THE CALCULATION OF ILLUMINATION BY THE FLUX OF LIGHT METHOD[1]

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It is the object of this paper to give certain methods employed by the authors for a number of months past in calculating the illumination of large interiors and to show the practical application of the excellent suggestions made by Dr. Clayton H. Sharp in his presidential address before the first annual convention of the Illuminating Engineering Society at Boston July 30, 1907. The authors expressed the opinion at that time that the simple methods suggested by Dr. Sharp would be of considerable practical value, and immediately upon the close of the convention worked out methods of applying them in every-day engineering practice.

In this paper no attempt will be made to describe in detail the methods of calculating illumination which were common previous to the time Dr. Sharp delivered his address, and which are now generally used. The most common method where accurate calculations were made was to plot a rectangular curve of illumination with the polar photometric curve of a single lamp as a basis. If there were a number of sources of light in a room the distance of each from any given point was determined, preferably by a drawing made to scale. By adding together the illumination obtained from the various sources the total illumination on any one point could be found. This point-by-point method has been fully described in Chapter IV of "Practical Illumination," by the authors, and in other engineering literature. Its principal drawback is the amount of labor involved. It may be in order here to call attention to an error of application which has frequently crept into calculations by the point-by-point method and also into actual measurements. To approximate the true average illumination on a working plane, one must select a sufficient number of points equally spaced over the entire plane. The illumination must be calculated or measured for all these

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points. For example, in Fig. 1, suppose the source of light to be hung over the middle of the center circular area A. Assume the illumination of area A to be 4 foot-candles, B 3 foot-candles, C 2 foot-candles, and D 1 foot-candle. The incorrect method frequently used is to take the average of these, or 2.5 foot candles, as the average of the whole area. The correct method is to multiply the illumination by the area covered and take the mean of these results; or, in other words, to obtain the weighted mean

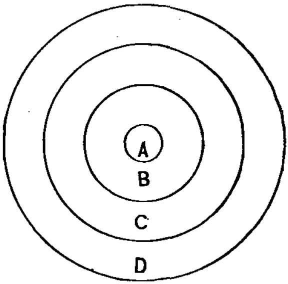


Fig. 1.—Diagram of Concrete Areas.

of the various circles. Let the diameter of A=1, B=3, C=5 and D=7.

$$\text{Then the area A} = \frac{\pi}{4}\,1^2 \qquad = \frac{\pi}{4}\,1$$

$$B = \frac{\pi}{4}(3^2 - 1^2) = \frac{\pi}{4}\,8$$

$$C = \frac{\pi}{4}(5^2 - 3^2) = \frac{\pi}{4}\,16$$

$$D = \frac{\pi}{4}(7^2 - 5^2) = \frac{\pi}{4}\,24.$$

The weighted mean of the illumination will then be

$$\frac{\frac{\pi}{4}\left[(1\times 4 \text{ ft.-candles})+(8\times 3 \text{ ft.-candles})+(16\times 2 \text{ ft.-candles})+(24\times 1 \text{ ft.-candles})\right]}{\frac{\pi}{4}\,49} = 1.72.$$

The true average is therefore 1.72 foot-candles instead of 2.5 foot-candles.

The suggestion advanced by Dr. Sharp was to the effect that the total amount or flux of light given out from the lamps be calculated in lumens to determine the amount of light available, and that the average intensity of illumination in foot-candles on the working plane it is desired to illuminate be determined from the number of lumens falling on that plane.

The lumen, according to English standards, may be defined as the amount of light falling on a surface 1 ft. square, when it is evenly illuminated to an intensity of 1 foot-candle. Thus, if a source of light giving 1 candle power in all directions be placed in the center of a sphere of 1 ft. radius, each square foot surface of that sphere would receive one lumen; and since the surface of a sphere is 4π, or 12.57, times the square of the radius, a sphere of 1 ft. radius will have a surface of 12.57 sq. ft. A lamp of 1 mean spherical candle power illuminating 12.57 sq. ft. with an illumination of 1 foot-candle would, therefore, be giving out 12.57 lumens. The number of lumens given out by a lamp is, therefore, the mean spherical candle power multiplied by 12.57. In illuminating engineering calculations one is concerned principally with the flux of light or in other words with the number of these lumens which fall upon a certain plane commonly referred to as the working plane. For example, in a large store, the working plane would be considered as being even with the tops of the counters, or about 42 ins. from the floor. In a large general office room it would be the plane of the desk tops, about 30 in. from the floor.

It is desired to determine as accurately as possible in advance the average illumination in foot-candles which will be obtained on the working plane. The lumen being that flux of light which will produce an average illumination of 1 foot-candle over a surface of 1 ft. square, it easily follows that if the total number of lumens emanating in the direction of the working plane and the number of square feet in the plane are known, we can easily arrive at the average foot-candle intensity by dividing the total number of lumens by the square feet in the room or plane. Thus, if there are 300 lumens falling on a plane of 100 sq. ft. area there is an average intensity of 3 foot-candles over that plane.

To make practical application of these principles in the calculation of illumination, it is necessary to determine with approximate accuracy the number of lumens given out from the lamps in the direction of the plane to be illuminated. When this is determined it is then necessary only to add together the number of lumens directed toward the working plane by all of the lamps in a room and to divide this total by the number of square feet in the room to get the average foot-candles on the working plane.

This method, of course, takes no account of the lumens directed toward the ceiling and walls, a part of which ultimately reach the working plane by reflection. This element of uncertainty, however, is the same as that which accompanies other methods of calculation heretofore used. Of course, in the majority of cases a certain proportion of the total number of lumens given out by the lamps is needed to illuminate ceilings and walls. Except where opaque reflectors are used, this portion of the problem can usually be neglected, bcause the ordinary opal, prismatic and sand-blasted reflectors let through enough of the total lumens given out by a lamp to take care of ceiling and wall illumination.

This method further takes no account of the variations between maximum and minimum illumination which may exist in different parts of a room. If there is any question as to whether the illumination will be uniform enough for practical purposes, this method should be supplemented by the older point-by-point method before referred to, in order that the probable variations between maximum and minimum can be determined. This method, however, is most generally applicable to large interiors with many lamps, these lamps being spaced at sufficiently short distances so that the variations in illumination are not great. In a large low room with a few very large sources of light, this method would not be of much value. Fortunately the number of rooms lighted in this way is on the decrease.

With this method it is evident that the key to the problem is the determination of the probable number of lumens directed toward the working plane. To do this with accuracy for all the lamps in a room would be a mathematical task out of the question for common engineering work. It is possible, however, to make some assumptions which are approximately correct and which render the calculation very simple.

For example, in the lighting of a large store or general office room, calculations seem to show that one may consider as effective all the lumens given out from a lamp within an angle of about 75 degrees from vertical. In the case of lamps located near the walls this relation will not be strictly true because of the lumens which will first strike the walls. Some of these, of course, will be recovered by reflection. In smaller rooms where lamps are placed close to walls, the lumens falling within 60 degrees of a vertical line through the lamp may be assumed as striking the working plane. The accuracy of these assumptions will be discussed later in the paper.

It is well to describe rapid methods for determining the number of lumens given out in different zones about a lamp. Assuming that the distribution of light about each lamp is practically symmetrical with reference to the axis of the lamp and that the lamps are to be placed with axes vertical (these being the conditions now commonly prevailing with high candle-power incandescent lamps, Nernst lamps and arc lamps) it is very easy to determine the number of lumens given out within a certain number of degrees from the lamp axis or perpendicular. One of the quick ways to determine the lumens is to plot a Rousseau diagram. It will not be necessary here to discuss the principle of the Rousseau diagram, as it has been described in a number of works dealing with photometry, and in the paper by Mr. J. E. Woodwell, printed in the TRANSACTIONS of the Illuminating Engineering Society, Nov., 1906, Page 248. It will be sufficient here to explain that it is a curve, Fig. 2, the height of which at any point represents the candle-power at the angle corresponding to that point and the area of which is proportional to the flux of light or lumens from the lamp.

The method of preparing Rousseau diagram paper is also indicated by Fig. 2, in which the full lines represent the preferable Rousseau diagram ruling and the dotted lines indicate the method by which the ruling is obtained. Rousseau diagram paper already ruled is now easily obtainable. In Fig. 2 is shown a Rousseau curve plotted for a Gem lamp of 50 mean horizontal candle-power in a prismatic bowl reflector. The polar-co-ordinate candle-power curve, commonly known as the photometric curve of the same lighting unit is also shown in Fig. 2. The method of plotting the Rousseau illumination curve from the polar candle-power curve will be apparent upon brief inspection. For example, looking on the polar photometric curve, it is seen that the candle-power at ten degrees from the vertical is 65. A dot should be placed on the ten-degree line of the Rousseau diagram at the height 65 corresponding to the candle-power. At 20 degrees from the vertical the candle-power shown on the photometric curve is 80; a dot should be placed on the 20-degree vertical line of the Rousseau diagram at a height corresponding to

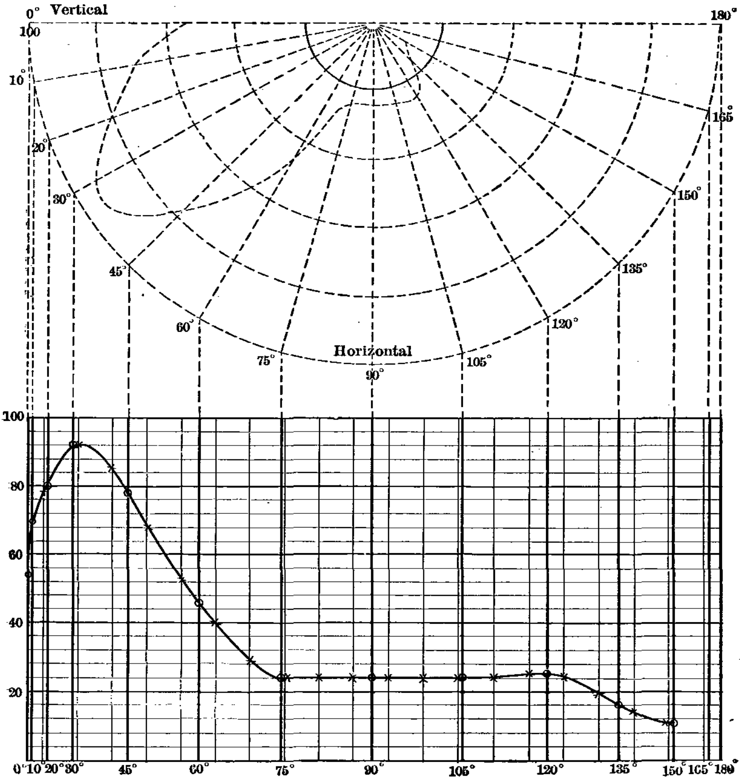


Fig. 2.—Rousseau Diagram for Gem Lamp with Bowl Reflector.

80 candle-power. When this has been done for the entire half circle, these dots should be joined to form the curve called the Rousseau curve or diagram. The area of this diagram between any two vertical lines representing degrees corresponds to the total flux of light in lumens given out in the zone bounded by those degrees. For example, the area enclosed by the curve between zero and 90 degrees (90 degrees being horizontal) repre-

sents graphically the number of lumens given out in the lower hemisphere; while the area enclosed by the whole curve from zero to 180 degrees represents the total number of lumens given out. If the average height of this curve be found by measuring its height at equal intervals one obtains the mean spherical candle-power. To determine the number of lumens from the Rousseau diagram it is necessary only to ascertain the area enclosed by the curve expressed in proper terms. In the Rousseau curve shown in Fig. 2, the ruling is such that there are twenty equal divisions of the diagram as measured horizontally. As the average height of the curve represents candle-power, the horizontal distance instead of 20 should be 12.57, if the area is to represent lumens. The constant by which the sum of the candle-power readings must be multiplied in order to get the lumens in this case is therefore, $\frac{12.57}{20}$, or 0.628.

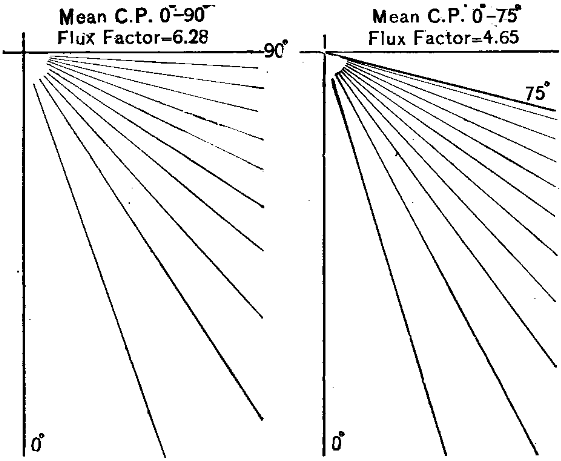
Another and preferable method of determining the number of lumens in any given zone is to measure the average height of the curve within that zone, which height gives the mean zonular candle-power. Multiplying this mean zonular candle-power by a certain factor which may be called a flux factor, will give the number of lumens within that zone. This flux factor is the area of the zone of a sphere of unit radius. Since the flux in lumens is equal to the area in square feet multiplied by the mean foot-candle intensity, and since 1 foot-candle intensity will be given anywhere on the inner surface of a sphere 1 ft. in radius by a light of 1 candle-power, the area of the zone of a sphere need merely be multiplied by the mean zonular candle-power to determine the lumens within that zone. The area of the zone of a sphere of unit radius equals $2\pi\ (1 - \cos a)$ where a equals the angle between the pole or vertical and the edge of the zone. The flux factors calculated by this formula for angles from 5 to 90 degrees are as follows:

Degrees	Flux factor
5	0.025
10	0.094
15	0.0220
20	0.376
25	0.590
30	0.842

Degrees	Flux factor
35	1.13
40	1.47
45	1.84
50	2.24
55	2.68
60	3.14
65	3.62
70	4.18
75	4.65
80	5.18
85	5.73
90	6.28

The above considerations lead to a further short cut which obviates the necessity of plotting a Rousseau curve. From a study of Fig. 2, it will be evident that from the polar curve one can obtain the candle-power values for the points indicated by crosses on the Rousseau curve if the radial lines on the polar curve correspond in position to the fine equally-spaced lines of the Rousseau curve. It is a simple matter of drafting to provide a set of flux polar diagrams with radial lines corresponding to the positions of the equally-spaced lines on the Rousseau diagram. Such a flux polar diagram for an angle of 90 degrees is shown in Fig. 3. If this diagram is made on transparent celluloid or tracing cloth, it can be laid over the ordinary polar-photometric curve with the zero line at the vertical and the 90-degree line at the horizontal. By noting and averaging the candle-power of the photometric curve at the ten points where the radial lines (not including the zero and 90-degree lines) cross the photometric curve, the mean hemispherical candle-power is obtained. As there are ten readings to be averaged, it is only necessary to add the readings and transfer the decimal point one figure to the left to obtain the average. This mean value of hemispherical candle-power multiplied by the flux factor 6.28 for the 90-degree zone, gives the total number of lumens in the lower hemisphere. Similar flux polar diagrams can be prepared for other zones. For example, Fig. 4 is arranged for obtaining the mean zonular candle-power and lumens in a zone extending 75 degrees from the vertical. This diagram also is divided into ten parts so that the average of the readings is obtained by dividing their sum by 10 and the lumens are obtained by multiplying this average by the flux factor 4.65 for 75 degrees. The same thing is true of Figs. 5 and 6 which are for 60 and 30 degrees, respectively. With a set of these flux polar diagrams, the illuminating engineer is in a position very quickly to find the lumens from any symmetrical photometric curve within any of the zones which are of practical use to him.

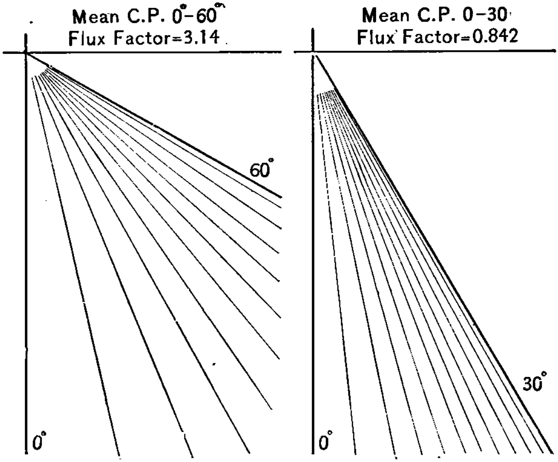
To show the practical application of the above described method, on applying the 60-degree flux polar diagram to the polar photometric curve shown in Fig. 2, it is found that 236 lumens are given out by the lamp between 60 degrees and the vertical.



Figs. 3 and 4.—Flux Polar Diagrams, 90 and 75 Degrees.

The curve of Fig. 2 relates to a 125-watt Gem lamp in a bowl reflector, so the value for lumens can be reduced to watts per lumen by dividing the wattage of the lamp; thus there is 0.53 watt per lumen. Therefore, if 0.53 watt is expended per square foot of floor space, one lumen per square foot is obtained. Since the average intensity in foot-candles is equal to the lumens divided by the square feet and the area in this case is 1 sq. ft., the average foot-candle value will be 1. In other words, if the average foot-candle intensity be multiplied by the value of watts per lumen in the zone under consideration, obtained with the desired lamp and reflector, the value of the watts per square foot necessary to illuminate the room will be obtained. The rule or formula then simply becomes:

Average foot-candles × watts per lumen = watts per sq. ft.



Figs. 5 and 6.—Flux Polar Diagrams, 60 and 30 Degrees.

It now remains to determine how closely the results calculated as above check with actual results obtained in installations which have been photometered. It is evident that the main chances of error in the above method lie in the reflection from walls and in assuming too large or too small an angle from the vertical in which to determine the flux of useful light. In a small room with very dark walls and the lamps placed high, the zone of useful light as measured by the angle from the vertical would be small, sometimes not over 50 or 60 degrees. On the other hand, in a very large room with few obstructions and with light-colored walls, the light which emanates from the source at a considerable angle from the vertical ultimately falls on the working plane and becomes useful. The area illuminated by each lamp is very large in such a case and each point in the working plane receives illumination from many lamps.

There is given herewith a table of constants based on all of the results of reliable tests on efficiency of illumination which have come to the notice of the authors. This table is the result of careful consideration of all the evidence obtainable, and is believed to be approximately correct. The table shows the watts per lumen, or in other words, the watts per square foot required to give one foot-candle average illumination in several types of lighting installations.

To apply it use the rule.

Watts per sq. ft. = foot-candle intensity × constant from table.

TABLE SHOWING NUMBER OF WATTS PER SQUARE FOOT OF FLOOR AREA REQUIRED TO PRODUCE AN AVERAGE OF 1 FOOT-CANDLE OF ILLUMINATION. (WATTS PER LUMEN.)

Incandescent Lamps.	
Tungsten lamps rated at 1.25 watts per horizontal candle power; clear prismatic reflectors, either bowl or concentrating; large room; light ceiling; dark walls; lamps pendant; height from 8 to 15 feet	0.25
Same with very light walls	0.20
Tungsten lamps rated at 1.25 watts per horizontal candle power; prismatic bowl reflectors enameled; large room; light ceiling; dark walls; lamps pendant, height from 8 to 15 feet	0.29
Same with very light walls	0.23
Gem lamps rated at 2.5 watts per horizontal candle power; clear prismatic reflectors either concentrating or bowl; large room; light ceiling; dark walls; lamps pendant; height from 8 to 15 feet	0.55
Same with very light walls	0.45
Carbon filament lamps rated at 3.1 watts per horizontal candle power; clear prismatic reflectors either bowl or concentrating; light ceiling; dark walls; large room; lamps pendant; height from 8 to 15 feet	0.65
Same with very light walls	0.55
Bare carbon filament lamps rated at 3.1 watts per horizontal candle power; no reflectors; large room; very light ceiling and walls; height from 10 to 14 feet	0.75 to 1.5
Same; small room; medium walls	1.25 to 2.0
Carbon filament lamps rated at 3.1 watts per horizontal candle power; opal dome or opal cone reflectors; light ceiling; dark walls; large room; lamps pendant; height from 8 to 15 feet	0.70
Same with light walls	0.60
Arc Lamps.	
5-ampere, enclosed, direct-current arc on 110-volt circuit; clear inner, opal outer globe; no reflector; large room; light ceiling; medium walls; height from 9 to 14 feet	0.50

It is interesting to compare the results shown in the table with the results which would be obtained in the calculation of the illumination of similar interiors by the flux-of-light method which is described herein. For example, taking the first two items on the table, the constant obtained from practical experience is 0.20 to 0.25 while the calculated flux of light from tungsten lamps with bowl reflectors within 75 degrees from the vertical corresponds to about 0.20 watt per lumen. The calculated flux within 60 degrees corresponds to 0.23 watt per lumen. For this item it would seem that the zone of 75 degrees is the one which gives the most accurate results with light walls while the 60-degree zone would be best for dark walls. Referring to the seventh item on the table, by calculation carbon-filament lamps with bowl prismatic reflectors, take 0.495 watt per lumen below 75 degrees and 0.63 watt per lumen below 60 degrees. The table gives 0.65 watt per lumen. For conservative estimates the 60-degree zone should usually be taken.

The methods described above are only approximate at best, but probably they are no more open to error than other methods. They are certainly much more simple and rapid. The authors have used them for the past year.

DISCUSSION.

Mr. R. L. Lloyd:—The authors conclude their paper with a paragraph beginning as follows:

"The methods described above are only approximate at best."

I am a member of this Society, not as a scientist or investigator, but as one who wants some practical facts to work on. Probably there are many others who are in the same boat as myself, and to show how very practical the tables really are, a little experience of mine cannot be amiss. Out in the burgh, where I live, we have recently attained the dignity of a concert hall, called an auditorium. Naturally, I was interested at the start in the project, and as soon as I was able got hold of the plans looked into the arrangements for the lighting. To my mind, they were very meagre and insufficient, and after talking the matter over with those in charge, I volunteered to lay out a lighting plan for them.

I did not know much about the matter, but had entire confidence in the tables and formulae which I gathered from our Society. With these as a basis, I recommended six tungsten lamps in the ceiling, (the ceiling probably a little higher than in this room—about 25 feet)—which should give about one candle-power per