Interreflections in Asymmetrical Rooms

By PHILIP F. O'BRIEN

Available empirical and analytical data for the predetermination of luminance and illumination distributions are principally reported for rooms that display axial symmetry. An analytical technique for the prediction of lighting in rooms with luminous inputs and surface reflectances that depart from axial symmetry is outlined in this paper. The Luminous Analogue Computer which is a network or lumped parameter representation of the photic field is used for the numerical results reported here. A cubical room and a large room are analyzed for luminous inputs and reflectances which are non-uniform over the walls, ceiling and floor.

During the past forty years a large body of experimental and analytical data necessary to the calculation of luminances and illuminations in rooms has been prepared for the lighting designer. These design data have been reported largely for rooms whose luminous flux inputs and surface reflectances are located symmetrically with respect to a vertical axis at the room center. Both experimental and analytical studies of lighting systems have considered a uniform flux input at the ceiling and floor and a wall input which is constant at a particular height on all walls of the room. Similarly, the luminous reflectances of the floor, walls and ceiling are uniform over these areas. In order to extend the lighting design-data to rooms whose flux inputs and surface reflectances are not arranged uniformly with respect to a vertical axis of symmetry, an analytical method for the prediction of lighting in these asymmetrical rooms is described in this paper.

Lighting systems that employ asymmetrical luminous-flux inputs and reflectance distributions are widely used in modern lighting. Coves, coffers, troffers, and other forms of built-in lighting elements may be located asymmetrically. Windows, glass-block walls and their associated light controls are asymmetrical light sources. Ceilings can exhibit non-uniform reflectances associated with pipes, structural elements or built-in lighting elements, while the apparent reflectance of the region below the work plane can vary in a non-uniform manner from furniture tops to the cavities between furniture. Although the use of an overall equivalent or apparent reflectance of the ceiling and floor may be justified, the application of this technique to wall areas with reflectances that differ widely may lead to large errors in the calculation of illumination, particularly on vertical planes.

Several extensive experimental studies of flux inputs located asymmetrically at the walls and ceiling have been reported but the results of these studies cannot be extended to cover the range of lighting systems required by modern architectural practice. Daylighting data in rooms with windows located in one or two walls have been published by Griffith[1*] and others. Similarly, a non-uniform flux-input at the ceiling as represented by cove lighting has been considered in an excellent experimental study by Brown and Jones.[2] Experimental studies, even as extensive as those identified above, cannot treat all conditions of flux-input, room geometry and surface reflectance. A comprehensive analytical method for the prediction of the luminous conditions in rooms which depart from axial symmetry is clearly required for the rational design of lighting systems.

Analytical Methods

Modern analytical treatments of the distribution of radiant flux in a room or enclosure have been based on a continuity or flux-balance expression of the following form:

$$L_1 = L_{01} + \rho_1 E_1 \qquad \text{Eq. 1}$$

$$= L_{01} + \frac{\rho_1}{\pi A_1} \int_{A_1} \int_{A_n} L_n \frac{\cos\theta_1 \cos\theta_n}{r^2_{1-n}} dA_n \, dA_1$$

where:

L_1 — total average luminous emittance of

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*Superscript numbers indicate literature references.

finite surface A_1 *including* all interreflections (lumens/ft.²)

L_{01} — initial or "input" luminance of surface A_1 *excluding* interreflections (lumens/ft.²)

ρ_1 — luminous reflectance of surface A_1

E_1 — total illumination of surface A_1 including interreflections (lumens/ft.²)

L_n — total luminous emittance, including interreflections, of a surface element dA_n which may be viewed directly from A_1 (lumens/ft.²)

$\cos\theta_1 \cos\theta_n$ — cosines of the angles between the normals to surface elements dA_1 and dA_n and the ray connecting these elements

r_{1-n} — length of ray connecting the surface elements dA_1 and dA_n.

If the emittance L_n is postulated to be uniform over some finite area A_n Equation 1 may be rewritten as follows:

$$L_1 = L_{01} + \rho_1 (L_2 F_{1\to 2} + L_3 F_{1\to 3} + \ldots + L_n F_{1\to n}) \qquad \text{Eq. 2}$$

where

$L_2, L_3, \ldots$ — total luminous emittances of finite areas A_2, A_3, etc., which are viewed from A_1 (lumens/ft.²)

$$F_{1\to n} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_n} \frac{\cos\theta_1 \cos\theta_n}{r^2_{1-n}} dA_n \, dA_1$$

The quantity $F_{1\to n}$ is the shape modulus, a dimensionless measure of the shape of A_n relative to A_1. Specifically, $F_{1\to n}$ is the fraction of diffuse flux leaving finite area A_1 that is directly intercepted by finite area A_n. Although the shape modulus for finite areas has been described by Moon[3] in some detail, this parameter has not been employed in the lighting literature to the extent represented by the literature of heat-transfer. An excellent compilation of shape moduli has been prepared by Hamilton and Morgan[4] as a contribution to the analysis of radiant power-transfer systems but the data are equally applicable to the analysis of lighting systems. In a recent paper, Jones and Neidhart[5] have employed the shape modulus in the development of the "Algebraic Method" for the calculation of flux transfer in a room.

Several important contributions to lighting design have been based on Equations 1 and 2. Moon and Spencer[6] have published a comprehensive set of data describing flux distributions in symmetrical rooms. The luminous emittances reported by these authors were obtained by the solution of a set of simultaneous integral equations similar to Equation 1 but the kernel of the integrals was approximated by an exponential function. More recently, Spencer and Stakutis[7] have outlined a technique for the application of Equation 1 to asymmetrical inputs such as windows, but the method has not been extensively developed. European authors seem to favor the finite difference form of Equation 2; for example, Caracciolo[8] of Italy has published data for symmetrical rooms based on the solution of three simultaneous equations of the form of Equation 2. Centeno and Zagustin[9] of Venezuela have suggested that spaces with asymmetrical inputs and reflectance distributions may be represented by a system of equations similar to Equation 2 and an iterative process for the solution of these equations was outlined.

By the rearrangement of Equation 2, Kirchhoff's node equation for radiant-flux flow is obtained as follows (note: Reference 10 outlines a derivation of Equation 3):

$$\frac{[L_{01}/(1-\rho_1)] - L_1}{\rho_1/(1-\rho_1) A_1} = \frac{L_1 - L_2}{1/A_1 F_{1\to 2}} + \frac{L_1 - L_3}{1/A_1 F_{1\to 3}} + \ldots \frac{L_1 - L_n}{1/A_1 F_{1\to n}} \qquad \text{Eq. 3}$$

Although Equation 3 is merely another form of Equation 2, it defines an equivalent circuit or network which can be employed to describe a room with any distribution of inputs and reflectances. The application of this network method to lighting systems has recently been applied to both symmetrical and asymmetrical enclosures by O'Brien[10, 11] and Howard.[12, 13] As shown in Equation 3 the potentials of the network are the luminous emittances while the surface reflectances and shape moduli are related to network impedances which are the denominators of the fractions of Equation 3.

The Luminous Analogue Computer

An electrical analogue computer, the Luminous Analogue Computer,[14] has recently been constructed in the Department of Engineering, University of California, Los Angeles. This device is simply a non-planar network of variable electrical-resistance elements which are interconnected according to ten simultaneous equations similar to Equation 3. The schematic diagram, Fig. 1, of the Luminous Analogue Computer shows variable electrical power-supplies that are used to set a current input proportional to a luminous flux input at a surface which forms one part of the boundary of a room. If a surface does not supply a direct luminous flux to the room, the input or boundary node

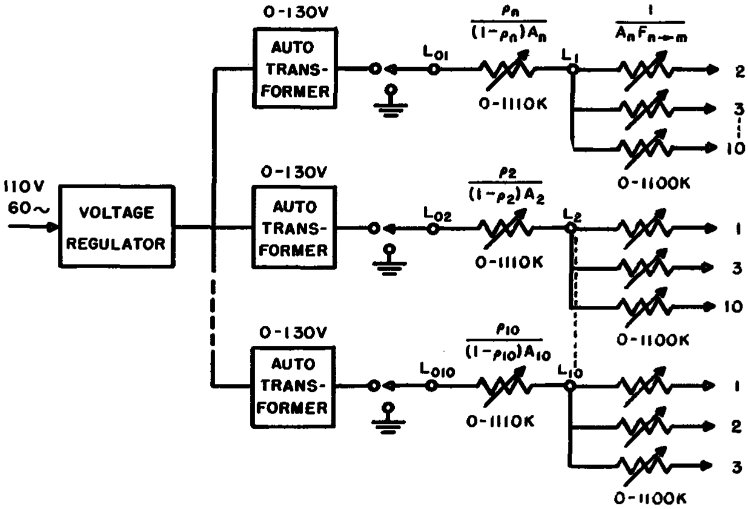


Figure 1. Luminous Analogue Computer of the Department of Engineering, University of California; dashed lines indicate a repetition of identical input units to a total of ten; variable resistance elements expressed in kilo-ohms.

representing that surface is connected to ground potential. As shown in Fig. 1 a variable resistance proportional to the surface reflectance ρ_n (note: surface reflectance resistance of area A_n equals $\rho_n/(1-\rho_n)A_n$) is located between the input node L_{on} and the total luminous emittance node L_n. This reflectance resistance is zero for a black surface and infinite for a perfectly white surface. The infinite resistance setting is obtained, in practice, by opening the circuit between the input and total emittance nodes. On the right side of Fig. 1 the variable resistors which define a room geometry are shown to be located between the total luminous emittance nodes. Because the shape modulus can vary between zero and unity the geometrical resistor (note: the geometrical resistor connecting total emittance node L_n to node L_m is equal to $1/A_n F_{n\rightarrow m}$) has a minimum value approaching $1/A$ for large surfaces in proximity and a maximum value approaching infinity for small distant surfaces.

An important feature of the analogue or network representation of a lighting system is the clear separation of room geometry from the boundary conditions of surface-reflectance and luminous input. By simply changing the setting on a variable network element the effect on the total lighting system of changing the size, reflectance, or initial luminous input of a surface can be determined.

Asymmetrical Lighting Conditions in a Cubical Room

In order to illustrate the application of the network representation to asymmetrical lighting conditions, the cubical room depicted in Fig. 2 was programmed for the Luminous Analogue Computer. The cubical room was divided into ten surface areas so that the emittance distribution from ceiling to floor on one wall might be computed. Subscripts 2 and 3, as in Fig. 2, refer to the ceiling and floor, respectively, while subscripts 1, 4 and 5 indicate the larger wall surfaces of area equal to the floor and ceiling.

The program for the solution of this problem on the computer consisted in the computation of the geometrical resistances, $1/A_n F_{n\rightarrow m}$, and the resistances proportional to surface reflectance, $\rho_n/(1-\rho_n)A_n$. Table I is a list of these resistances with the absolute values which were set on the machine. Because areas A_6 through A_{10} lie in the same plane the shape modulus between these areas is zero and the geometrical resistance is infinite as listed in the right-hand column of Table I. Resistance values on the order of kilo-ohms were chosen so that inexpensive two-watt ratio-type potentiometers could be used in the computer.

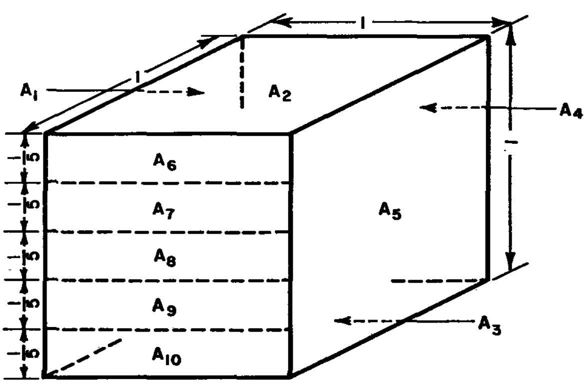


Figure 2. Cubical room with one wall divided into five equal areas; A_2 — ceiling; A_3 — floor; A_1, A_4, A_5 — full-size walls; A_6, A_7, . . . A_{10} — 1/5 wall areas.

TABLE I — Surface Reflectance and Geometrical Resistance Values for the Cubical Room of Fig. 2; Resistances Are Expressed in Kilo-Ohms.

Surface Reflectance Resistances $[\rho_n/(1-\rho_n)A_n] \times 10{,}000$			Geometrical Resistances $[1/A_nF_{n\rightarrow m}] \times 10{,}000$ Ft.^{2} Ohms $= R_{n-m}$							
Area	ρ	K-Ohms	R_{n-m}	K-Ohms	R_{n-m}	K-Ohms	R_{n-m}	K-Ohms	R_{n-m}	K-Ohms
$A_1 = 1$	0.8	40	1 - 2	50	2 - 4	50	3 - 7	416	5 - 6	282
$A_2 = 1$	0.8	40	1 - 3	50	2 - 5	50	3 - 8	311	5 - 7	230
$A_3 = 1$	0.5	10	1 - 4	50	2 - 6	129	3 - 9	190	5 - 8	218
$A_4 = 1$	0.8	40	1 - 5	50	2 - 7	190	3 - 10	129	5 - 9	230
$A_5 = 1$	0.8	40	1 - 6	282	2 - 8	311	4 - 5	50	5 - 10	282
$A_6 = .2$	0.8	200	1 - 7	230	2 - 9	416	4 - 6	287	6 - 7	∞
$A_7 = .2$	0.8	200	1 - 8	218	2 - 10	667	4 - 7	240	6 - 8	∞
$A_8 = .2$	0.8	200	1 - 9	230	3 - 4	50	4 - 8	212	⋮	⋮
$A_9 = .2$	0.8	200	1 - 10	282	3 - 5	50	4 - 9	240	⋮	⋮
$A_{10} = .2$	0.8	200	2 - 3	50	3 - 6	667	4 - 10	287	9 - 10	∞

The luminous emittance distribution on the wall from ceiling to floor is plotted in Fig. 3 for several asymmetrical lighting conditions. These cases were chosen to demonstrate the ability of the network representation to predict emittance distributions across a room surface. For Case 1 (Fig. 3) the luminous input is via a uniform ceiling panel with reflectance of 0.8 while ρ wall is 0.8 and ρ floor is 0.5. Because the floor is darker than walls and ceiling, the wall emittance near the floor is about half the value near the ceiling. As indicated on Fig. 3, Case 1 was also computed by Moon and Spencer[6] and their data are plotted as a dashed curve. The per cent difference between these analytical results is less than 10 per cent.

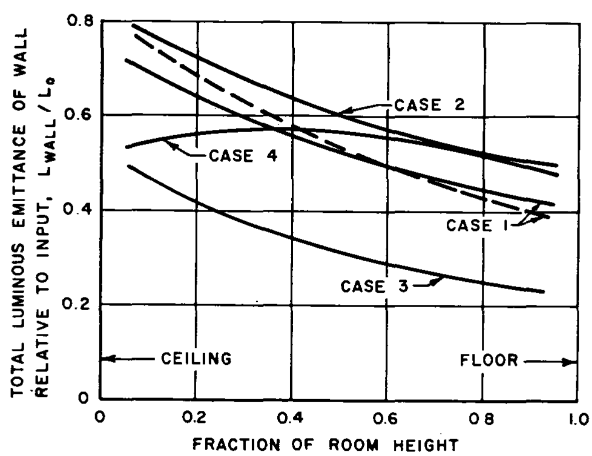


Figure 3. Luminous emittance distribution on one wall ($A_6 + A_7 + \ldots + A_{10}$) of cubical room shown in Fig. 2.

Case 1 — Ceiling input L_{02}; $\rho_2 = \rho_{1,4,5,6} \cdots {}_{10} = 0.8$; $\rho_3 = 0.5$; dashed curve for same conditions as computed by Moon and Spencer in "Lighting Design," (Addison Wesley Press, 1948) pp. 203.

Case 2 — Same as Case 1 except A_4 is perfectly white, $\rho_4 = 1.0$.

Case 3 — Same as Case 1 except A_4 is perfectly black, $\rho_4 = 0$.

Case 4 — One wall input, L_{04}; reflectances as in Case 1.

Cases 2 and 3 for the cubical room of Figs. 2 and 3 indicate the effect of changing the reflectance of one wall A_4 from zero to unity on the luminous emittance distribution at the opposite wall. In this small room the emittance at the wall mid-point is reduced by nearly 50 per cent when the opposite wall is made black.

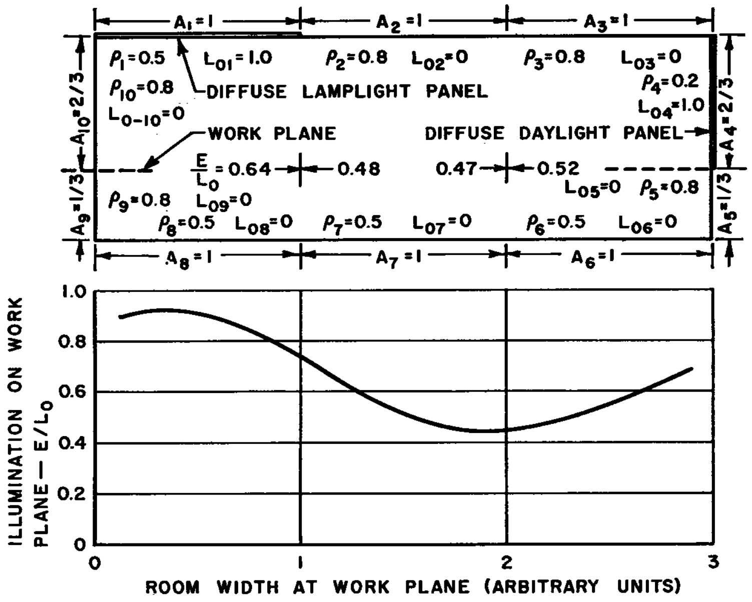
In Case 4 the luminous input is located at one wall A_4 which is opposite the wall composed of the five equal surface elements. The emittance distribution on the wall as plotted in Fig. 3 shows that the maximum emittance is located four tenths down the wall from the ceiling. This maximum point is nearer the ceiling because the floor reflectance is lower than the ceiling reflectance. With the luminous wall panel of Case 4, the luminous emittance of the opposite wall varies less than twenty per cent from floor to ceiling.

Cases 2, 3 and 4 of Fig. 3 are lighting systems with reflectances and inputs that are located asymmetrically with respect to the vertical axis of the room. Asymmetrical lighting conditions cannot be described by an analytical treatment that postulates axial symmetry. In addition, the room coefficient and similar geometrical parameters are useful for the analysis of symmetrical systems but their application to asymmetrical lighting is limited.

Asymmetrical Lighting Conditions in a Long Room

An infinitely long room whose height is one unit and width is three units (see Fig. 4) was programmed for the Luminous Analogue Computer by the same method used for the cubical room. As shown in Fig. 4 a luminous ceiling panel and a wall panel of a size and location approximating a window are the luminous inputs to the room. The ceiling panel A_6 and wall panel A_4 exhibit identical initial luminous emittances (*i.e.* $L_{01} = L_{04} = 1$) but the total flux streaming from the wall panel is two

Figure 4. Elevation view of a long room with combined daylighting and lamplighting; initial luminous emittances L_{01} and L_{04} equal unity; illuminations on vertical planes at work plane level expressed relative to initial luminous emittances.



thirds that from the ceiling panel because the panel areas are in this ratio. The floor and ceiling are each divided into three equal areas and the walls are divided into two areas representing a dado and an upper wall or window area. For the case treated in Fig. 4 the reflectances of the non-input areas of the ceiling, walls, and floor are 0.8, 0.8 and 0.5, respectively. The ceiling panel displays an effective reflectance of 0.5 and the daylighting wall-panel has a reflectance of 0.2.

The total luminous emittances of all surface areas of this long room are tabulated as Case I in Table II. Based on these emittance values, the illumination distribution across the room at work plane level was computed and is plotted in the lower half of Fig. 4. In addition to the work plane illumination distribution, the illuminations on vertical planes at work plane level are reported in Fig. 4. These values of illumination suggest that a reduction in the initial luminous emittance L_{01} of the ceiling panel will increase the uniformity of work plane illumination and will also reduce shadow forming conditions which are related to the vertical plane illuminations.

Additional asymmetrical lighting conditions in the long room are listed as Cases 2, 3 and 4 of Table II. The reduction of luminous emittances when the dado areas A_5 and A_9 are made black is shown as Case 2 of Table II. Although the dado areas are relatively small, the emittances of several room areas are reduced twenty per cent when the dado reflectances are changed from 0.8 to 0. Cases 3 and 4 of Table II show the fractional contribution of the ceiling and wall panels to the total luminous emittances of the room surfaces. The emittances of Case 3 were obtained with the input at the lamp-lighting ceiling-panel and the reflectances as shown on Fig. 3. In Case 4 the input is at the daylighting wall-panel.

Conclusions

An analytical method for the prediction of luminous emittance and illumination distributions in rooms with asymmetrical lighting conditions is out-

TABLE II — Ratio of Luminous Emittance to Input Emittance, L_n/L_o, for the Surfaces of the Long Room Shown in Fig. 4.

Case 1 — reflectances and inputs as in Fig. 4.
Case 2 — same as Case 1 except dado areas A_5 and A_9 are black, $\rho_5 = \rho_9 = 0$.
Case 3 — reflectances of Case 1 but input at lamplight panel A_1 only.
Case 4 — reflectances of Case 1 but input at daylight panel A_4 only.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
Case 1	1.21	.29	.38	1.08	.32	.26	.29	.38	.49	.58
Case 2	1.16	.23	.33	1.07	0	.21	.26	.33	0	.53
Case 3	1.14	.16	.10	.04	.18	.09	.18	.31	.29	.41
Case 4	.07	.13	.28	1.04	.14	.17	.11	.07	.20	.17

lined in this paper. The variety of light-inputs, reflectance distributions and room geometries which are employed in contemporary architectural practice points out the need for a comprehensive analytical method for the predetermination of the performance of lighting systems. Tabulations of lighting data, whether obtained from experiment or analysis, cannot meet all the needs of the progressive lighting engineer. The solution of systems of linear simultaneous equations similar to Equation 2 of this paper will allow the designer to predict the flux-transfer characteristics of rooms with unique lighting conditions. These equations may be solved with a specialized analogue computer such as the Luminous Analogue Computer or with general-purpose high-speed digital computers. These modern aids to numerical analysis have become routine tools for the engineering designer in many technical areas.

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DISCUSSION

PHELPS MEAKER:* We have reviewed this latest paper with much interest. The following thoughts may be worthy of mention.

One might wish for some tables showing the application of this approach to some typical situations. Three outstanding ones are as follows:

1. The typical variation of wall brightness from top to bottom under indirect or luminous ceiling-lighting.
2. The typical variation of wall brightness from bottom to top with downlighting.
3. The variation of ceiling brightness with filament and fluorescent indirect lighting.

An endless number of tables would not be necessary to show the order of difference resulting from the assumption of complete uniformity over these surfaces. As Mr. O'Brien has said, tables to cover every detailed practical situation are out of the question. But until a general survey has been made, covering end results, it is difficult to know how much the data are needed.

Surely the do-it-yourself aspect of this development has excited the enthusiasm of many listeners. Is this the objective of the author; that many groups start working with such a computer? If so, are detailed specifications to be made available, with instructions such that one might derive the proper dial settings and determine a result, without delving deeply into the mathematical and physical theory on which the analog network is based?

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DOMINA EBERLE SPENCER:* The author is to be congratulated on having developed a valuable tool which will be a useful supplement to the interflection method. The results obtainable with the Luminous Analogue Computer are, however, in the class of numerical solutions, that is, only one combination of room shape and reflectances can be analyzed at a time. The method employing integral equations is more powerful: a solution for an entire class of problems is obtained at one fell stroke.

The great advantage of the O'Brien analogue computer is its ability to handle *asymmetrical* problems. When the lack of symmetry is such that the mathematics is quite intractable, the Luminous Analogue Computer takes over with ease.

A limitation on the accuracy with which the O'Brien analogue computer can represent actual distributions of light is that it is incapable of handling any continuous variation. A continuous variation on a room surface must always be approximated by a finite number of uniform surfaces. Fig. 3 indicates that this discrepancy is less than ten per cent in the example cited. However, better agreement with the interflection method would have been obtained if the distribution on all four walls had been permitted to vary. As the number of elements employed in the analogue computer is increased, the accuracy of the approximation to actual distributions of light is improved.

At first sight, the O'Brien analogue computer with its facility in handling asymmetric problems, might be hailed as the key to daylighting calculations. But has it been fully realized how much continuous variation of the distribution on ceiling and floor is needed in daylighting? A clear window is not a perfectly diffusing uniform source of light, nor can it be represented by a finite combination of such sources. Viewed from ceiling and from floor, it has an entirely different character. The treatment of windows containing prismatic glass blocks, louvers, or venetian blinds is likewise complicated. It is only in the very special case of fenestration completely covered with diffusing media that the O'Brien procedure, as outlined here, is directly applicable. This omits the great majority of interesting daylighting problems.

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J. STUART FRANKLIN:* Dr. O'Brien is to be commended for doing pioneering work in the field of electrical analogies for light, the use of standard electrical network theory, and its practical application to our every-day asymmetrical lighting installations.

My primary questions relate to the use of these principles and techniques in outdoor lighting problems. The Luminous Analogue Computer seems to be applicable to the solution of many outdoor lighting problems such as underpasses, tunnels, downtown areas of streets with building fronts, etc. By making some reasonable engineering assumptions dealing with geometry of system, reflectances, etc., perhaps residential and highway lighting systems could be so programmed. Is such possible?

To make allowances for more variables, and thus approximate an actual situation to a greater degree, is it feasible perhaps, on a digital rather than analog computer, to simulate more closely actual pavement reflectance characteristics, fog, haze, etc.? How would the author suggest approaching such a problem?

My final question relates to the Luminous Analogue Computer constructed by the author. What is the approximate cost, let us say, of one of the indentical input units; the actual cost, then, depending upon the total number of input units provided?

ROBERT S. WISEMAN:** Dr. O'Brien is to be complimented for putting to use the modern tools of today's scientist to show how to solve a very practical type of problem which heretofore has been untouchable because of its complications. Certainly, many of the important, unsolved problems in illuminating engineering will be resolved when more scientists begin applying their knowledge and new techniques.

Because of the complications of even solving the physics problems associated with light transmission, only at first were the unprofessional cook book methods practical to use to calculate the illumination levels. Although the interflectance method has been available for almost a decade, I wonder how many have made use of the important contributions of Dr. Spencer and Professor Moon which permit one to calculate the brightness distribution in a room? Certainly, the alert organization which will no longer guess, but will use these modern methods, will profit from their scientific approach to illuminating engineering.

It is to be hoped that scientific techniques for the study of the physiological and psychological effects of the luminous environment will enable the illuminating engineer to approach this part of the problem with the proper professional attitude. The solutions to our problems which are to provide luminous environments for humans will not come easily, and the application of the scientific methods as done by Dr. O'Brien and Mr. Horton and Dr. Zaphyr should be encouraged for the betterment of the life of man, which should be the goal of our profession.

JOHN O. KRAEHENBUEHL:† The author of this paper, and of previous papers published elsewhere, has developed a tool which can be of practical use to the illuminating engineer. To this fact I wish to bear witness with some of the experience gathered at the laboratories of the University of Illinois through investigations made by graduate students. The tool may be relatively inexpensive to yield results within the requirements of practical application. Once the method has been assimilated, with very little effort the necessary brightness of surfaces may be determined, also the average work-surface illumination. This applies to the usual problems with which the illuminating engineer is confronted. With little effort, it may be extended to rooms where the various surfaces have different brightnesses or act as sources.

For unusual shapes and for daylighting many assumptions are made and the design of such lighting belongs to the consultant who is acquainted with analytical methods and has the experience to modify the end results of hypothetical cases and assumptions, in particular when daylight is considered. What is daylight? This in itself is such a variable quantity that any assumption will be even less accurate than the ideal cases assumed in analytical discussions.

At Illinois, studies were being made on using circuit theory in interreflection studies of enclosures when a paper by Mr. O'Brien appeared in the *Journal of the Optical Society of America* (June 1955) which answered the circuit question. Using this basic information, the work was centered upon determining the benefits of this approach to actual design in the field of brightness ratio prediction. The equipment used was a discarded suit-case network analyzer which had been replaced by digital and analogue computers of the most modern type.

In one investigation, a room was selected which was in the

TABLE I — Surface Brightness.

	6-Node			7-Node			Moon & Spencer		
	W	C	F*	W	C	F*	W	C	F*
Room E									
General Diffusing	3.58	6.72	0.81	3.70	6.27	0.80	3.63	6.89	0.80
Indirect	0.61	2.59	0.14	0.68	2.30	0.14	0.62	2.64	0.14
Direct	0.52	0.32	0.23	0.54	0.44	0.23	0.52	0.32	0.23
Room A									
General Diffusing	7.15	9.50	1.54	7.35	9.28	1.53	6.94	9.21	0.98
Room I									
General Diffusing	2.71	6.06	0.44	2.74	5.70	0.44	2.74	6.37	0.42

*W = Wall; C = Ceiling; F = Illuminated Surface.

Room Size:		Surface Reflectance:	
A	60′ x 80′ x 12′	Ceiling	80%
E	20′ x 30′ x 12′	Walls	50%
I	8′ x 10′ x 12′	Floor	10%

Equipment Distribution

Indirect $I_o \cos \theta_{90°\text{-}180°}$

Direct $I_o \cos \theta_{0°\text{-}90°}$

General Diffusing $I_o \sin \theta + I_o \cos \theta_{90°\text{-}180°} + I_o \cos \theta_{0°\text{-}90°}$

Mounted 3 feet from ceiling on 9-foot centers.

TABLE II — Coefficients of Utilization.

	Network	Moon & Spencer	Harrison & Anderson	Jones & Neidhart
Room E				
General Diffusing	0.51	0.52	0.53	0.56
Indirect	0.48	0.51	0.44	0.50
Direct	0.22 (?)	0.75	0.78	0.79
Room A				
General Diffusing	0.42 (?)	0.73	0.71	0.77
Room I				
General Diffusing	0.17	0.23	0.32	0.29

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medium size class, with equipment of standard type, a room which could readily have been either an office or a classroom — it was the latter. Here none of the brightnesses were uniform, the surfaces did not obey the cosine law; therefore, the results were determined for average conditions. The values could be determined by direct measurement and information was available for developing the proper circuit, the parameters of which were then placed upon the analyzer and illumination and brightness determined. The results checked within reasonable practical application. It is understood that one room does not prove the proposition, but it is indicative of hopeful future developments.

As graduate work,[1] three rooms were selected and three types of theoretical luminaire distributions were chosen. The results are given in Tables I and II. Table I gives a summary of the surface brightness as computed by the Moon and Spencer method, and as determined by a six and seven node network. Table II lists comparative coefficients of utilization as determined by four methods. Again, one investigation does not prove a case, and there are some values which should be carefully rechecked for the deviation does not seem reasonable, but the work does indicate what might be expected from a method which uses an electrical network to represent an illuminated enclosure.

1. Fairbanks, Kenneth E., University of Illinois.

Philip F. O'Brien:* The author wishes to thank the discussers for their helpful and stimulating comments.

The suggestion that the finite difference or network method be applied to predict the luminance distribution on the walls and ceiling of various lighting systems is made by Mr. Meaker. As shown in Fig. 3 of the paper, the luminance distribution on the walls of a particular cubical room agrees closely with the analytical results of Moon and Spencer. It seems reasonable that other rooms with symmetrical reflectances and inputs will indicate equally close agreement between the two analytical approaches to the problem. However, conditions 2 and 3 of Mr. Meaker's comments clearly define lighting systems with asymmetrical flux inputs to the room surfaces. Typical downlighting systems introduce an initial flux to the walls which is non-uniform with respect to distance down the wall and also across the wall at a particular level. In addition, lighting provided by downlights produces an initial illumination of the floor which is not uniform. Indirect lighting of the ceiling produces a non-uniform input at the ceiling that has been described for restricted cases by Iijima[1] using the Fredholm integral equation. These asymmetrical inputs are most conveniently considered by the use of finite difference equations, and the solution of typical problems is certainly required to gain some concept of the relative effect of changes in the system parameters on the luminance patterns. Some concept of the luminances associated with non-uniform input at ceiling could be obtained by applying a luminous input at surfaces A_7 and A_9 of Fig. 2 of the paper.

More detailed instructions for use of the Luminous Analogue Computer were not included in order to limit the length of the paper. Fig. 1 indicates the order of magnitude of the electrical potentials and resistance values used in the computer. In typical problems, the total current flowing in the network is less than 100 milliamperes because resistances on the order of kilo-ohms are employed. The total resistance of the network is chosen to be small compared to the input impedance of the precision vacuum tube voltmeter used to set the inputs and also to read the output potentials which are directly proportional to the total luminances. A precision resistance bridge is used to set the relatively inexpensive carbon potentiometers. Linear helical potentiometers would improve the elegance of the machine but would also increase the cost by a factor of about ten. Experience with the computer over a period of about two years indicates that a machine with a fifteen-surface capacity would be much more useful than the existing ten-surface model. A rapid means of displaying the output voltages as on an oscilloscope screen would also be a useful addition to the machine, particularly for the synthesis of luminance patterns. For example, the voltages corresponding to the total luminances of the surface elements A_6 through A_{10} of Fig. 2 could be displayed on a screen so as to produce a plot of luminance *vs.* distance down the wall of the cubical room. The reflectance distribution on the wall that would produce a uniform luminance could be quickly determined by merely changing potentiometer settings corresponding to the reflectances. That is, the desired final luminance distribution could be synthesized in terms of the surface reflectances and inputs. The author would be pleased to answer any specific question regarding the details of construction of the Luminous Analogue Computer.

*Author.

The luminous-emittance distributions and luminous transfer-functions (also termed "interflectance factors") provided by the Luminous Analogue Computer are "in the class of numerical solutions" as pointed out by Professor Spencer. However, numerical solutions are highly useful when the differential and integral equations of classical mathematical-physics resist closed-form solution because of asymmetrical geometries, non-uniform boundary conditions, and non-uniformly distributed internal sources and sinks. The value of numerical analysis is illustrated by the unprecedented development of computing devices since World War II.

The non-diffuse characteristic of flux streaming through clear glass windows or other fenestration with light control devices was not specifically treated in the paper. In 1951, Professor Spencer suggested that for non-diffuse fenestration it is possible "to calculate the actual distribution of light (including interflections) on wall, floor and ceiling, when the direct component of the distribution is known or obtainable from photometric data on a particular type of fenestration."[2] That is, the initial distribution of flux from the non-diffuse fenestration is merely regarded as a luminous input after the initial reflection from each diffuse surface element in the room. Direct flux-transfer kernels or shape moduli are available to aid the calculation of the initial distribution before interflections occur.

Mr. Franklin asks about the application of the finite-difference method to the predetermination of luminance patterns in large outdoor semi-enclosures with one boundary which is, in general, nearly black in effective reflectance. These spaces are ideally suited to analysis by this method because the length is usually many times the width and the geometry may be idealized as a semi-infinite hallway which is effectively a two-dimensional problem. One interesting problem that can be effectively solved with the Luminous Analogue Computer is the prediction of luminances at tunnel and underpass entrances where combined daylighting and lamplighting exist.

The use of digital rather than analog computers for the treatment of non-diffuse reflectances and space attenuators (*e.g.*, fog, haze) is suggested by Mr. Franklin. If non-diffuse reflectances and non-isotropic scattering and attenuation of fluids must be treated, the problem is probably best approached with the Monte Carlo technique, which is effectively applied with large scale digital computers. However, if the system properties may be idealized as diffuse and isotropic, analog or digital computers are equally applicable and the effort to formulate the problem for the two types of machines is identical. Cost of machine time, computing time, ease of repetitive calculation, storage modes, display methods and other factors related to economics will probably determine the choice of an analog or digital computer for a particular application.

In answer to Mr. Franklin's last question, the cost of the Luminous Analogue Computer with a capacity of ten surfaces was about $500 for materials and labor in the university shops. This cost does not include the vacuum tube voltmeter and resistance bridge which are necessary to the operation of the machine. Roughly, the total cost of the computer is related to the square of the number of surfaces by the following empirical relationship:

$$\text{Total Cost (dollars)} = A + 20n + 4(n)^2$$

where:

A = cost in dollars of precision vacuum tube voltmeter and resistance bridge

n = total number of surface elements that can be represented by machine.

As suggested by Messrs. Meaker and Wiseman, the alert engineering organization with a responsibility for lighting design may wish to apply the finite-difference equations to lighting systems which are not adequately described by existing analytical and experimental data. Whether relaxation methods, digital computers, analogue computers or some other aids, to numerical analysis are employed by the engineer to solve the set of finite difference equations will be determined by the knowledge and economics of the particular design situation. The two major steps in the performance of a lighting calculation are (1) the idealization and formulation of the system in terms of a set of finite difference equations, and (2) the solution of the equations for the set of luminances that constitute the problem solution. Item 2 can be accomplished by a technician with appropriate routine instructions, but Item 1 cannot be effected without a fundamental understanding of the physics and mathematics of the luminous system. Briefly, the Luminous Analogue Computer can be an effective aid to the lighting designer who is grounded in the "scientific basis of illuminating engineering."

References

1. Iijima, T.: "Theory of Inter-reflection between Two Infinite Parallel Planes," *The Japan Science Review — Series I. Engineering Sciences*, Vol. I, No. 1, pp. 9-14 (March 1949).

2. Spencer, D. E., *et al.*: "Glass Block Fenestration and the Interflection Method," ILLUMINATING ENGINEERING, Vol. XLVI, No. 9, pp. 445-447 (September 1951).

Edge Lighting — For General Illumination

This is said to be the only known large-area edge lighting installation in the country. It is one of the demonstrations of lighting techniques shown at the Chicago Lighting Institute. Individual pieces of Plexiglas, 36 inches long, 3 inches wide and ¾ inch thick have been arranged on one-inch centers to form "panels." The spacers are aluminum channel. The plenum chamber above these panels contains 48-inch, 430 ma lamps, placed crosswise to the panels, edge-lighting the plastic, which provides high levels of general illumination.