Luminous-Ceiling Lighting

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The paper deals with the design of complete luminous ceilings lighted from above. Such lighting compares favorably in coefficient of utilization with other methods and has definite advantages over the louverall ceiling. For brightness uniformity, the normal spacing between rows of fluorescent lamps should not exceed twice the distance between lamps and translucent plates. Also, the outside rows of lamps should be spaced three quarters of normal spacing, and the distance from them to the sides of the lightbox should be one-quarter of normal spacing rather than the one-half generally used in the past.

ILLUMINATING engineers are becoming increasingly conscious of the necessity for planned lighting that will provide ideal visual conditions. As a step in this direction the I.E.S. Committee on Standards of Quantity and Quality of Interior Illumination[1] has recommended a 3:1 brightness (helios) ratio* for all rooms in which the visual task is severe. The recently developed interflection method[2] allows the brightness (helios) ratios to be predetermined for any room and gives a foundation for the design of really satisfactory lighting installations. A previous paper[3] presented a series of charts showing under what conditions the 3:1 ratio can be satisfied. Exposed fluorescent lamps give a brightness (helios) ratio of 50:1 or more** and therefore should not be used in offices, drafting rooms, school rooms, or other places where close visual work is encountered. The same brightness (helios) ratio is obtained with louvered fixtures and with the louverall ceiling, which are therefore potential sources of reflected glare.

Ideal lighting is thus narrowed down to two possibilities:

1. Ceiling lighting, Type IIb, where the entire ceiling acts as a secondary source.[4] The only satisfactory hanging luminaire for this type of lighting is of the luminous indirect type, the bottom of the luminaire having approximately the same brightness (helios) as the ceiling.
2. Luminous-ceiling lighting, Type IIa, utilizing a hung ceiling of translucent plastic or other diffusing material lighted from above.

Even with IIa and IIb lighting, the 3:1 criterion is satisfied only if high enough reflectances are used. With a floor reflectance of 0.10, the interflection method (Fig. 1a) shows that the 3:1 ratio is never satisfied, though a ratio of less than 10:1 is usually obtained. By raising the floor reflectance to 0.30 and employing a wall reflectance of at least 0.50, however, one obtains excellent visual conditions (Fig. 1b), at the same time utilizing the light most economically.[5]

Type IIb lighting is employed in many installations today, though generally with such low reflectances that the potential excellence of the lighting system is obscured. Luminous-ceiling lighting (IIa) offers interesting possibilities and merits further study. The purpose of the present paper is to begin such a study by investigating the design of luminous ceilings.

Fig. 2 shows one arrangement for ceiling lighting. Continuous rows of fluorescent lamps are attached to the ceiling. No reflectors are employed and the ceiling is painted white. At distance l below the lamps is a grid of metal bars. Since the grid supports only a small weight and is hung from the ceiling at frequent intervals, it can be of very light construction. Resting on the grid are flat panels of diffusing plastic which can be slid to the side or removed for relamping and cleaning. A less-expensive construction employs, in place of each plastic panel, a simple frame covered with thin diffusing plastic, tracing paper, or glass cloth. Convenient dimensions are $s = 0.6$m (24 in.), $l =$

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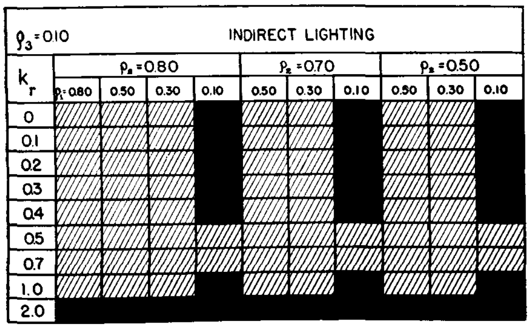
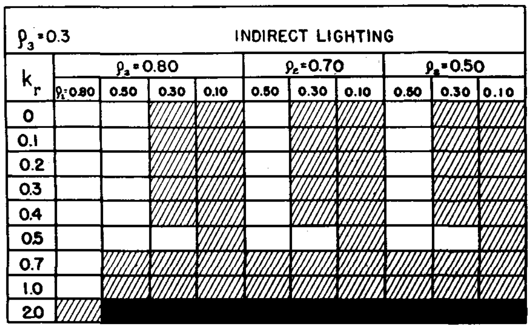
*The 3:1 ratio does not mean that the maximum brightness (helios) in the room is limited to 3 times the minimum. It does mean that the adaptation brightness (helios), for any orientation of the eyes, must not exceed 3 times the adaptation brightness H for the work; and the adaptation brightness of the work must not exceed 3 times the minimum adaptation brightness for any orientation of the eyes. Since the adaptation brightness associated with an extended surface is essentially equal to the brightness of the surface, one may express the criterion as

$$H_{max}/H_A < 3,$$

$$H_A/H_{min} < 3.$$

**The 40-watt T-12, 3500K white fluorescent lamp has a brightness (helios) of 18,000 blondel and the 96-in., T-8, 4500K white lamp has approximately the same brightness (helios). But this value is so much greater than any other brightness (helios) in the room that poor visual conditions are sure to result if louvered fixtures are used. A high value of illumination (incident pharosage) is 500 lumen m^{-2} (47 lumen ft^{-2}); and with ordinary printed matter having an average reflectance of 0.65, the eyes will adapt to the brightness (helios) $H_A = 500 \times 0.65 = 325$ blondel. Thus the brightness (helios) ratio is $H_{max}/H_A = 18000/325 = 55$.





Figures 1a and 1b. Charts for ceiling lighting, showing in what cases the 3:1 brightness (helios) ratio can be obtained. Blank areas indicate that adaptation brightness (helios) ratios for ceiling, walls, and floor are less than 3:1. Cross-hatched areas show that the highest ratio is more than 3 but less than 10. Black areas indicate a maximum brightness (helios) ratio of more than 10. Wall reflectance is denoted by ρ_1, ceiling reflectance by ρ_2, and floor reflectance by ρ_3. Domance k_r specifies the shape of the room and is evaluated by means of the equation,

$$k_r = S_1/4S$$

where S_1 is the total wall area and S is the floor area.

0.3m (12 in.). The individual panels can be approximately 0.6 x 0.6m (24 x 24 in.). The appearance of a room lighted by this type of construction is indicated in Fig. 3.

Design

For any type of lighting, the average illumination (pharosage) is expressed by the well-known equation,

$$D_{AV} = k_u \frac{F_L}{S} = fg \frac{F_L}{S}$$

where D_{AV} = average incident illumination (pharosage) (lumen m^{-2}) on principal surface,
k_u = coefficient of utilization,
F_L = total lumen (pharos) from lamps,
S = floor area (m^2),
f = interflectance of room,
g = efficiency (logance) of luminaire.

For simplicity, we shall consider only rooms that are so large ($k_r \rightarrow 0$) that the effect of the walls may be neglected. Further analysis[6] shows that the *general conclusions* obtained in this limiting case apply to all rooms though numerical values may vary somewhat. For an infinite room lighted by a luminous ceiling,

$$f = \frac{1}{1 - \rho_2' \rho_3}. \qquad (2)$$

Thus from Equation (1),

$$D_{AV} = \frac{g}{1 - \rho_2' \rho_3} \frac{F_L}{S}. \qquad (3)$$

Also, the brightness (helios) of ceiling and floor are

$$H_2 = \frac{g F_L}{(1 - \rho_2' \rho_3) S} \qquad (4)$$

$$H_3 = \frac{\rho_3 g F_L}{(1 - \rho_2' \rho_3) S} \qquad (5)$$

where ρ_2' = apparent reflectance of luminous ceiling, as viewed from below,
ρ_3 = reflectance of floor,
H_2 = average brightness (blondel) of luminous ceiling, as viewed from below,
H_3 = average brightness (blondel) of floor.

Equations (2) to (5) were obtained by mentally following the light as it bounces about between floor and ceiling.[7] The interflections in the luminaire (upper portion of the room, including translucent plates, Fig 4) were not considered specifically. We now bring the luminaire into the picture and find how its characteristics change as the reflectance and transmittance are altered. The equation for D_{AV}, derived in Appendix A of the present paper, is

$$D_{AV} = \frac{\tau(1 + \rho_5)}{2(1 - \rho_2 \rho_5)} \frac{1}{1 - \rho_3 \left[\rho_2 + \frac{\rho_5 \tau^2}{1 - \rho_2 \rho_5} \right]} \frac{F_L}{S}. \qquad (6)$$

Here τ = transmittance of translucent plates,
ρ_2 = reflectance of translucent plates,
ρ_5 = reflectance of ceiling (See Fig. 4).

Comparison of equations (3) and (6) shows that the efficiency (logance) of the luminaire is

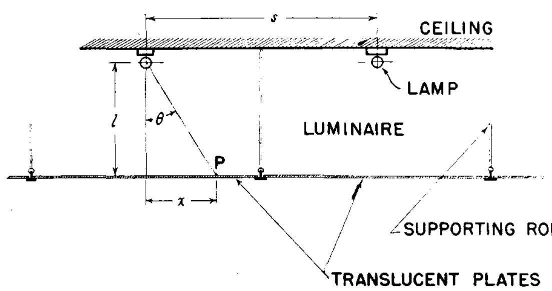


Figure 2. Cross-section of a luminous ceiling.

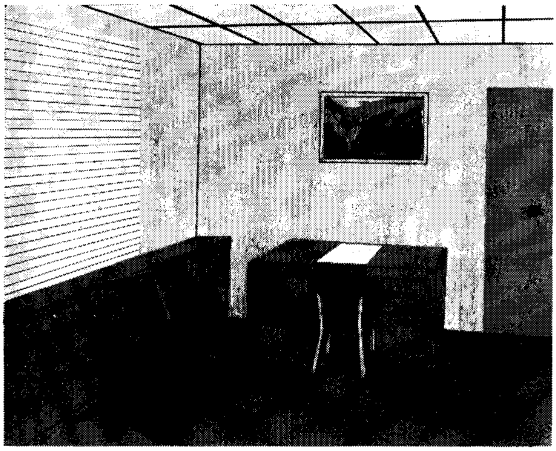


Figure 3. Appearance of a room lighted by means of a luminous ceiling (Type IIa lighting).

$$g = \frac{\tau(1+\rho_5)}{2(1-\rho_2\rho_5)} \qquad (7)$$

and the apparent reflectance of the luminous ceiling, as seen from below, is

$$\rho_2' = \rho_2 + \frac{\rho_5\tau^2}{1-\rho_2\rho_5}. \qquad (8)$$

Equation (7) shows that the efficiency (logance) increases as the transmittance of the translucent plates is increased and depends also on the reflectances of the luminaire surfaces. Equation (8) indicates that the apparent reflectance of the ceiling, as seen from below, is equal to the true reflectance ρ_2 of the translucent plates plus a term that includes luminaire reflectances and transmittance.

Equations (7) and (8), in conjunction with equations (2) to (5), give complete information for the design of luminous ceilings. If $k_r \to 0$, the average illumination (pharosage) is given by equation (3) and the values of brightness (helios) by equations (4) and (5), where g and ρ_2' are obtained

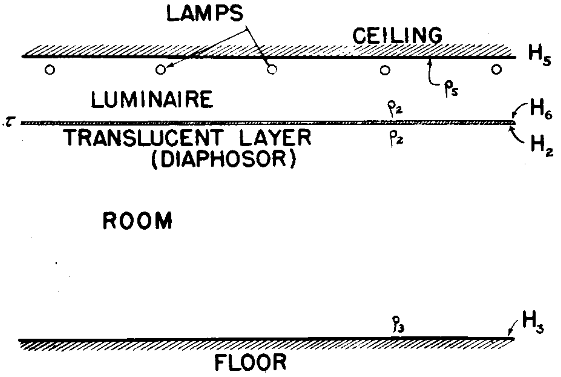


Figure 4. Cross-section of a room lighted by a luminous ceiling.

from equations (7) and (8). If $k_r \to 0$, equations (7) and (8) still give approximate values of efficiency (logance) and apparent reflectance which can be used in the interflection tables.[8]

Fig. 5 shows how the apparent reflectance of the translucent plates varies as the transmittance is varied. The assumption is made that

$$\rho_2 \doteq 0.90 - \tau,$$

which agrees with experimental data on the best translucent materials.[9] The curves were calculated by use of equation (8). If the ceiling were black ($\rho_5 = 0$), the apparent reflectance ρ_2' would be the same as the reflectance ρ_2 of the translucent plate. When the ceiling is painted white, however, the apparent reflectance rises. For $\tau = 0.50$, the apparent reflectance is not equal to $\rho_2 = 0.40$ but

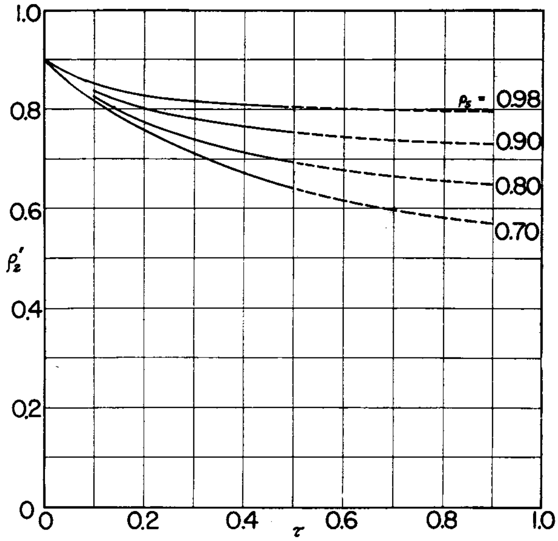


Figure 5. Apparent reflectance of the luminous ceiling, as a function of the transmittance τ of the translucent plates and of the reflectance ρ_5 of the surface above the lamps.

varies from 0.65 to 0.80 depending on the reflectance ρ_5.

The efficiency (logance) of the luminaire is plotted in Fig. 6. The curves show again that the reflectance ρ_5 should be as high as possible. They also indicate that the transmittance of the translucent plates should be as high as is consistent with good diffusion.

Fig. 7 gives the coefficient of utilization for a room with ceiling lighting and having a floor reflectance of 0.30 and a k_r of zero. Note that a coefficient of utilization of 0.80 is easily obtainable,*

*Note that these values of k_r are for an infinite room. For other rooms, the coefficient of utilization is lower and can be obtained from the interflection tables, Reference 3.

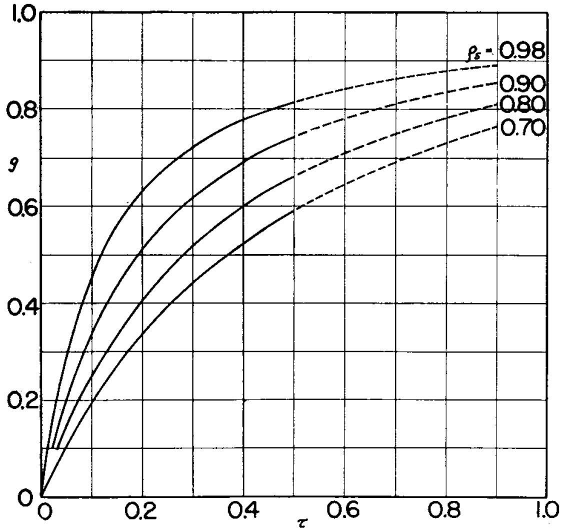


Figure 6. "Efficiency" (logance) of a luminous ceiling.

and values of more than 1.00 are possible by raising the reflectances ρ_3 and ρ_5. Thus luminous-ceiling lighting compares favorably with any other form of lighting in effectiveness. One might think that the louverall ceiling, being a "direct" lighting system, would have a higher coefficient of utilization than the luminous ceiling, but such is not the case. With a 45° cut-off, each cell of the louverall ceiling acts like a miniature cubical room interposed between the lamps and the large room. And lighting in a cubical room is always inefficient[8]. This fact accounts for the low values of k_u (0.25 to

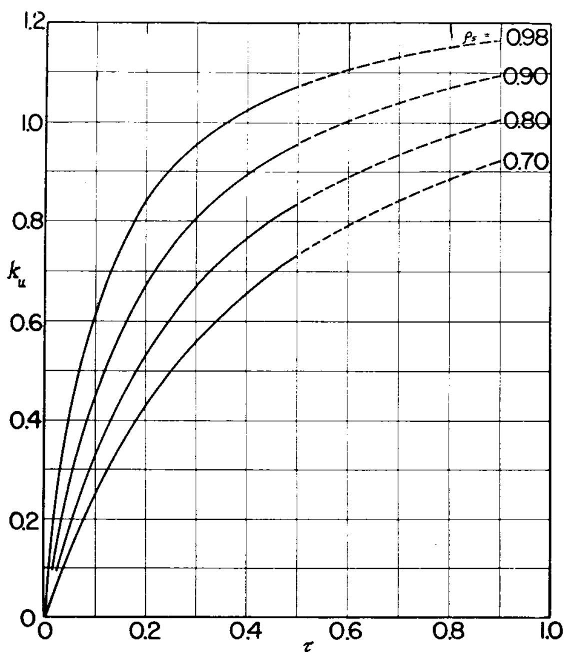


Figure 7. Coefficient of utilization of an infinite room with IIa lighting, $k_r = 0$, $\rho_3 = 0.30$.

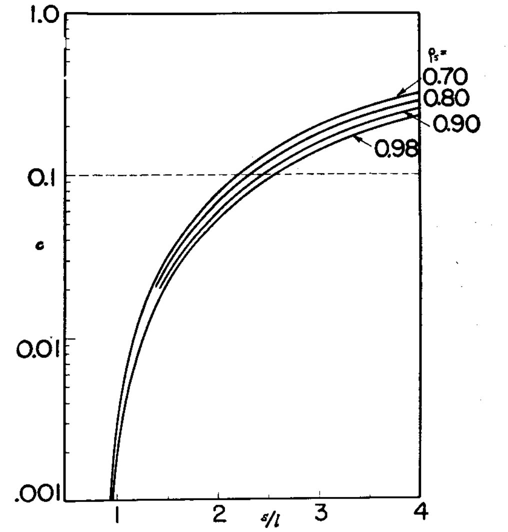


Figure 8. Brightness (helios) variation of the translucent plates for an infinite luminous ceiling, as affected by the lamp spacing s and the distance l between lamps and translucent plates. The contrast c is

$$c = \frac{H_{max} - H_{min}}{H_{max}}.$$

The horizontal line at $c = 0.10$ represents the minimum perceptible variation. Thus spacings of approximately $2l$ will give a ceiling that appears to be uniform.

0.40 according to C. L. Amick[10]) obtained experimentally on louverall installations.

Spacing of Lamps

Consider now the spacing between rows of lamps that will give visual uniformity for the luminous ceiling. Since the translucent plates are assumed to be perfectly diffusing, uniformity in brightness (helios) requires merely that the incident pharosage (including interflections) on the top of the plates be uniform. Appendix B gives equations for the maximum and minimum incident pharosage and for the contrast:

$$c = \frac{H_{max} - H_{min}}{H_{max}} = \frac{D_{max} - D_{min}}{D_{max}}.$$

The results are plotted in Fig. 8, which shows how c increases when the spacing between lamps is increased. The analysis includes the direct light from the lamps and the infinite number of interflections within the luminaire. According to W. E. K. Middleton,[11] a contrast of 0.10 is just detectable in a gradual transition from one brightness to another. Using this criterion, we see that a spacing of $2l$ between rows of lamps is allowable. That is, in a very large room, *the spacing between rows of fluorescent lamps may be made twice the distance from the lamps to the translucent plates.* This rule presupposes a diffusing upper surface with high reflectance ρ_5.

TABLE I — Dimensions for Luminous-Ceiling Lighting with normal spacing of $s = 0.61$m (24″), $l \geqq 0.30$m (12″).

Width of available space (w)		Number rows (n)	Spacing of outside rows = ¾ s		Distance to sides of luminaire = ¼ s	
(meter)	(ft)		(m)	(in.)	(m)	(in.)
0.91	3	2	......	......	0.15	6.0
1.52	5	3	......	......	0.15	6.0
1.83	6	4	0.46	18	0.15	6.0
2.44	8	5	0.46	18	0.15	6.0
3.05	10	6	0.46	18	0.15	6.0
3.66	12	7	0.46	18	0.15	6.0
4.27	14	8	0.46	18	0.15	6.0
4.88	16	9	0.46	18	0.15	6.0
5.49	18	10	0.46	18	0.15	6.0
6.10	20	11	0.46	18	0.15	6.0

For other spacings, change all dimensions in proportion. The total width is

$w = (n - ½)s$ for $n = 2$ or 3,

$w = (n - 1)s$ for $n \geqq 4$.

The distance from lamp to panel is

$l \geqq s/2$.

The above study of lamp spacing has been for an infinite room. There still remains the question of lamp spacing at the sides of the room. Should the lamp spacing remain uniform throughout or should there be a reduction in spacing near the sides of the room? Also, what spacing should be employed between the final row of lamps and the side of the luminaire? The equations are given in Appendix C.

For a narrow panel containing only two rows of lamps, the illumination (pharosage) without interflections is easily calculated. The results are shown in Fig. 9 for spacings of 1.5, 2, and 3 times the distance l. The spacing $s = 2l$ gives approximately 20 per cent variation without interflections, but this value is reduced to about 10 per cent by multiple reflections. The graph shows also that to keep the edges of the translucent plate at sufficiently high helios, *the distance between lamp and end of luminaire must be approximately one-fourth of the normal spacing between lamps.* Similar conclusions apply to 3, 5, and 7 rows of lamps. A trial-and-error process showed that when more than three rows are employed, *the outside rows should be moved toward the center so that their spacing is three-quarters of normal spacing.* This arrangement is found to give better uniformity than with normal spacing ($s = 2l$) throughout. An illumination (pharosage) plot for seven lamps spaced in this way is shown in Fig. 10.

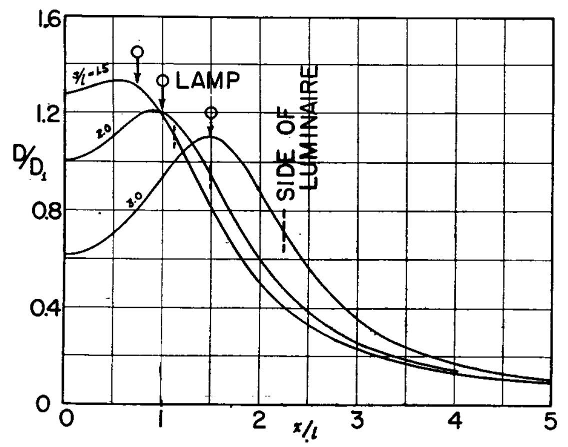


Figure 9. Incident illumination (pharosage) on translucent plate without interflections). Two infinitely long rows of fluoresent lamps. Brightness (helios) at the edge of plate is made approximately equal to brightness (helios) in the center ($x = 0$) by using ¼ normal spacing between lamps and sides of luminaire.

Conclusions

Equations and graphs have been given for the design of luminous ceilings. It is found that the coefficient of utilization compares favorably with the best values obtained with other lighting systems. In particular, the complete luminous ceiling costs no more than the louverall ceiling, has a higher coefficient of utilization, and eliminates that great defect of the louverall — *reflected glare.* Properly designed, the luminous ceiling gives ideal lighting that satisfies the 3:1 brightness (helios) criterion and all eight factors of lighting.[12]

Best results are obtained under the following conditions:

1. The translucent plates should have the highest transmittance that is consistent with complete diffusion. The optical absorptance of the material should be as low as possible.
2. The reflectance ρ_5 of the upper surface in the luminaire (Fig. 4) should be at least 0.80.
3. Floor reflectance ρ_3 in the room should be at least 0.30 and average wall reflectance should be at least 0.50.
4. The normal spacing s between rows of lamps should not exceed twice the distance l between lamp centers and translucent plates.
5. With more than 3 rows of lamps, the spacing of the outer rows should be three-quarters of normal spacing.

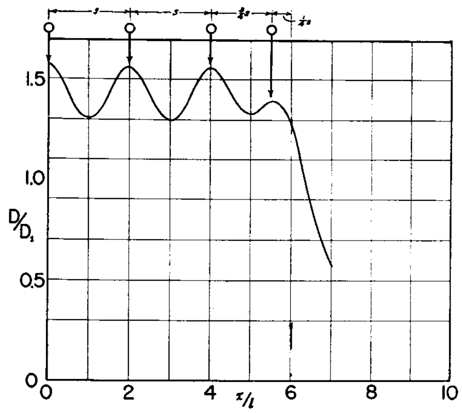


Figure 10. Incident illumination (pharosage) on translucent plate (without interflections). Seven infinitely long rows of fluoresent lamps.

6. Spacing between outside rows of lamps and side walls of the luminaire should be one-fourth of normal spacing between lamps. The old rule has been to use one-half normal spacing, but the present analysis shows that one-fourth is definitely superior.

7. Each row of lamps should run to the extreme end of the luminaire. In fact better brightness uniformity is obtained by making the length of the luminous panel slightly less than the total length of a row of lamps.

Some suggested dimensions are given in Fig. 11 and Table I. The table shows how many rows n of fluorescent lamps are needed for different available widths w. The tabulated values are for a normal spacing of 0.61m (24 in.), but dimensions for any other spacing are easily obtained. The spacing s is determined by the desired illumination (pharosage) D_{AV}, and Table II is included as an aid in determining this spacing. Values of D_{AV} have been calculated for various lamp spacings and for two rooms, one very large ($k_r \rightarrow 0$) and one of rather small size ($k_r = 0.70$). Most rooms will fall between these two extremes and may be investigated in greater detail by use of the interflection tables.[3] Table IIa gives the illumination (pharosage) in metric units and Table IIb gives the same data in English units.

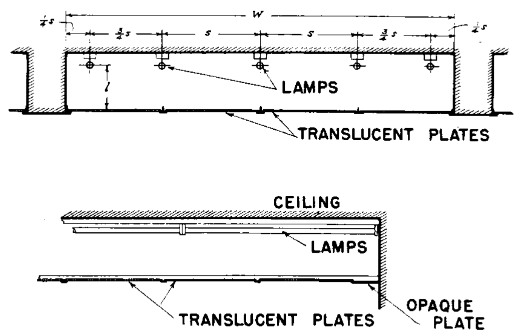


Figure 11. Portion of a luminous ceiling, showing how the spacing between lamps is reduced near the sides of the luminaire. The lower view indicates how the lamps extend beyond the end of the translucent plates to make the brightness (helios) more nearly uniform.

Lighting in accordance with the foregoing specifications has been installed in one room,[13] and other installations are in the process of construction. Tests indicate that the theoretical analysis is sound. There remain many questions of best ar-

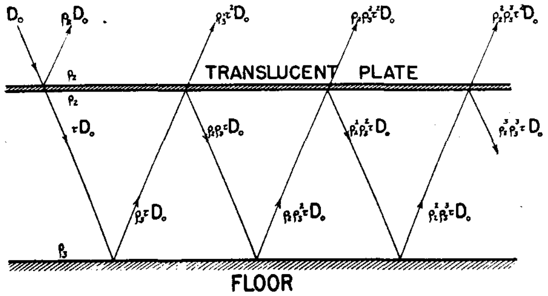
TABLE IIa — Average Illumination (Pharosage) as a Function of Lamp Spacing

Normal Spacing (s)		Average Illumination (Pharosage) (lumen m^{-2}) in Service					
		$k_r \rightarrow 0$			$k_r = 0.70$		
(m)	(in.)	T-12, 40w	T-8, 200 ma	T-8, 100 ma	T-12, 40w	T-8, 200 ma	T-8, 100 ma
0.30	12	4090	2710	1600	1790	1190	700
0.46	18	2730	1800	1070	1240	790	470
0.61	24	2040	1360	800	900	600	350
0.76	30	1640	1090	640	720	480	280
0.92	36	1370	900	530	600	400	230
1.22	48	1030	680	400	450	300	170

The lamps are the 40-watt, 48″, T-12, 3500K white fluorescent lamp (2300 lumen), and the T-8, 96″, 4500K white lamp operating at 200 ma (3050 (lumen) and 100 ma (1800 lumen). Reflectances are $\rho_3 = 0.30$, $\rho_1 = 0.50$, $\rho_5 = 0.80$. Also, $\tau = 0.40$, $k_m = 0.87$, $k_u = 0.76$ for $k_r \rightarrow 0$, $k_u = 0.333$ for $k_r = 0.70$.

TABLE IIb — Average Illumination (Pharosage) as a Function of Lamp Spacing

Normal Spacing (s)		Average Illumination (Pharosage) (lumen ft^{-2}) in Service					
		$k_r \rightarrow 0$			$k_r = 0.70$		
(m)	(in.)	T-12, 40w	T-8, 200 ma	T-8, 100 ma	T-12, 40w	T-8, 200 ma	T-8, 100 ma
0.30	12	380	250	150	170	110	65
0.46	18	250	170	100	110	74	44
0.61	24	190	130	74	84	55	33
0.76	30	150	100	60	67	44	26
0.92	36	130	84	50	55	37	22
1.22	48	95	63	37	42	28	16



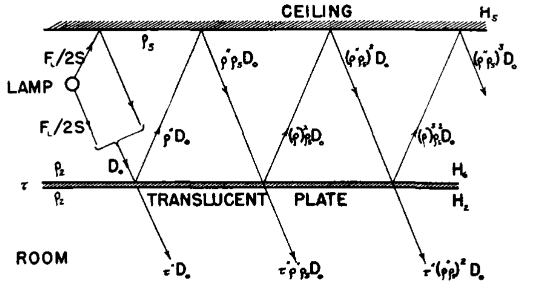


Figure 12. (Left). Schematic diagram showing interflections of light in an infinite room with arbitrary incident illumination (pharosage) D_0 applied to top of the translucent layer. The actual reflection and transmission is, of course, diffuse but is drawn schematically in one direction for simplicity in representation. The values are rigorously correct for infinite rooms with perfectly diffusing surfaces. Figure 13. (Right). Schematic diagram showing interflections in an infinite luminaire.

rangements for practicability and economy, and it is hoped that the present paper will stimulate further work in the development of ceiling lighting.

Appendix A

Interflections in an Infinite Room

Consider an infinite room (Fig. 4) lighted by means of a luminous ceiling. It is proposed to investigate the interflections in the complete assembly. All surfaces are assumed to be perfectly diffusing, and (F_L/S) is the total lamp lumens (pharos) per unit area of ceiling.

(I) First we formulate the interflections in the room itself, with arbitrary incident illumination (pharosage) D_0 on the top of the panel and no interflections in the luminaire. Fig. 12 gives a schematic diagram of the multiple reflections within the room. Adding all the components that go upward through the translucent plate, we obtain the brightness (helios) of the plate as seen from above:

$$H_6 = D_0 \left[\rho_2 + \rho_3 \tau^2 (1 + \rho_2\rho_3 + \rho_2{}^2\rho_3{}^2 + \ldots)\right]$$
$$= D_0 \left[\rho_2 + \frac{\rho_3 \tau^2}{1 - \rho_2\rho_3}\right].$$

But $H_6/D_0 = \rho'' =$ apparent reflectance of plate, as seen from above. Thus when light is incident on the upper side of the translucent plate (or "diaphosor"), it is not reflected according to the usual reflectance ρ_2 of this plate but according to the enhanced reflectance,

$$\rho'' = \rho_2 + \frac{\rho_3 \tau^2}{1 - \rho_2\rho_3}. \tag{9}$$

The apparent reflectance ρ'' is higher than ρ_2 because of the light reflected within the room.

Similarly, the brightness (helios) of the translucent plate, as seen from below, is obtained by adding all the contributions (Fig. 12), or

$$H_2 = \tau D_0 \left[1 + \rho_2\rho_3 + (\rho_2\rho_3)^2 + (\rho_2\rho_3)^3 + \ldots\right]$$
$$= \frac{\tau D_0}{1 - \rho_2\rho_3}.$$

But $H_2/D_0 = \tau'' =$ apparent transmittance of ceiling, with no interflections within the luminaire. Thus,

$$\tau'' = \frac{\tau}{1 - \rho_2\rho_3}. \tag{10}$$

The apparent transmittance of the diaphosor is higher than the measured transmittance τ because of multiple reflections within the room.

(II) The second step is to employ equations (9) and (10) in calculating the interflections in the luminaire (Fig. 13). Irrespective of where the fluorescent lamps are placed in the luminaire, exactly half of their luminous output is incident directly on the ceiling and half on the translucent plate. The total incident illumination (pharosage) without multiple reflections, is evidently

$$D_0 = \frac{F_L}{2S}(1 + \rho_5).$$

Adding the components reflected upward (Fig. 13), we obtain

$$H_6 = D_0 \rho'' \left[1 + (\rho''\rho_5) + (\rho''\rho_5)^2 + \ldots\right]$$
$$= \frac{D_0 \rho''}{1 - \rho''\rho_5} = \frac{F_L (1 + \rho_5)\rho''}{2S(1 - \rho''\rho_5)}. \tag{11}$$

And the transmitted components give

$$H_2 = D_0 \tau'' \left[1 + (\rho''\rho_5) + (\rho''\rho_5)^2 + \ldots\right]$$
$$= \frac{D_0 \tau''}{1 - \rho''\rho_5} = \frac{F_L (1 + \rho_5)\tau''}{2S(1 - \rho''\rho_5)}. \tag{12}$$

Also,

$$H_5 = \frac{\rho_5 F_L}{2S} + \rho''\rho_5 D_0 \left[1 + (\rho''\rho_5) + (\rho''\rho_5)^2 + \ldots\right]$$
$$= \frac{\rho_5 F_L}{2S}\left[1 + \frac{\rho''(1 + \rho_5)}{1 - \rho''\rho_5}\right]. \tag{13}$$

(III) We now obtain equations for coefficient of utilization, logance, and apparent reflectance of the complete luminous ceiling. Since the room is infinite in extent, the average illumination (pharosage) on the principal surface is

$$D_{AV} = H_2$$

or from equation (12),

$$D_{AV} = \frac{(1 + \rho_5)\tau''}{2(1 - \rho''\rho_5)} \frac{F_L}{S} = \frac{\tau(1 + \rho_5)}{2(1 - \rho_2\rho_3)} \frac{1}{1 - \rho_5\left[\rho_2 + \frac{\rho_3\tau^2}{1 - \rho_2\rho_3}\right]} \frac{F_L}{S} \tag{14}$$
$$= \frac{\tau(1 + \rho_5)}{2(1 - \rho_2\rho_5)} \frac{1}{1 - \rho_3\left[\rho_2 + \frac{\rho_5\tau^2}{1 - \rho_2\rho_5}\right]} \frac{F_L}{S}.$$

But in any room,

$$D_{AV} = k_u \frac{F_L}{S}.$$

Thus the coefficient of utilization k_u for an infinite room with Type IIa lighting is

$$k_u = \frac{\tau(1+\rho_5)}{2(1-\rho_2\rho_5)} \frac{1}{1-\rho_3\left[\rho_2+\frac{\rho_5\tau^2}{1-\rho_2\rho_5}\right]}$$

$$= \frac{\tau(1+\rho_5)}{2[(1-\rho_2\rho_3)(1-\rho_2\rho_5)-\rho_3\rho_5\tau^2]}. \qquad (15)$$

Comparison of equations (3) and (6) shows that the efficiency (logance) of the luminaire is

$$g = \frac{\tau(1+\rho_5)}{2(1-\rho_2\rho_5)} \qquad (7)$$

and the apparent reflectance of the translucent ceiling, as seen from below, is

$$\rho_2' = \rho_2 + \frac{\rho_5\tau^2}{1-\rho_2\rho_5}. \qquad (8)$$

A comparison of equations (8) and (9) indicates that the apparent reflectance of the translucent ceiling is different when viewed from above and from below. The two expressions differ only in the substitution of ρ_5 instead of ρ_3. It is easy to show that

$$(1-\rho_2\rho_3)(1-\rho''\rho_5) = (1-\rho_2\rho_5)(1-\rho'\rho_3). \qquad (16)$$

Appendix B

Brightness (Helios) Uniformity for Infinite Room

The pharosage from a single infinite row of uniform fluorescent lamps is

$$D = \frac{F_L s}{2\pi S l}\cos^2\theta$$

where the quantities are indicated in Fig. 2. But

$$\cos^2\theta = \frac{1}{1+(x/l)^2}.$$

Thus for $\theta = 0$, the incident pharosage produced by an infinite number of lamps (no interflections), with uniform spacing s between rows, is

$$D_{\max} = \frac{F_L(s/l)}{2\pi S}\left\{1+2\sum_{n=1}^{\infty}\frac{1}{1+(ns/l)^2}\right\}. \qquad (17)$$

Midway between rows of lamps,

$$D_{\min} = \frac{F_L(s/l)}{2\pi S} 2\sum_{1}^{\infty}\frac{1}{1+(n-½)^2(s/l)^2}. \qquad (18)$$

Equations (17) and (18) give the illumination (pharosage) produced directly by the lamps. To these values must be added the illumination (pharosage) from the upper surface of the luminaire. This added illumination (pharosage) is equal to H_5 and is assumed to be uniform over the entire diaphosor. Then, from equations (13), (17), and (18),

$$c = \frac{1-2\sum_{1}^{\infty}\frac{(n-¼)(s/l)^2}{[1+(ns/l)^2][1+(n-½)^2(s/l)^2]}}{1+2\sum_{1}^{\infty}\frac{1}{1+(ns/l)^2}+\frac{A}{(s/l)}} \qquad (19)$$

where

$$A = \pi\rho_5\left[1+\frac{\rho''(1+\rho_5)}{1-\rho''\rho_5}\right].$$

These equations were used in computing the curves of Fig. 8.

Appendix C

Brightness (Helios) Variation, Small Number of Rows

Consider now the incident illumination (pharosage) on the top of the translucent plates, produced directly by a small number of rows of lamps, neglecting interflections. The incident illumination (pharosage) produced by one row of lamps is

$$D = \frac{F_L(s/l)}{2\pi S}\frac{1}{1+(x/l)^2}.$$

Directly beneath the lamp, $x = 0$ and the pharosage is

$$D_1 = \frac{F_L(s/l)}{2\pi S}$$

At any other point,

$$D/D_1 = \frac{1}{1+(x/l)^2}. \qquad (20)$$

Equation (20) was used in plotting the curve for a single row of lamps. Superposition of these curves gave Figs. 9 and 10.

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