Light Flux Distribution in a Rectangular Parallelepiped and its Simplifying Scale

By KIYOSI HISANO

Translated from the Japanese by HARRY SHIRAMIZU*, edited by PARRY MOON**

The following is a translation of Research Bulletin No. 394 of the Electrotechnical Laboratory, Tokyo, which was published in 1936. The research makes a fundamental contribution to lighting design and thus its English translation should be of interest to illuminating engineers. The translation is believed to be a faithful mirror of the original ideas, though we have condensed the text slightly and have omitted some of the figures. (Ed.)

1. Introduction

INTERIOR-lighting is an important branch of illuminating engineering, a branch in which the space under consideration is usually enclosed in a rectangular parallelepiped. A rigorous treatment of the rectangular parallelepiped, however, would involve extremely complicated calculations, and the problem has therefore remained unsolved to this day.

During the past ten years, lighting research has advanced in a striking manner so that today we can calculate light distributions from large sources and can deal theoretically with daylight illumination. Methods have been obtained for the calculation of illumination from large surface sources and of total flux from one surface to another. Furthermore, the basic theory of interreflections has been developed and the interreflection theory for the circular cylinder has been completed. The distribution of flux in a rectangular parallelepiped, however, still remains an unsolved problem.‡

Observing that the luminous flux distribution in a rectangular parallelepiped is of great practical importance, the author investigated the possibility of expressing the result in simple form. He obtained the *simplifying scale* described in this paper. Calculations and experiments prove that the use of

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Author: Electrotechnical Laboratory, Tokyo: Japan.

* Harvard University, Cambridge: Mass.

** Massachusetts Institute of Technology, Cambridge, Mass.

‡ For more recent work on this problem, see

W. T. White, Calculation of light distribution in lightwells, *Journal of the Optical Society of America*, 31, 1941, p. 308.

P. Magnusson, Numerical methods of solving integral equations in two independent variables, *Journal of Mathematics and Physics*, 21, 1942, p. 250.

Parry Moon, Interreflections in finite cylinders, *Journal of the Optical Society of America*, 31, 1941, p. 223;

Interreflections in lightwells, *Journal of the Optical Society of America*, 31, 1941, p. 301;

Interreflections in rooms, *Journal of the Optical Society of America*, 31, 1941, p. 374. (Ed.)

this simplifying scale allows every rectangular parallelepiped to be replaced by a rectangular parallelepiped with square base (equivalent square room). Thus experiments or calculations need be made only on square rooms.

This study was made while the author was connected with the third section of the Electrotechnical Laboratory. The author desires to express his deep gratitude to his associates of the same laboratory for their painstaking calculations and their assiduous effort in carrying through the experimental work. He thanks particularly Dr. Ziro Yamauti for his advice.

2. Definitions and Postulates

Let a and b be the dimensions of the ceiling of the rectangular parallelepiped and let h be the height. The harmonic mean of the two sides a and b is

$$\lambda = \frac{2ab}{a+b}. \tag{1}$$

The quantity λ is called the *simplifying scale* because if it is used to measure the length, the problem of the rectangular parallelepiped is greatly simplified. That is to say, within the practical range of room dimensions, the flux distribution in a parallelepiped with ceiling $a \times b$ and height h is essentially the same as the distribution in a room whose ceiling is 1×1 and whose height is h/λ. The height z, measured by the simplifying scale, is

$$z = h/\lambda. \tag{2}$$

The quantity z is called the *equivalent height* and the room whose square floor measures 1×1 and whose height is z is called the *equivalent square room*.

The following postulates are used in this treatise:

(1) All interior surfaces of the rectangular parallelepiped are perfectly diffusing.
(2) Ceiling, walls, and floor—each of the three has a uniform reflectance.
(3) Brightness of the ceiling is uniform.
(4) Illumination at a given height on the wall surface is uniform.
(5) Illumination of the floor is uniform.

Also, since the purpose of the paper is to simplify practical problems, dimensions whose actual occurrence is uncommon will not be considered. Parallelepipeds have been restricted to the range.

$$z \leqq 2, \quad k = b/a \leqq 4. \tag{3}$$

As the object is the application to lighting design, the aim will be an accuracy of $\pm$ 10 per cent.

3. Primary Flux Distributions

Flux to Floor from Ceiling

It is known[1] that the total luminous flux direct from a ceiling ($a \times b$) having uniform unit brightness*, to a floor at distance h from the ceiling, is

$$F' = 4 \left\{ b^2 \left[\varphi_2 \left(\tan^{-1} \frac{b}{\sqrt{a^2 + h^2}} \right) - \varphi_2 \left(\tan^{-1} \frac{b}{h} \right) \right] \right.$$
$$\left. + a^2 \left[\varphi_2 \left(\tan^{-1} \frac{a}{\sqrt{b^2 + h^2}} \right) - \varphi_2 \left(\tan^{-1} \frac{a}{h} \right) \right] \right\}. \tag{4}$$

Here the function φ_2 is defined as

$$\varphi_2(\omega) = \tfrac{1}{2} \left(\omega \cot \omega - \tfrac{1}{2} \log \sin \omega + \tfrac{1}{2} \cot^2 \omega \log \cos \omega \right). \tag{5}$$

Hence U, the ratio of the flux incident on the floor to the flux from the ceiling, is $F'/\pi ab$, (in other words[2], U is the floor illumination when the ceiling has unit luminosity), or

$$U = \frac{4}{\pi} \left\{ \frac{b}{a} \left[\varphi_2 \left(\tan^{-1} \frac{b}{\sqrt{a^2 + h^2}} \right) - \varphi_2 \left(\tan^{-1} \frac{b}{h} \right) \right] \right.$$
$$\left. + \frac{a}{b} \left[\varphi_2 \left(\tan^{-1} \frac{a}{\sqrt{b^2 + h^2}} \right) - \varphi_2 \left(\tan^{-1} \frac{a}{h} \right) \right] \right\}. \tag{6}$$

As shown by Eq. (6), U is determined by a/h and b/h. The equation can be expressed also in terms of the ratio $k = b/a$ and the equivalent height z, for

$$\frac{a}{h} = \frac{a}{z} \frac{a + b}{2ab} = \frac{1 + k}{2kz}, \qquad \frac{b}{h} = \frac{b}{z} \frac{a + b}{2ab} = \frac{1 + k}{2z}. \tag{7}$$

If the floor is square, $k = 1$ and

$$\frac{a}{h} = \frac{b}{h} = \frac{1}{z}. \tag{8}$$

Then Eq. (6) assumes the simplified form:

$$U = \frac{8}{\pi} \left\{ \varphi_2 \left(\tan^{-1} \frac{1}{\sqrt{1 + z^2}} \right) - \varphi_2 \left(\tan^{-1} \frac{1}{z} \right) \right\}, \qquad (k = 1). \tag{9}$$

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* (Throughout the paper, brightness is expressed in *candles per unit area*. Hisano could have saved himself the annoying necessity of shifting π's about if he had used the concept of *helios* or if he had expressed his brightness in equivalent lumens per unit area. Ed.)

[1] Z. Yamauti, Researches of the Electrotechnical Laboratory, Tokyo, No. 250, 1929.

[2] In addition to the above definition, U may be designated as follows:

"U is the average illumination on the floor when the ceiling has unit luminosity", or

"U is the ratio of the average illumination on the floor to the illumination on the floor that would be produced if the ceiling extended indefinitely."

Thus U is the *daylight factor*.

If U is considered as a function of a and b, Eq. (6), U will vary greatly as the ratio k is changed, even if h is constant. This variation is illustrated by a chart in the author's previously published research[3]. But if U is considered as a function of z and k, then k has comparatively little effect, as shown in Fig. 1. In fact, curve I of Fig. 1 shows that over the practical range the effect of k may be neglected. Within this range, the value of U for a given z is independent of k to within approximately 10 per cent.

Accordingly, for the degree of accuracy and for the ranges of k and z considered in this paper, Eq. (9) is sufficient to determine U. This quantity is considered to be a function of z only, and thus it is possible to eliminate one of the independent variables required in determining the flux incident on the floor directly from the ceiling.

For a square room having the side *a*,

$$\lambda = a.$$

The *equivalent square room* is one whose floor dimensions are 1×1 and whose height is $z = h/a$. Therefore in determining experimentally the U pertaining to various parallelepipeds, it is sufficient to experiment with a square room $1 \times 1 \times z$.

Direct Illumination on the Wall

It has been established (footnotes 4 to 9) that the following equation expresses the illumination on the wall, produced by a ceiling source of unit brightness:

$$E_b = \tan^{-1} \frac{b/2}{h} - \frac{h}{\sqrt{a^2 + b^2}} \tan^{-1} \frac{b/2}{\sqrt{a^2 + b^2}}.$$

Here the distance h is measured vertically from the ceiling to the point on the center line of the wall, where the illumination is desired, and b is the width of the wall. Thus

$$U_b = \frac{1}{\pi} \left\{ \tan^{-1} \frac{b/2}{h} - \frac{h}{\sqrt{a^2 + b^2}} \tan^{-1} \frac{b/2}{\sqrt{a^2 + b^2}} \right\}. \tag{10}$$

These equations are in terms of a/h and b/h only. But by Eq. (7), U_b can be expressed as a function of z and k. A numerical analysis shows that, like U, U_b also is independent of k, to the desired degree of accuracy and within the

[3] K. Hisano, Researches of the Electrotechnical Laboratory, Tokyo, No. 367, Fig. 2 in Appendix.

[4] H. S. Swan and G. H. Tuttle, *Architectural Forum*, Nov. 1918.

[5] H. H. Higbie and A. Levin, *Transactions* of the Illuminating Engineering Society, 21, 1926, p. 273.

[6] H. H. Higbie and W. T. Szymanowski, *Transactions* of the Illuminating Engineering Society, 25, 1930, p. 213.

[7] Giiti Onomoto, *Journal of the Society for Study of Illumination* (Japan), 8, p. 126.

[8] Z. Yamauti, Researches of the Electrotechnical Laboratory, No. 194.

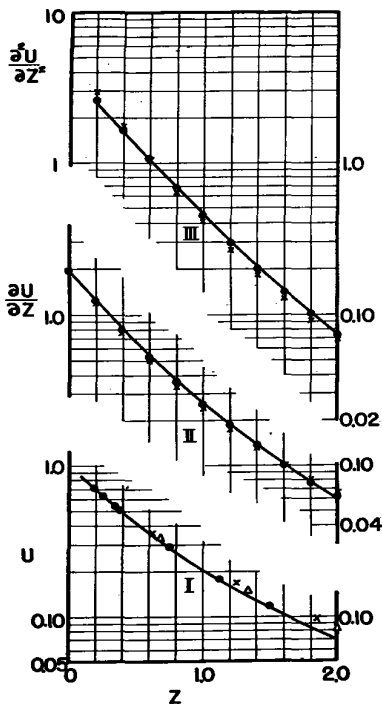[9] K. Hisano, Researches of the Electrotechnical Laboratory, No. 367.

Fig. 1—Direct flux from ceiling to floor (and its first and second derivatives). The curves are drawn for square rooms and the points indicate calculated values for various rectangular floors. —, $k = 1$; ●, $k = 2$; △, $k = 3$; x, $k = 4$.

practical ranges of k and z. That is, this problem also can be treated by means of the equivalent square room. Equation (10) then becomes

$$U_b = \frac{1}{\pi}\left\{\tan^{-1}\frac{1}{2z} - \frac{z}{\sqrt{1+z^2}}\tan^{-1}\frac{1}{2\sqrt{1+z^2}}\right\}, \quad (k = 1). \quad (11)$$

Average Illumination at Given Distance from Ceiling

Since U is the flux to the floor, produced by unit ceiling flux, the quantity $-\dfrac{\partial U}{\partial b}\,db$ represents the flux from the ceiling to the four elementary wall strips found by two horizontal surfaces at distances b and $(b + db)$ from the ceiling. Furthermore,

$$-\frac{\partial U}{\partial b}\,db/2(a + b)\,db$$

is the average illumination on these elementary wall strips.

But for an infinite ceiling of brightness $1/(\pi ab)$, the illumination on a horizontal surface is $1/ab$. Therefore the average wall illumination at distance h below the ceiling, expressed as a daylight factor, is

$$-\frac{\partial U}{\partial h}\,dh\,\frac{ab}{2(a+b)\,dh} = -\frac{1}{4}\frac{\partial U}{\partial h}\lambda = -\frac{1}{4}\frac{\partial U}{\partial z}. \tag{12}$$

The formula for $-\partial U/\partial h$ was obtained by Yamauti[10]:

$$-\frac{\partial U}{\partial z} = \frac{4}{\pi(a+b)}\left\{b\left[\tan^{-1}\frac{b}{h} - \frac{h}{\sqrt{a^2+h^2}}\tan^{-1}\frac{b}{\sqrt{a^2+h^2}}\right]\right.$$
$$+ a\left[\tan^{-1}\frac{a}{h} - \frac{h}{\sqrt{b^2+h^2}}\tan^{-1}\frac{a}{\sqrt{b^2+h^2}}\right] \tag{13}$$
$$\left.- h\ln\frac{(a^2+h^2)(b^2+h^2)}{h^2(a^2+b^2+h^2)}\right\}.$$

This, too, is a function of a/h and b/h only; and the possibility of expressing it in terms of k and z is the same as for U. If $k = 1$, Eq. (13) becomes

$$-\frac{\partial U}{\partial z} = \frac{4}{\pi}\left\{\tan^{-1}\frac{1}{z} - \frac{z}{\sqrt{1+z^2}}\tan^{-1}\frac{1}{\sqrt{1+z^2}} - z\ln\frac{1+z^2}{z\sqrt{2+z^2}}\right\}, \tag{14}$$
$$(k = 1)$$

Calculated values of $-\partial U/\partial z$ are plotted in Curve II, Fig. 1. As shown, this quantity becomes a function of z only, practically independent of k. In other words, for practical purposes Eqs. (12) and (13) are sufficient to determine the average illumination on the walls at any height, using the ceiling as light source. Evidently an experimental investigation with a square room will suffice to give all necessary information.

Interreflections between Walls

The flux to an elementary wall strip of width dh and distance h from a ceiling of unit brightness is $-\dfrac{\partial F'}{\partial h}dh$. Reciprocally, the flux to the ceiling from an elementary wall strip of unit brightness is also $-\dfrac{\partial F'}{\partial h}dh$. Hence when the wall strip is of unit luminosity, the flux incident to the ceiling is

$$-\frac{1}{\pi}\frac{\partial F'}{\partial h}\,dh$$

and the average ceiling illumination is

[10] Z. Yamauti, Researches of the Electrotechnical Laboratory, No. 250.

$$-\frac{1}{\pi ab}\frac{\partial F'}{\partial b}\, db = -\frac{\partial U}{\partial z}\, dz. \tag{15}$$

This quantity, too, can be treated as a function of z only, as shown in the previous section.

When this elementary wall strip of width db has unit luminosity, the flux from it to a second elementary wall strip of width db_2 at distance b from the first strip is

$$\frac{1}{\pi}\frac{\partial^2 F'}{\partial b^2}\, db\, db_2 \frac{1}{2(a+b)\, db_2} = \frac{\partial^2 U}{\partial b^2}\, db\, \frac{ab}{2(a+b)} = \frac{1}{4}\frac{\partial^2 U}{\partial z^2}\, dz.$$

This equation constitutes the basis for the determination of interreflections between wall surfaces. Yamauti[10] has shown that

$$\frac{\partial^2 U}{\partial z^2} = \frac{8ab}{\pi(a+b)^2}\left\{\frac{a^2 b}{(a^2+b^2)^{3/2}}\tan^{-1}\frac{b}{\sqrt{a^2+b^2}}\right.$$
$$\left. +\frac{ab^2}{(b^2+b^2)^{3/2}}\tan^{-1}\frac{a}{\sqrt{b^2+b^2}} + \ln\frac{(a^2+b^2)(b^2+b^2)}{b^2(a^2+b^2+b^2)}\right\}. \tag{17}$$

Just as with $\partial U/\partial z$, this new function can be expressed in terms of k and z. For $k = 1$, Eq. (17) becomes

$$\frac{\partial^2 U}{\partial z^2} = \frac{4}{\pi}\left\{\frac{1}{(1+z^2)^{3/2}}\tan^{-1}\frac{1}{\sqrt{1+z^2}} + \ln\left(\frac{1+z^2}{z\sqrt{2+z^2}}\right)\right\},\quad (k=1). \tag{18}$$

That $\partial^2 U/\partial z^2$ will, for practical purposes, be independent of k is indicated by Curve III of Fig. 1.

Flux to the Floor from a Point Source

Previous sections have been devoted to the luminous flux from surface to surface, but this section considers the flux to one surface from a point light source. Assume that the source is in the center of the room and that its height above the working plane is h_s. The problem can be handled easily if the intensity is uniform or if the source is a portion of perfectly diffusing surface in a horizontal plane.

The ratio of the luminous flux incident on the working plane to the total flux from the luminaire is called the *intrinsic utility factor*[*] K'. The problem is to find the relation between K', $z_s = h_s/\lambda$, and k. For a uniform point source of intensity I, the total flux is $4\pi I$ and the flux incident on the working plane is

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[*] Intrinsic utility factor is the coefficient of utilization pertaining to the light emitted by luminaires. Needless to say, the relation between the two quantities is

Coefficient of utilization = (Intrinsic utility factor) × (Luminaire efficiency).

$$4I \tan^{-1} \frac{(a/2)(b/2)}{b_s\sqrt{b_s^2 + (a/2)^2 + (b/2)^2}}.$$

Thus the intrinsic utility factor is

$$K' = \frac{1}{\pi} \tan^{-1} \frac{ab}{4b_s\sqrt{b_s^2 + (a/2)^2 + (b/2)^2}}. \tag{19}$$

When the light source is a perfectly diffusing surface of area ΔS and of unit brightness, placed horizontally in the center of the ceiling, the flux from the source is $\pi\Delta S$. The flux to the working plane from this source is, by reciprocity, the same as the flux to ΔS when the whole working plane is a source of uniform brightness. This can be expressed as follows:

$$4\Delta S\left[\frac{1}{2} \frac{a/2}{\sqrt{b_s^2 + (a/2)^2}} \tan^{-1} \frac{b/2}{\sqrt{b_s^2 + (a/2)^2}}\right.$$
$$\left.+ \frac{1}{2} \frac{b/2}{\sqrt{b_s^2 + (b/2)^2}} \tan^{-1} \frac{a/2}{\sqrt{b_s^2 + (b/2)^2}}\right]$$

Therefore the intrinsic utility factor is

$$K' = \frac{2}{\pi}\left[\frac{a/2}{\sqrt{b_s^2 + (a/2)^2}} \tan^{-1} \frac{b/2}{\sqrt{b_s^2 + (a/2)^2}}\right.$$
$$\left.+ \frac{b/2}{\sqrt{b_s^2 + (b/2)^2}} \tan^{-1} \frac{a/2}{\sqrt{b_s^2 + (b/2)^2}}\right]. \tag{20}$$

Equations (19) and (20) may be expressed in terms of a/b_s and b/b_s. Thus, as in previous cases, these quantities may be written as functions of k and z_s. Fig. 2 indicates the relation between K' and z_s for the values of k specified in the diagram. Curve I corresponds to Eq. (19) and Curve II corresponds to Eq. (20).

Since this preliminary survey was based on the theory of only two representative light distributions, it is necessary to examine the validity of the method with light distributions in general. Next, therefore, a survey was made using the so-called Trojan luminaire. The exterior of the fixture was diffusing and was not ornamented in any way. Measurements were made by means of a photocell.

First, to determine the flux directly to the rectangle, all surfaces of the room were painted black and the luminaire was suspended in the center of the ceiling. Three different horizontal rectangles were used ($k = 1, 2, 4$), with the luminaire at height b_s above them. The flux to these rectangles was measured and the ratio to the condition for $k = 1$ was determined. The
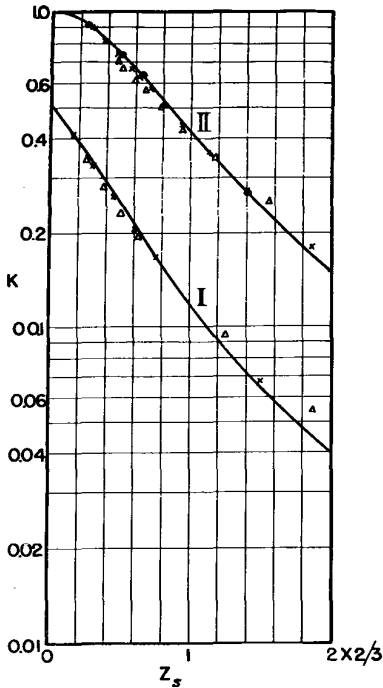
FIG. 2—Calculated flux to the floor from luminaires. Curve I is for a rectangular room lighted by a single luminaire whose candle-power is constant in all directions, $I(\Theta) = I$. Curve II is for a room lighted by a small, perfectly diffusing panel in the center of the ceiling, $I(\Theta) = I \cos \Theta$. $-$, $k = 1$; x, $k = 2$; $\triangle$, $k = 4$.

source was located above the center of each rectangle. The result of these measurements is shown in Table I. For practical purposes, it is unnecessary to consider $z_s = 2$. Accordingly, if the range is limited to $z_s \leqq 1$, the results are independent of k even with a light distribution such as that given by the Trojan luminaire.

Results obtained in Table I, as well as the theoretical investigation, were for a light source above the center of all rectangles. Subsequent work dealt with the source off center. But in this experiment, interreflections also were introduced. White paint with a reflectance of 0.871 was applied uniformly to the walls above the working plane and to the ceiling, and black paint was applied to the walls below the working plane and to the floor.

The working plane was divided into 36 squares 1m × 1m, and the illumination was measured at the center of each square. This value of illumination was considered to be the average for the square, and the addition of these values gave the total flux to the working plane. When the source was in a central position, the final flux to the working plane was called unity. For other positions, the values were 0.996 and 0.977. That is to say, in a room

TABLE I

k	$z_s = 2$	$z_s = 1$	$z_s = 0.6$	$z_s = 0.3$
1	1.00	1.00	1.000	1.000
2	1.09	1.04	0.996	0.967
4	1.36	1.11	0.965	0.918

where $z_s = 0.35$, the light source may be considered to be in the center of the ceiling, even when it is actually displaced, and the error will be negligible. This property is convenient in dealing with practical problems.

Final Flux Distribution

Previous sections showed that, for the range of practical interior lighting and within the necessary accuracy, the primary light distributions in a rectangular parallelepiped (flux from ceiling to floor or to walls, flux from wall to ceiling or to wall, flux from a point source to the working plane) are all functions of z only, k having negligible effect. Hence it is to be expected that with final flux distributions (including interreflections) in interior electric lighting the same relation will apply and that it will hold likewise for daylight illumination.

This section proposes to show experimentally that this anticipation is correct. The experimental results of Harrison and Anderson [11], [12] on interior lighting and the data of Meacock and Lambert [13] on lightwells will be used, also the daylighting experiments of the author.

To determine the *coefficient of utilization* by the Harrison-Anderson triple light-distribution principle, the first requisite is the *room index* which is fixed by the shape of the room. *But a precise definition of room index has been given only for square rooms.* If the side of the square is a and $x = a/b_s$, the room index $R(x,x)$ is

$$R(x,x) = \frac{a/2}{b_s} = x/2 . \tag{21}$$

In this problem, the simplifying scale is $\lambda = a$, and thus

$$z_s = \overline{b_s/a} = 1/x .$$

Therefore,

$$R(x,x)z_s = 1/2 . \tag{22}$$

[11] W. Harrison and E. A. Anderson, *Transactions* of the Illuminating Engineering Society, 11, 1916, p. 67.

[12] W. Harrison and E. A. Anderson, *Transactions* of the Illuminating Engineering Society, 15, 1920, p. 97.

[13] Meacock and Lambert, Illumination Research, Technical Paper No. 11 (London).

TABLE II—Harrison's Room Indices

a	b	Edition	$h_s = 7$	8	9	10	12	14	16	20	24
10	10	1	0.6	0.6	0.6	0.6					
		2	1.2	1.0	0.8	0.6	0.6				
	20	1	0.8	0.8	0.6	0.6	0.6	0.6			
		2	1.2	1.0	0.8	0.6	0.6	0.6			
	30	1	1.0	0.8	0.8	0.6	0.6	0.6	0.6		
		2	1.5	1.2	1.0	0.8	0.6	0.6			
	40	1	1.25	1.0	1.0	0.8	0.6	0.6	0.6	0.6	
		2	1.5	1.2	1.2	1.0	0.8	0.6	0.6	0.6	
30	30	1	2.0	2.0	1.5	1.5	1.25	1.0	1.0	0.8	0.6
		2	3.0	2.5	2.5	2.0	1.5	1.2	1.2	1.0	0.8
	60	1	2.5	2.5	2.0	2.0	1.5	1.25	1.25	1.0	0.8
		2	3.0	3.0	2.5	2.5	1.5	1.5	1.5	1.0	1.0
	100	1	3.0	2.5	2.5	2.0	2.0	1.5	1.5	1.0	0.8
		2	4.0	3.0	3.0	2.5	2.0	2.0	2.0	1.5	1.2
	140	1	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.25	1.0
		2	4.0	3.0	3.0	2.5	2.0	2.0	2.0	1.5	1.2
60	60	1	4.0	4.0	3.0	3.0	2.5	2.0	2.0	1.5	1.2
		2				4.0	3.0	2.5	2.5	2.0	1.5
	100	1	5.0	4.0	4.0	3.0	3.0	2.5	2.0	1.5	1.5
		2				4.0	3.0	3.0	3.0	2.5	2.0
	200	1	5.0	4.0	4.0	3.0	3.0	3.0	2.5	2.0	2.0
		2				4.0	3.0	3.0	3.0	2.5	2.0

That is to say, the room index of a square room may be determined by the equivalen height. Suppose that h_s is 2/3 h, as in ordinary lighting with diffusing globe luminaires. Then if $z = h/\lambda = h/a$, Eq. (21) becomes

$$R(x,x)\, z = 3/4. \tag{23}$$

The definition of Eq. (21) provides the room index for direct lighting, bu the room index of a square room with semi-indirect or indirect lighting i

$$R(x,x) = \frac{a/2}{2\,h/3}. \tag{24}$$

but in this problem $x = \dfrac{2}{2\,h/3}$ and the relation between R(x,x) and z can b expressed by Eq. (23).

Although the room index for rectangular rooms has been considered i many books, its definition is very vague. Presumably it was determined a follows. Given a rectangular room with dimensions $a \times b \times h$ and wit reflectances ρ_c and ρ_w. Take a square room $a' \times a' \times h$ with the same coeffi

TABLE III—Harrison's Room Indices

a/h_s	Edition	$b/h_s = 1.0$	2.0	3.0	5.0	7.0	10.0	20.0
1.0	1	0.6	0.6	0.6	0.8	1.0		
	2	0.6	{0.9, 0.8}	{0.6, 0.8}	{0.8, 1.0}	{0.8, 1.0}	{0.8, 1.0, 1.2}	
2.0	1		1.0	1.25	1.5	1.5	1.5	
	2		{1.0, 1.2, 1.5}	{1.2, 1.5, 2.0}	{1.2, 1.5, 2.0}	{1.2, 1.5, 2.0}	{1.5, 2.0, 2.5}	
3.0	1			1.5	1.5	2.0	2.0	2.0
	2			{1.2, 1.5, 2.0, 2.5}	{1.5, 2.5}	{1.5, 2.0, 2.5}	{1.5, 2.0, 2.5, 3.0}	{2.5, 3.0}
5.0	1				2.5	3.0	3.0	3.0
	2				{2.5, 3.0, 4.0}	{3.0, 4.0}	{3.0, 4.0, 5.0}	5.0
7.0	1					3.0	4.0	4.0
	2					5.0	5.0	5.0
10.0	1						5.0	5.0
	2						5.0	5.0

cient of utilization and with the same reflectances.   Finally, assign the room index R(x', x') of the square room to the rectangular room.   If derived in this way, the concept seems to be of little value.

Moreover, the early room indices given by Harrison (who wrote Chapter V in Cady and Dates, *Illuminating Engineering*, first edition, 1925, p. 296) differ greatly from those he gives in the second edition of the same book (1928, p. 300).   The difficulty is particularly noticeable in the second edition, where *different values are assigned to the room index when the absolute value of the room height is varied, even though the room shape is the same.*   A comparison is given in Table II.   The table shows that drastic changes, which cannot be thought of as simple revisions, have been made.   Then, too, the room indices of the second edition are not uniform, even though the relative dimensions of the room are the same.   In fact, a table for a/h_s and b/h_s (Table III) shows the existence of from two to four room indices for identical values of a/h_s and b/h_s.

This fact does not tally with Harrison's own notion of room index and imposes considerable strain on his concept.   The author's feeling is that *room index should be determined absolutely by the relative dimensions of the room.*   If additional variables must be considered, they should be introduced by means

TABLE IV—Room Indices for Rectangular Rooms, R (x, y)

a/h_s	$b/h_s = 1.0$	1.5	2.0	3.0	4.0	5.0	6.0	7.0	10.0	15.0	20.0
0.8		0.6	0.6	0.6	0.6	0.8 (0.69)	0.8	0.8	0.8	1.0 (0.76)	1.0 (0.7
1.0	0.6	0.6	0.6	0.6 (0.75)	0.8	0.8	0.8	1.0 (0.88)	1.0	1.0	1.2 (0.9
1.5		0.8	0.8	1.0	1.0	1.25	1.25	1.25	1.5 (1.30)	1.5	1.5
2.0			1.0	1.25	1.25	1.5	1.5	1.5	1.5	1.5 (1.77)	1.5 (1.8
3.0				1.5	1.5	1.5 (1.87)	2.0	2.0	2.0 (2.31)	2.0 (2.50)	2.0 (2.6
4.0					2.0	2.0	2.5	2.5	2.5 (2.85)	2.5 (3.16)	2.5 (3.3
5.0						2.5	2.5	3.0	3.0	3.0 (3.75)	3.0 (4.0
6.0							3.0	3.0	3.0 (3.75)	3.0 (4.28)	3.0 (4.6
7.0								3.0 (3.5)	4.0	4.0 (4.77)	4.0 (5.1
10.0									5.0	5.0 (6.00)	5.0 (6.6

of another factor rather than by trying to hang them onto the room inde
To illustrate, Harrison has given no consideration whatever to the spacir
of luminaires. Nor in his experiments is the mounting height of the lum
naires kept constant at $h_s = 2b/3$. Because of the existence of these defect
it can be surmised that Harrison found it necessary to make the revisions th
are found in the second edition.

A comparison of the room indices (first edition) and the simplifying sca
will now be made. If the values of the first edition are to be changed, tl
new values are indicated in parentheses in Table IV. Assume that the rel
tion,

$$h_s = 2b/3 \tag{2}$$

holds. Then if relations similar to Eqs. (22) and (23) are valid, and $x = a/l$
$y = b/h_s$, the room index for the rectangular room is

$$R(x, y) = \frac{1}{2z_s} = \frac{3}{4z} . \tag{2}$$

The values of room index calculated from Eq. (26) are indicated in pare
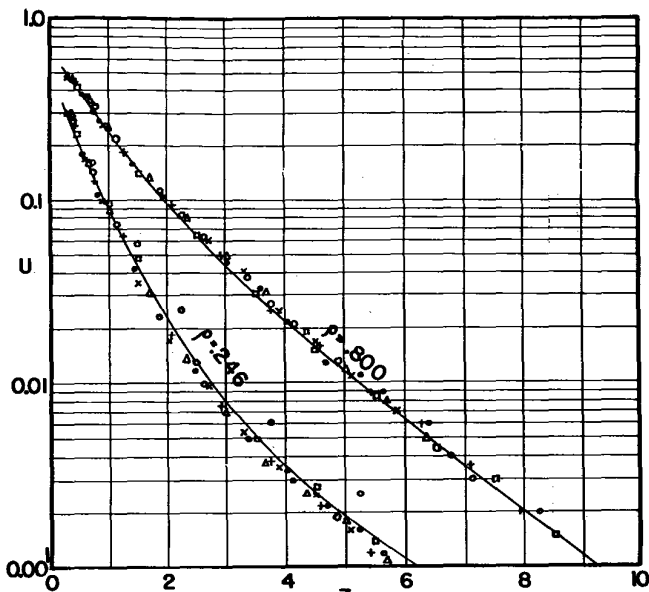theses in Table IV. For the values not enclosed in parentheses, the ca

FIG. 3—Meacock and Lambert's data for the light distribution in rectangular lightwells.   ●,
: = 0.5; Curve □, $k = 1.0$; +, $k = 1.5$; ○, $k = 2.0$; △, $k = 3.0$; x, $k = 5.0$; ●, $k = 10.0$.

:ulated results when rounded off to 0.6, 0.8, 1.0, 1.25, 1.5, 2.0, 3.0, 4.0, and
;.0 will agree with Harrison's values. A close examination of Table IV
eveals the following:

(1) Of 73 room indices, only 26 do not agree;
(2) Of these 26, seven are at the boundaries where indices change. They tally with the room
index next to the right or next to the left in the table.
(3) Of these 26 room indices that do not agree, 10 are below 1.5. The remaining 16 are above
2.0 and when they are shifted to the next higher or next lower, there will still be no con-
siderable change in the coefficient of utilization.

Thus if the room index is defined by Eq. (26), most values will be essen-
ially correct. Even in the problem of very large rooms, where discrepancies
etween Eq. (26) and experiment are comparatively frequent, the error in the
oefficient of utilization is 10 per cent or less. Hence the room index of a
ectangular room, for which the definition has always been vague, can be
efined by Eq. (26) with results that are sufficient for practical purposes.

But the room index is nothing but a stepping stone for the determination of
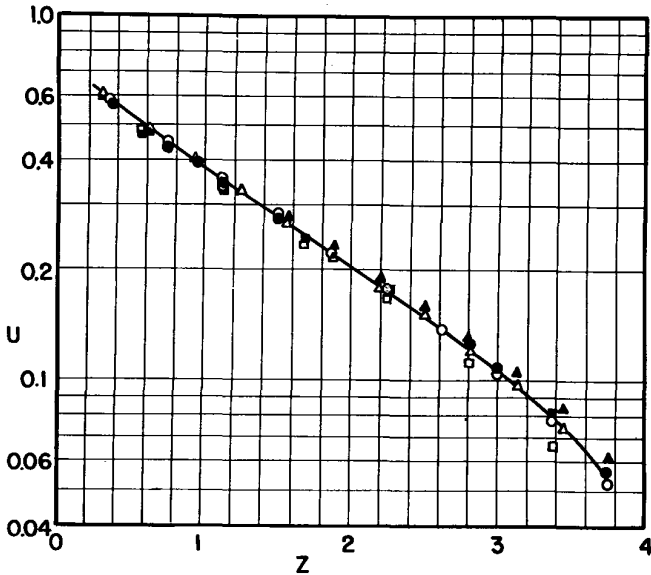he coefficient of utilization. This coefficient can be obtained directly from

FIG. 4—Author's data for light distribution in rectangular lightwells. Reflectance of walls = 0.857, depth $Z_0 = 4$. Curve, $k = 1$; $\bigcirc$, $k = 2$; $\triangle$, $k = 4$; $\square$, $k = 8$; $\bullet$, $k = \frac{1}{2}$; $\blacktriangle$, $k = \frac{1}{4}$; $\blacksquare$, $k = \frac{1}{8}$.

z_s, whose specification is exact, or by z, in which cases there is absolutely no necessity for considering the room index.

The determination of intrinsic utility factor does not require room indices. The possibility of using equivalent height instead of room index has been mentioned. The relation between z and the intrinsic utility factor was investigated for all the rooms (A, B, C, D, E, F_1, F_2, F_3, F_4)[14] measured by Harrison and Anderson. The results show that the intrinsic utility factor is determined by z only, independent of k.

The next investigation shows that the flux distribution in a lightwell, with sky as source, is also independent of k. The experimental results of Meacock and Lambert are used, as well as those of the author. Among the Meacock-Lambert data, two cases were chosen ($\rho = 0.800$ and $\rho = 0.246$) and the relation was studied between z and the final illumination along the center line of a wall. The results, expressed in terms of daylight factor y, are shown in Fig. 3.

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[14] F_1, F_2, F_3, F_4 are the four rooms cited on page 105, Table I of *Transactions* of the Illuminating Engineering Society, Vol. 15 (see Footnote 12).

In the wide range of $1 < k < 10$ and $z < 10$, all experimental points could be represented by one curve, proving that the final distribution (with interreflection) is independent of k.   The author's experimental results (Fig. 4) are similar.

The experimental details were as follows.   The room was 5.45×5.45 m (18 ft $\times$ 18 ft), the ceiling height was 3.64 m (12 ft), and the interior was equipped with four luminaires placed so that the ceiling was uniformly lighted.   Diffusing white paint of reflectance 0.80 was applied to the ceiling and walls and black paint of about 0.04 reflectance was applied to the floor. The ceiling represented the sky, and the lightwell erected in the center of the room was a model 1.8 m high.   Illumination was measured by means of a photocell, and daylight factor was determined as the ratio of the illumination on the walls of the model court to the illumination on a horizontal surface at the top of the court.   Experiments were made with reflectance of the walls of the lightwell ranging from approximately 0.15 to 0.85 but only a part of the results are presented here.

4. Conclusion

It has been shown that if the height of a room is measured by the simplifying scale, that is if the *equivalent height* is used, the treatment of flux distribution in a rectangular parallelepiped is greatly simplified.   When the floor is $a \times b$, determine the harmonic mean of a and b:

$$\lambda = \frac{2ab}{a + b}.$$

Then express the height of the rectangular parallelepiped by means of an equivalent height z:

$$z = h/\lambda.$$

Then both the primary flux distribution (without interreflections) and the final flux distribution (including interreflections) will be functions of z only, having no connection with $k = b/a$.   Thus the problem of the general rectangular parallelepiped resolves itself into the problem of a room whose dimensions are 1×1 and whose height is z: the *equivalent square room*.

Therefore, in both theoretical and experimental work, a consideration of square rooms gives all necessary information and the problem of the rectangular room is greatly simplified.   The range in which this simplification can be safely applied is

$$z \leqq 2, \, k \leqq 4$$

and the accuracy is within 10 per cent, which is close enough for practical purposes.