must determine whether the various different activities to be accommodated can be assigned to cells of the dissection in such a way that the specified adjacency relationships between activities are satisfied. Where the number of cells and activities is n. the number of potential assignments of activities to cells which must be considered for each dissection is n!. The value of n! grows quite rapidly, as shown in table 2, as n increases. Thus the task of checking all potential assignments could involve considerable amounts of computation, so the program relies on various tricks to rapidly eliminate large numbers of potential assignments from consideration.

For a given dissection and set of activities to be accommodated, the program discovers which of the following conditions exists:

(a) None of the potential assignments results in satisfaction of the specified adjacency relationships between the activities.

(b) One of more of the potential assignments results in satisfaction of the specified adjacency relationships.

In the former case, the program prints out a message to that effect, and in the latter case all the satisfactory assignments are printed out.

n	number of dissections	
1	1	
2	1	
3	2	
4	7	
5	22	
6	117	
7	700	(estimated)
8	10,000	(estimated)
9	250,000	(estimated)

Table 1: Numbers of different dissections of a rectangle into n rectangles

n	n!
1	1
2	2
3	6
4	24
5	120
6	720
7	5,040
8	40,320
9	362,880

Table 2: Values for n factorial

5. Finding dimensioning vectors which minimize heat loss

Once a satisfactory geometric arrangement, in minimum representation, has been selected, the next task is to discover dimensioning vectors which minimize heat loss. To illustrate how the problem of selecting such dimensioning vectors may be formulated, let us imagine that we have selected the geometric arrangement shown in figure 6(a) and assigned activities to spaces as illustrated. Functional requirements will give a minimum and maximum allowable area for each space, and minimum and maximum allowable dimensions. We may also wish to impose proportion constraints on spaces. Site dimensions may impose constraints on overall length and width. A set of plausible constraints, together with their expression in terms of the dimensioning vectors, are illustrated (figure 6(b) and 6(c)). The objective to be minimized, i.e., heat loss, is expressed in terms of the dimensioning vectors in figure 6(d).

As formulated, the problem now is to minimize a non-linear objective function subject to linear and non-linear constraints. The non-linear character arises out of the existence of interactive terms, representing areas and/or proportion ratios, in the objective and constraints. We may choose either to allow dimensions to vary continuously, or to require that they be integer multiples of some defined dimensional module. In the former case we have a standard non-linear programming problem, and in the latter an integer non-linear programming problem.

A number of different solution techniques are available for this type of optimization problem. Those to be considered here are:

(a) Exhaustive enumeration.
(b) Elimination of non-linear terms and use of standard linear programming techniques.
(c) Use of non-linear programming techniques.

Choice of a particular technique depends upon the specific character of the problem at hand.

Exhaustive enumeration, the most straightforward approach, may be suitable for relatively small problems for which an integer solution is desired. A standard dimensional increment, e.g., 1 foot, is set, then all the different combinations of values for elements of the dimensioning vectors are enumerated, and the combination resulting in minimum heat loss is selected. An advantage of exhaustive enumeration is that it generates <u>all</u> feasible solutions, not just the minimum heat loss solution. Obviously though, exhaustive enumeration becomes impractical for even moderately large problems.

If certain restrictions can be imposed on the problem, then it becomes possible to treat it as a linear programming problem, for which very efficient solution procedures exist. If for example the values for one of the dimensioning vectors can somehow be fixed, then the objective and constraints all become linear and the powerful simplex method of linear programming can be employed to find heat-loss-minimizing values for the remaining dimensioning vector very rapidly.

Non-linear programming problems are in general much more difficult to solve than linear programming problems, and as we might expect then, it is much more difficult to determine heat-loss-minimizing dimensions for a plan if both the x- and y-dimensioning vectors are free to vary. However some very powerful algorithms for solution of non-linear programming problems have been developed in recent years (Luenberger 1965). Figure 7 illustrates heat loss minimizing dimensioning vectors for the example problem, which were generated by one such algorithm (Clasen et al 1974). It solves the non-linear problem via a sequence of local linear programming problems, and is described as a local, gradient, stepwise correction descent algorithm. The FORTRAN-implemented version employed on the example problem generated the optimum solution in approximately 2 seconds of computation on a 360/91 computer at UCLA.

The heat loss minimizing design method which has been described may now be summarized as follows:

(a) An exhaustive "catalogue" of different geometric alternatives is generated and stored, utilizing the minimum representation, on magnetic tape.

(b) The tape is searched by computer to discover particular assignments of activities to spaces in particular geometric arrangements which satisfy specified adjacency requirements.

(c) Exhaustive enumeration, linear programming, or non-linear programming techniques are then utilized to discover heat loss minimizing configurations.

Although the discussion has been restricted to rectangular buildings composed of rectangular internal spaces, the same general approach can be extended to other families of forms (see for example Mitchell 1974).

6. Introduction of more realistic descriptions of buildings and their thermal behavior

In the models which have been discussed so far, I have utilized a very highly simplified view of the thermal behavior of buildings. Specifically, I have assumed that the fabric of the building is fairly homogeneous, that heat loss does not vary with orientation or height of a surface, that there is no heat loss through air exchange, and that transfer of heat through the external walls takes place under steady state conditions. These simplifying assumptions are quite reasonable within limits. For example the assumption of steady state heat transfer is appropriate where the heat storage capacity of the external walls is small in relation to the total heat flow (i.e., lightweight construction is used), and where the temperature difference across the external walls is large in relation to the temperature fluctuations on the inside and outside (Maver 1971). However many effective energy conserving design concepts depend upon a more subtle understanding of the dynamic interplay of heat sources and sinks within the without a complex building fabric (see Steadman 1974 for a convenient summary). Lately there has been considerable interest, for example, in buildings of high thermal inertia which (under suitable conditions) store incoming heat in the structure during the hot daytime hours and release it during the cool evening hours. Thus there is a need to develop more detailed and realistic models of buildings and their thermal behavior if sophisticated energy-conserving design techniques are to be allowed for.

The development of effective techniques for storage and manipulation of detailed building descriptions in a computer is a very general issue in computer aided architectural design, and there is now an extensive technical literature on the subject (see Mitchell 1974 for a summary). Most schemes which have been proposed for this purpose draw a distinction between the <u>project file</u> and the <u>component file</u>. Project files usually utilize some kind of fairly sophisticated list structure for describing building geometry, and incorporate pointers to specific items in the component file. Component files contain detailed data on the properties of specific building components and materials.

There has been substantial progress, in recent years, on development of effective computer-based thermal evaluation procedures which utilize detailed and realistic building descriptions, and evaluate the thermal behavior of the building over time rather than making a calculation for a single point in time. For example a thermal evaluation program developed for the United States Postal Service takes into account the effects of size, shape, orientation, wall and roof constructions, and window designs, and utilizes hourly Weather Bureau data (on magnetic tape) for the specific building location (Lokmanhekim 1971).

The problems with these more sophisticated models are firstly that the computations involved become very lengthy and complex, and secondly that it becomes difficult if not impossible to develop effective optimization procedures (but see Gupta (1971) for one implemented example). In the absence of an optimization procedure, trial and error must usually be employed; the designer generates a proposal, an evaluation is run, modifications are made on the basis of the results, a new evaluation is made, and so on. For this type of process to be effective, it must be relatively quick and easy for the designer to make modifications and receive feedback. This facility can be most effectively provided by an interactive computer graphics system equipped with convenient facilities for generating and modifying computer-stored building descriptions and for performing thermal evaluations on those descriptions. At least one such system has been implemented on an experimental basis (Hamer and Robinson 1973).

7. Conclusions

The highly simplified models which were initially discussed could readily be manipulated by hand, and "optimum" designs could be generated by use of simple techniques of algebra and calculus. Introduction of consideration of internal planning constraints led to a somewhat more complex model, and optimization required use of sophisticated computer techniques. Models of sufficient detail and richness to allow subtle energy conserving design techniques to be explored were seen to be exceedingly complex, and impossible to manipulate without the aid of a computer. Furthermore, optimization tends to become infeasible where rich and complex models are employed. However interactive computer graphics techniques can then be used to greatly facilitate the process of design by trial and error.

Future research into computer aided architectural design techniques can facilitate energy conserving design in the following ways:

1. By development of more effective data structures for the computer representation of building designs, and of computer graphics techniques for manipulating these representations.
2. By development of more complete and satisfactory models of the thermal behavior of buildings.
3. By development of more powerful design optimization procedures.

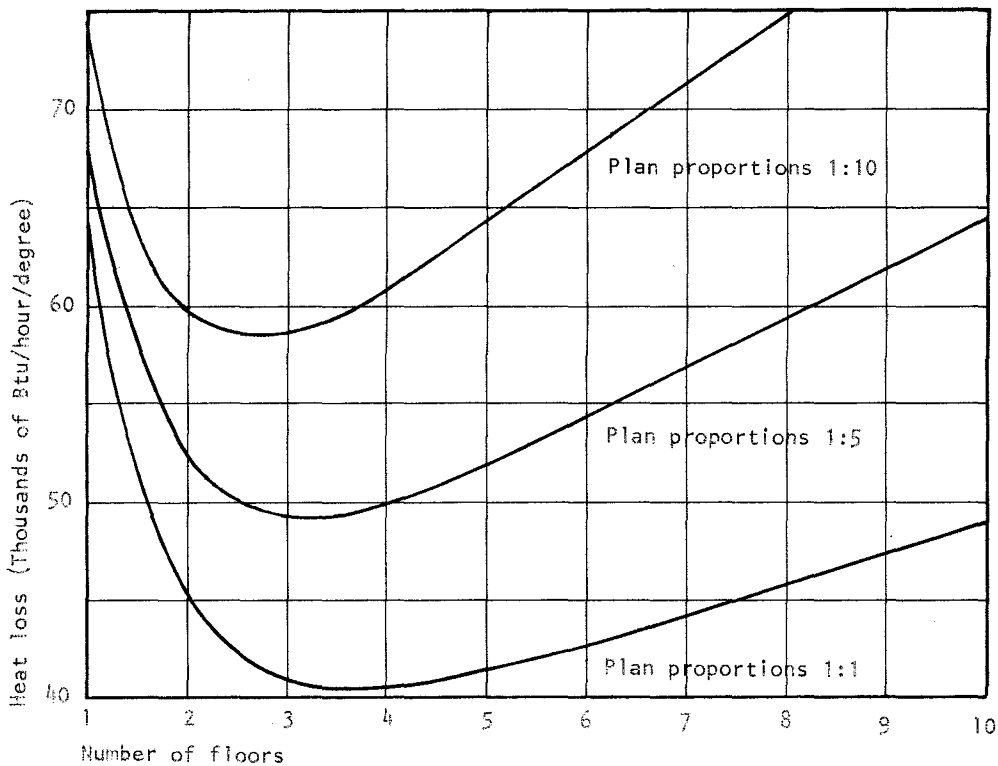
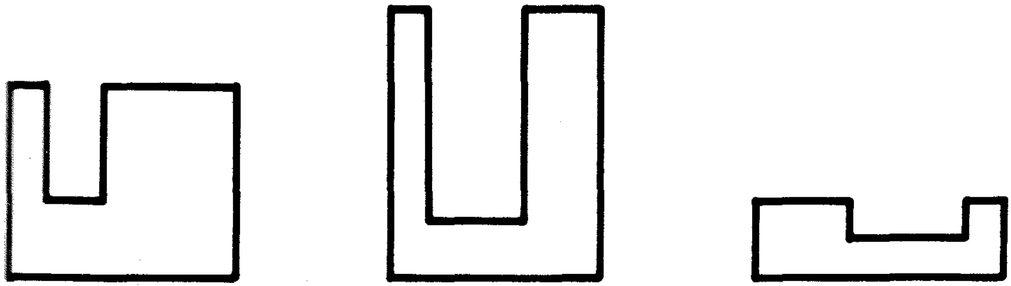
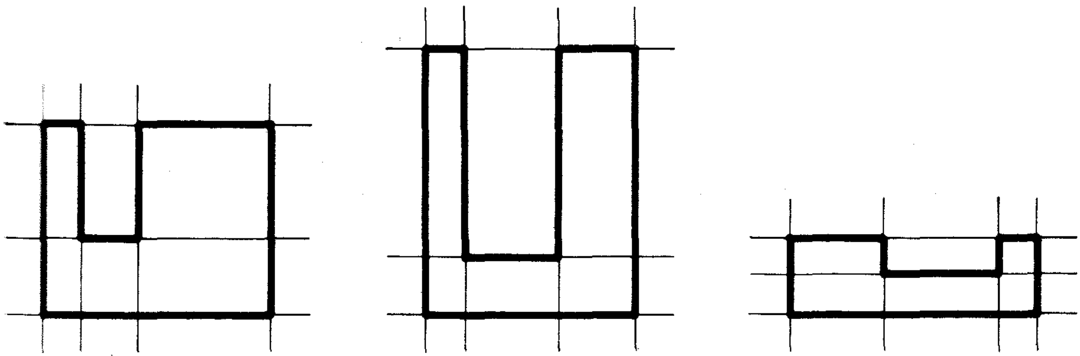


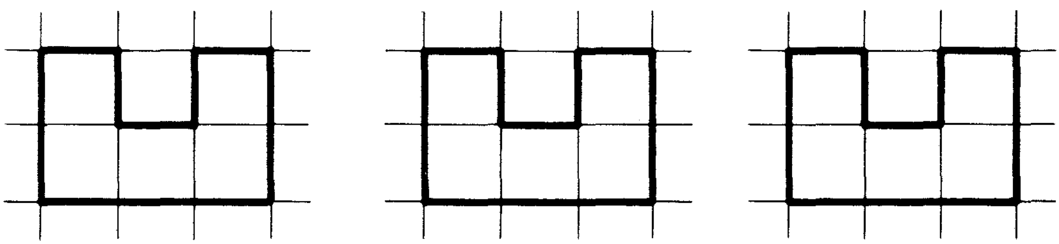
Figure 1: Graph showing comparative heat losses for rectangular buildings of equal floor area (20,000 sq. ft.) but differing proportions and numbers of floors (after Atkinson and Phillips)



2(a) Similar rectilinear forms

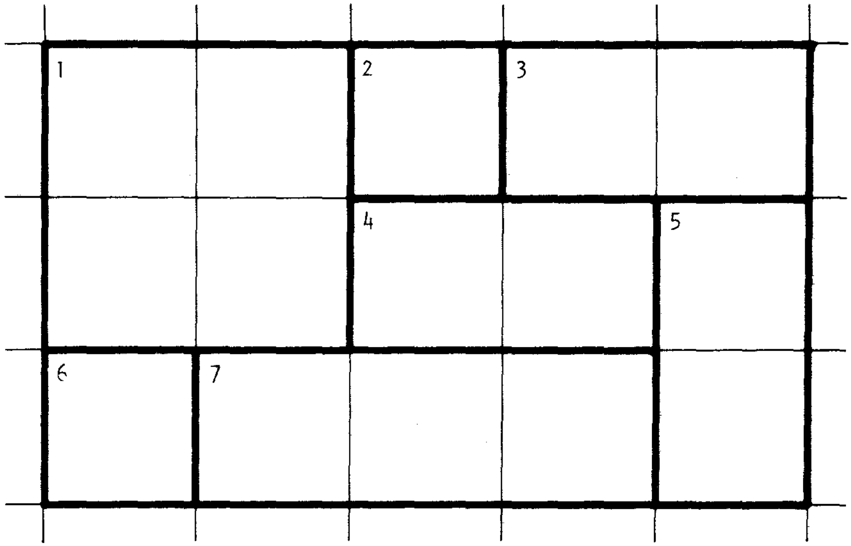


2(b) Imposition of minimum rectangular gratings



2(c) Minimum representations produced by reducing all cells in the gratings to squares

Figure 2: Minimum rectangular gratings and minimum representation of forms



3(a) Minimum representation of a rectilinear floor plan

1	1	2	3	3
1	1	4	4	5
6	7	7	7	5

3(b) Integer array form of minimum representation of the plan shown

Figure 3: Minimum representation of a floor plan

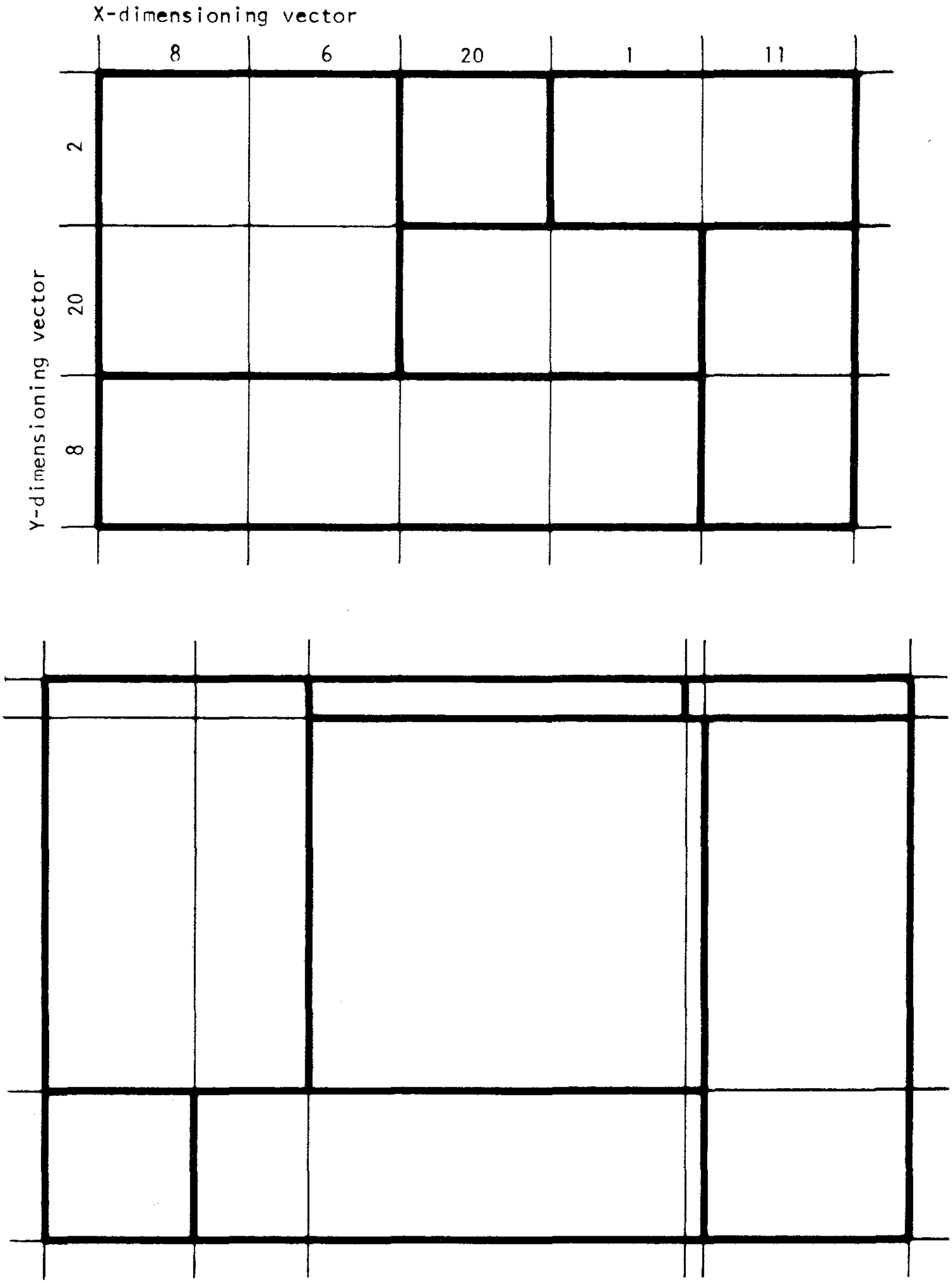


Figure 4: Dimensioning vectors applied to a minimum representation and the corresponding interpretation as a dimensioned layout.

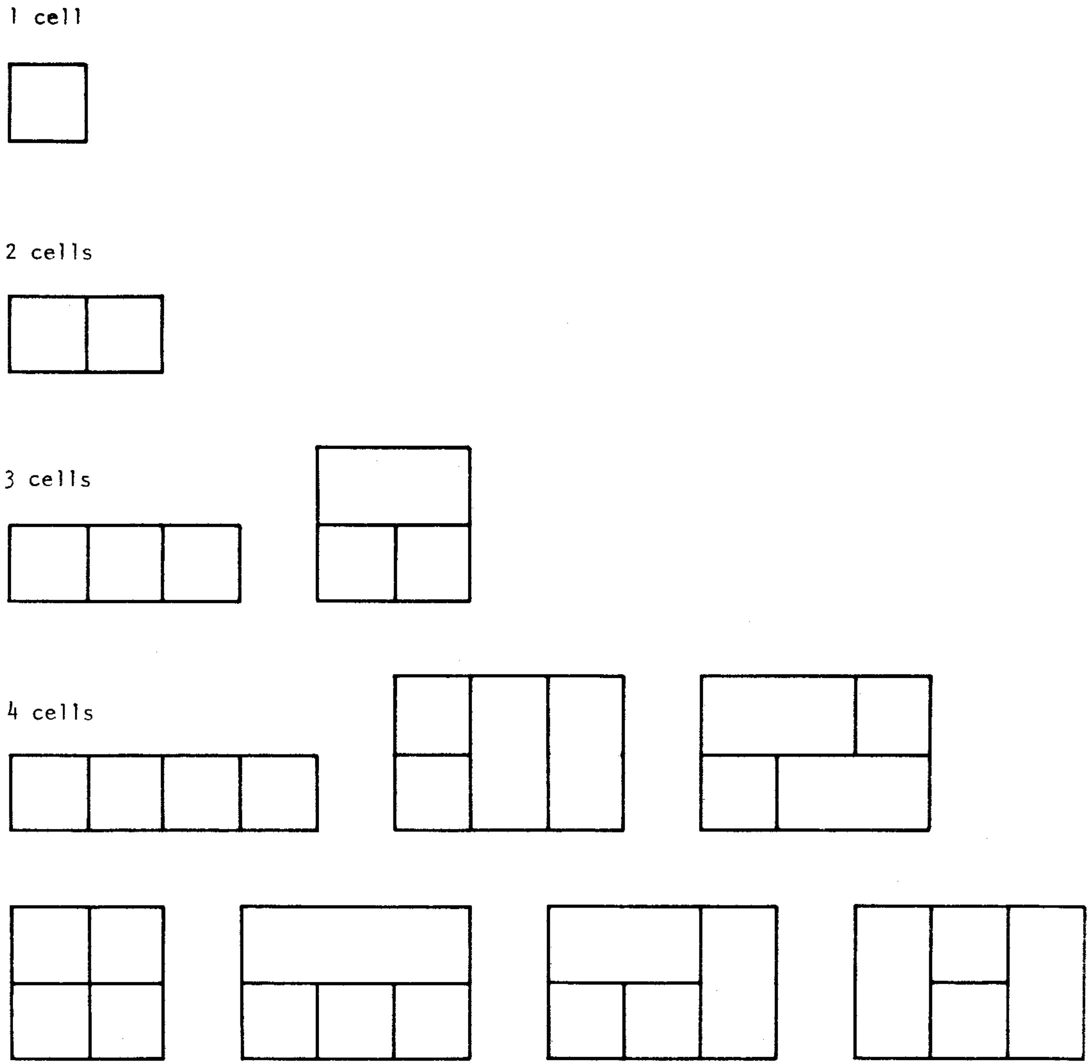
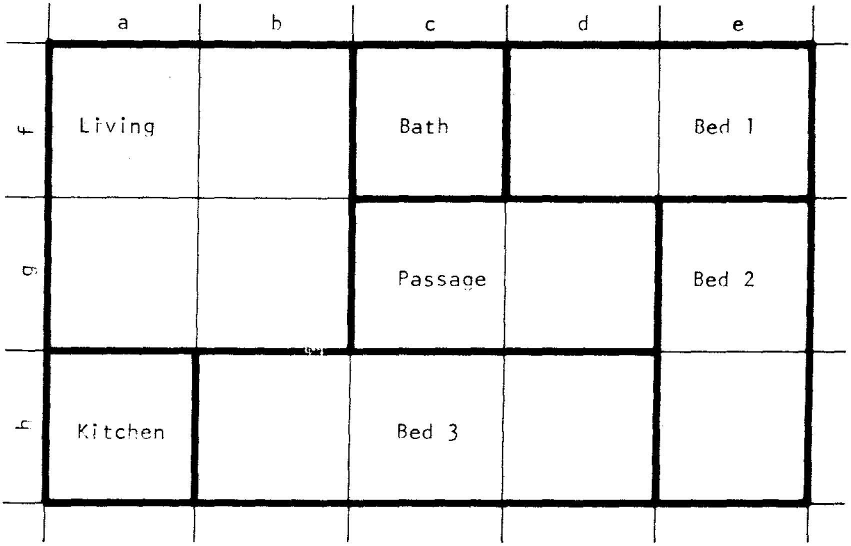


Figure 5: All distinct dissections of a rectangle into 4 or fewer rectangles (shown in minimum representation)



6(a) Minimum representation of an example arrangement

Room	Min. dimension	Max. dimension	Min. area	Max. area
Living	8	20	150	300
Bath	5.5	12	80	120
Bed 1	9	20	100	180
Bed 2	8	18	100	180
Bed 3	10	17	100	180
Passage	3.5	100	0	72
Kitchen	6	18	50	120

6(b) Dimensional constraints

Figure 6: Example dimensioning problem
(continued over page)

Living	$8 < (a + b) < 20$
	$8 < (f + g) < 20$
	$150 < (a + b)(f + g) < 300$
Bath	$5.5 < c < 12$
	$5.5 < f < 12$
	$80 < cf < 120$
Bed 1	$9 < (d + e) < 20$
	$9 < f < 20$
	$100 < (d + e)f < 180$
Bed 2	$8 < e < 18$
	$8 < (g + h) < 18$
	$100 < e(g + h) < 180$
Bed 3	$10 < (b + c + d) < 17$
	$10 < h < 17$
	$100 < (b + c + d)h < 180$
Passage	$3.5 < (c + d) < 100$
	$3.5 < g < 100$
	$0 < (c + d)g < 72$
Kitchen	$6 < a < 18$
	$6 < h < 18$
	$50 < ah < 120$

6(c) Constraints expressed in terms of dimensioning vectors

Heat loss $= 12(a+b+c+d+e) + 12(f+g+h) + 0.25(a+b+c+d+e)(f+g+h)$

Assuming U-value for external walls = 0.6

Height of external walls = 10 feet

Composite U-value for roof and floor = 0.25

6(d) Objective expressed in terms of dimensioning vectors

Figure 6: Example dimensioning problem (continued)

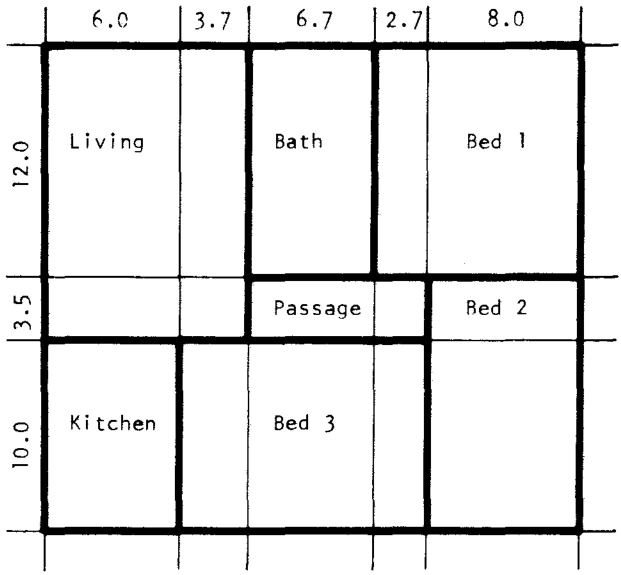


Figure 7: Heat-loss-minimizing plan found by non-linear programming

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