On the computation of equivalent sphere illumination

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Equivalent sphere illumination (ESI) can be calculated by an efficient technique that will make computation for many target locations in a room practical. The technique can be easily coded into a computer program, and results will be provided much faster than programs based on other computational techniques. Computations for the background and task luminance of a target are separated into two components: Part I of this paper considers the calculation of the direct components (the Appendices give a specific algorithm for their computation); Part II considers the computation of the reflected components.

Part I—Direct component computations

Calculation of the direct components begins by establishing a function to interpolate equally spaced, discrete, two variable data. The interpolating function used in the present work is a two variable trigonometric polynomial of cosines. It fits the discrete data values, $f(x_i, y_j)$, exactly (within roundoff noise) and assumes derivatives at each of the data points, defined by

$$\frac{\partial f(x_i,y_j)}{\partial x} \cong \frac{f(x_{i+1},y_j) - f(x_{i-1},y_j)}{x_{i+1} - x_{i-1}}$$

$$\frac{\partial f(x_i,y_j)}{\partial y} \cong \frac{f(x_i,y_{j+1}) - f(x_i,y_{j-1})}{y_{j+1} - y_{j-1}}$$

Other definitions for the derivatives could be chosen, but those given result in a convenient form for the function.

Using trigonometric polynomials that are constrained to fit not only data values but also derivatives, helps eliminate ripple—characteristic of trigonometric polynomials. This insures reasonable values of interpolation.

A paper presented at the Annual IES Conference, July 15–18, 1974, New Orleans, La. AUTHOR: Smith, Hinchman & Grylls Associates, Inc., Detroit, Mich.

Assume the following set of equally spaced data

$f(x_i,y_j)$ $i = 0, \ldots, M$ ($M + 1$ points of the x variable)
$j = 0, \ldots, N$ ($N + 1$ points of the y variable)

with $x_{i+1} - x_i = \Delta x$, for $i = 0, \ldots M - 1$ and $y_{j+1} - y_j = \Delta y$ for $j = 0, \ldots, N - 1$. In general, the increments Δx and Δy will not be equal. The interpolating function, $F(x,y)$ is given by

$$F(x,y) = \sum_{m=0}^{2M-1}{}' \sum_{n=0}^{2N-1}{}' a_{mn} \cos\left(\frac{2\pi}{2M} m \frac{M}{x_M} x\right) \times \cos\left(\frac{2\pi}{2N} n \frac{N}{y_N} y\right) \quad (1)$$

where a_{mn} = constant coefficient constructed from the data,
x_M = maximum value of the x variable,
y_N = maximum value of the y variable,
$0 \le x \le x_M$ and $0 \le y \le y_N$.

Since we are using a cosine polynomial, $F(x,y)$ be-

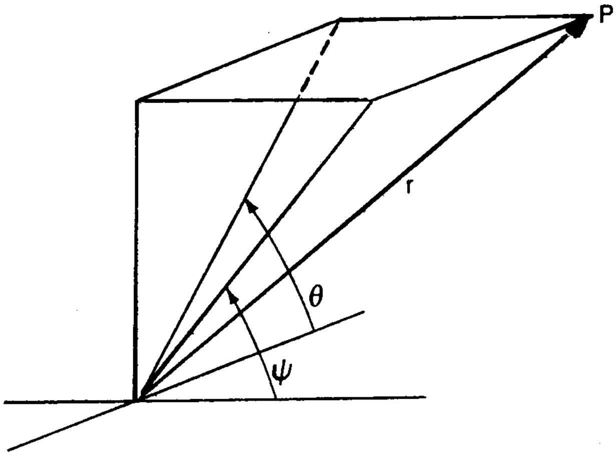


Figure 1. A modification of the perpendicular plane angular coordinate system.

haves as if the data is an even function of x_i with period $2M$, and an even function of y_j with period $2N$. The coefficients, a_{mn}, of (1) are given by

$$a_{mn} = \left[\frac{2(2M - m)}{(2M)^2} + \frac{\sin\left(\frac{\pi}{M}m\right)}{\pi(2M + 1)}\right] \times \left[\frac{2(2N - n)}{(2N)^2} + \frac{\sin\left(\frac{\pi}{N}n\right)}{\pi(2N + 1)}\right] \sum_{i=0}^{M}{}'' \sum_{j=0}^{N}{}'' \times 4f(i,j) \cos\left(\frac{\pi}{M}mi\right) \cos\left(\frac{\pi}{N}nj\right) \quad (2)$$

The double tick marks on the summations in (1) and (2) indicate that the first and last summends of the sums are halved before being added to the total.

A modification of the perpendicular plane angular coordinate system (see Fig. 1) is used throughout this procedure. The point P is located by the coordinates (θ,ψ,r). This coordinate system becomes indeterminent when θ and ψ are zero or pi.

Expressions are now derived for the task and background luminances of a visual target due to a single rectangular luminaire. The origin of the coordinate system is located at the target, and although it can assume any location, the orientation of the coordinate system is not changed. The planes in which θ and ψ are measured are always parallel and perpendicular to the same walls of the room. Fig. 2 shows two locations of the coordinate system specifying the position of Point P. Consider a particular task position, viewer orientation, and luminaire (geometric details are shown in Fig. 3). The viewing direction is called north.

A differential element of the luminaire, dA, is providing a luminous intensity, I_{dA} $(\tilde{\theta},\tilde{\psi})$, at the target. The intensity distribution of the element is specified by the coordinates $(\tilde{\theta},\tilde{\psi})$ of a perpendicular plane angular coordinate system with origin at the differential element. The incident direction to the target is specified by (θ,ψ). The Bidirectional Reflectance Distribution Function *(BRDF)* of the background is $\beta_b(\theta,\psi)$. The luminance of the background due to the element dA is given by

$$L_b = \beta_b(\theta,\psi) I_{dA}(\tilde{\theta},\tilde{\psi}) \cos(\xi)/r^2 \quad (3)$$

It is now assumed that the element of the luminaire has an intensity distribution proportional to the distribution of the entire luminaire. The proportionality constant is the ratio of the element's area to the entire luminaire's area. Thus,

$$I_{dA}(\tilde{\theta},\tilde{\psi}) = I(\tilde{\theta},\tilde{\psi})(dA/A)$$

where A = area of the entire luminaire
dA = differential element's area
I_{dA} $(\tilde{\theta},\tilde{\psi})$ = intensity of discrete element in direction $(\tilde{\theta},\tilde{\psi})$.
I $(\tilde{\theta},\tilde{\psi})$ = intensity of entire luminaire in direction $(\tilde{\theta},\tilde{\psi})$.

Substitution into (3) yields

$$L_b = \frac{1}{A}[\beta_b(\theta,\psi) I(\tilde{\theta},\tilde{\psi}) \cos(\xi)/r^2]dA \quad (4)$$

Since the plane of the luminaire is assumed parallel to the plane of the target, $\theta = \tilde{\theta}$ and $\psi = \tilde{\psi}$, which gives

$$L_b = \frac{1}{A}[\beta_b(\theta,\psi) I(\theta,\psi) \cos(\xi)/r^2]dA \quad (5)$$

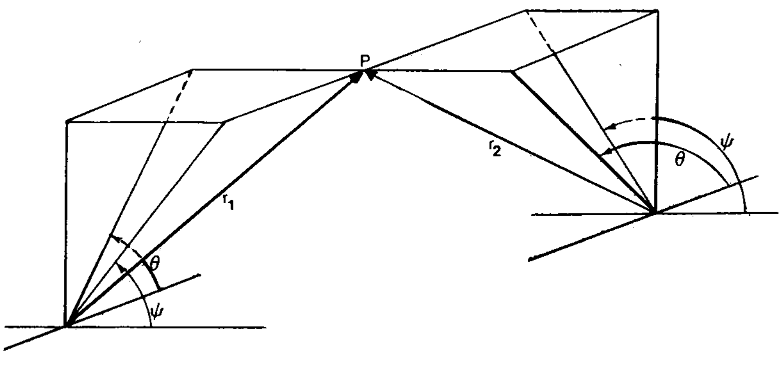


Figure 2. Two locations of the coordinate system specifying the position of Point *P*.

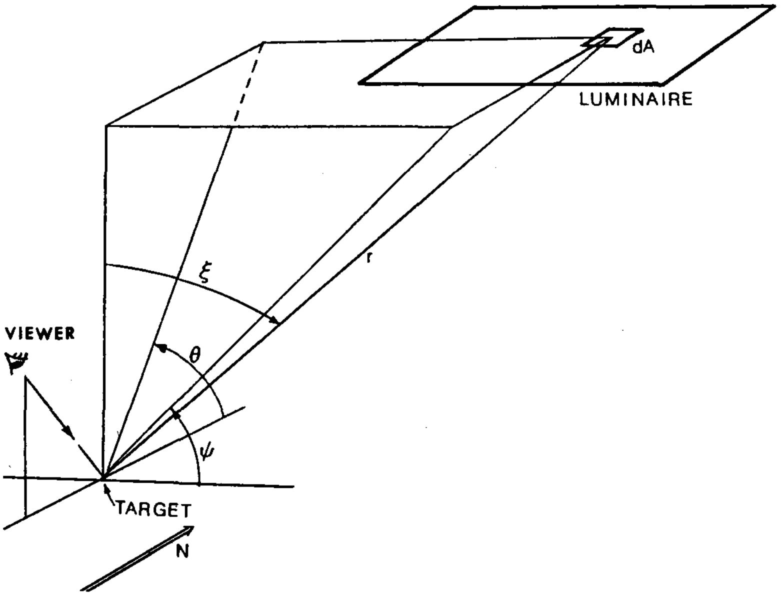


Figure 3. Geometric details of task position, viewer orientation, and luminaire.

Substitution is made for the quantities cos (ξ), $1/r^2$, and dA, in terms of the angular coordinates and (5) becomes

$$L_b = \frac{1}{A} \frac{\beta_b(\theta,\psi) I(\theta,\psi) \sin(\theta) \sin(\psi)}{[1 - \cos^2(\theta) \cos^2(\psi)]^{3/2}} \quad (6)$$

In order to account for the effects of the observer's body shadow, introduce function, $S(\theta,\psi)$, where $S(\theta,\psi) = 0$ where direction (θ,ψ) intersects the observer's body, and $S(\theta,\psi) = 1$ where direction (θ,ψ) does not intersect the observer's body. The desired effect is then produced by simply multiplying the right-hand side of (6) by $S(\theta,\psi)$. Integration of the entire quantity over the surface of the luminaire gives the background luminance produced by the entire luminaire,

$$L_b = \frac{1}{A} \int_{\Psi_1}^{\Psi_2} \int_{\Theta_1}^{\Theta_2} \times \frac{\beta_b(\theta,\psi) I(\theta,\psi) S(\theta,\psi) \sin(\theta) \sin(\psi)}{[1 - \cos^2(\theta) \cos^2(\psi)]^{3/2}} d\theta d\psi \quad (7)$$

where the limits of integration are defined in Fig. 4. Note that the limits of integration in the θ–variable are independent of those of the ψ–variable. This is the virtue of the perpendicular plane angular coordinate system.

Integral (7) is of little computational value since two of its elements, $\beta_b(\theta,\psi)$ and $I(\theta,\psi)$ are available only as sets of discrete values, $\beta_b(\theta_i,\psi_j)$ and $I(\theta_i,\psi_j)$, produced by photometric measurement. Thus, the integrand of (7) is replaced with a function $F_b(\theta,\psi)$, constructed from the available discrete values of the integrand, which allows simple integration. $F_b(\theta,\psi)$ will be the trigonometric polynomial of cosines previously described.

We choose a convenient increment size in θ and ψ, equal to $\pi/32$, and assume that the sets of data, $\beta_b(\theta_i,\psi_j)$ and $I(\theta_i,\psi_j)$ are available with this step size. The function takes the form

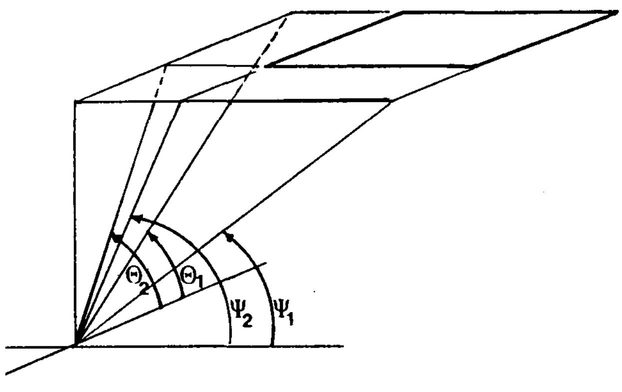
$$F_b(\theta,\psi) = \sum_{m=0}^{63} \sum_{n=0}^{63} a_{mn} \cos(m\theta) \cos(n\psi); \quad \theta,\psi \text{ in radians}, \quad (8)$$

with

$$a_{mn} = \left[\frac{2(64 - m)}{64^2} + \frac{\sin\left[\frac{\pi}{32}m\right]}{65\pi} \right] \times \left[\frac{2(64 - n)}{64^2} + \frac{\sin\left[\frac{\pi}{32}n\right]}{65\pi} \right] \sum_{i=0}^{32}{}'' \sum_{j=0}^{32}{}'' \times 4f(i,j) \cos\left[\frac{\pi}{32}mi\right] \cos\left[\frac{\pi}{32}nj\right]$$

$$m = 0, \ldots, 63 \qquad n = 0, \ldots, 63 \quad (9)$$

Figure 4. Limits of integration.



and

$$a_{m,o} \longleftarrow a_{m,o}/2, \qquad m = 1, \ldots, 63$$
$$a_{o,n} \longleftarrow a_{o,n}/2, \qquad n = 1, \ldots, 63 \quad (10)$$
$$a_{o,o} \longleftarrow a_{o,o}/4$$

The backward arrow in (10) means *replace with.* The operations on the coefficients indicated in (10) allow the summations in (8) to be taken without any of the summends being halved, thus the tick marks do not appear in (8). The data values indicated by $f(i,j)$ in (9) are the discrete values of the integrand of (7) and are expressed as functions of i and j by

$$f(i,j) = \frac{\beta_b\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) I\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) \times S\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) \sin\left(\frac{\pi}{32}i\right) \sin\left(\frac{\pi}{32}j\right)}{\left[1 - \cos\left(\frac{\pi}{32}i\right) \cos\left(\frac{\pi}{32}j\right)\right]^{3/2}} \qquad i = 1, \ldots, 31 \quad j = 1, \ldots, 31$$

Under the assumed behavior of the *BRDF* of the target's background, we define $f(i,j) = 0$ for $i = 0$, 32 or $j = 0$, 32. Function (8) is substituted into the integral (7) to give

$$L_b = \frac{1}{A} \int_{\Psi_1}^{\Psi_2} \int_{\Theta_1}^{\Theta_2} \sum_{m=0}^{63} \sum_{n=0}^{63} \times a_{mn} \cos(m\theta) \cos(n\psi) d\theta d\psi.$$

Term by term integration is allowed, which gives

$$L_b = \frac{1}{A} \sum_{m=0}^{63} \sum_{n=0}^{63} \times a_{mn} \left[\frac{\sin(m\Theta_2) - \sin(m\Theta_1)}{m}\right] \times \left[\frac{\sin(n\Psi_2) - \sin(n\Psi_1)}{n}\right] \quad (11)$$

Definition of the summends when m or n are zero is obtained from L'Hopital's rule:

$$\lim_{m \to 0} \left[\frac{\sin(m\Theta_2) - \sin(m\Theta_1)}{m}\right] = \Theta_2 - \Theta_1$$
$$\lim_{n \to 0} \left[\frac{\sin(n\Psi_2) - \sin(n\Psi_1)}{n}\right] = \Psi_2 - \Psi_1$$

Effectively then, when m or n is zero, the sine functions are to be ignored and the angles themselves are subtracted. Division by m or n is also ignored. This special handling when m or n is zero will be indicated by a slash through the summation sign. For other values of m and n, the coefficients can absorb the division by m or n, giving further permanent modifications to the coefficients:

$$a_{mn} \longleftarrow a_{mn}/mn; \; m = 1, \ldots, 63; \; n = 1, \ldots, 63$$
$$a_{o,n} \longleftarrow a_{o,n}/n; \quad n = 1, \ldots, 63$$
$$a_{m,o} \longleftarrow a_{m,o}/m; \quad m = 1, \ldots, 63$$

Finally, grouping all the operations together, Equations (12), (13), and (14) are obtained

$$L_b = \sum_{m=0}^{63} \sum_{n=0}^{63} a_{mn}[\sin(m\Theta_2) - \sin(m\Theta_1)][\sin(n\Psi_2) - \sin(n\Psi_1)] \quad (12)$$

with

$$a_{mn} = \frac{1}{A}\left[\frac{2(64-m)}{64^2} + \frac{\sin\left(\frac{\pi}{32}m\right)}{65\pi}\right]\left[\frac{2(64-n)}{64^2} + \frac{\sin\left(\frac{\pi}{32}n\right)}{65\pi}\right]\sum_{i=0}^{32}{}'' \sum_{j=0}^{32}{}'' \times 4f(i,j)\cos\left(\frac{\pi}{32}im\right)\cos\left(\frac{\pi}{32}jn\right)$$
$$a_{m,0} \longleftarrow \frac{a_{m,0}}{2m}, \qquad m = 1, \ldots, 63$$
$$a_{0,n} \longleftarrow \frac{a_{0,n}}{2n}, \qquad n = 1, \ldots, 63 \quad (13)$$
$$a_{0,0} \longleftarrow \frac{a_{0,0}}{4},$$
$$a_{mn} \longleftarrow a_{mn}/mn; \; m = 1, \ldots, 63; \; n = 1, \ldots, 63$$

and

$$f(i,j) = \frac{\beta_b\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) I\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) S\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) \sin\left(\frac{\pi}{32}i\right) \sin\left(\frac{\pi}{32}j\right)}{\left[1 - \cos^2\left(\frac{\pi}{32}i\right) \cos^2\left(\frac{\pi}{32}j\right)\right]^{3/2}} \qquad i = 1, \ldots, 31 \quad j = 1, \ldots, 31 \quad (14)$$

The a_{mn} defined in (13) and (14) are unique to particular sets of *BRDF* data, $\beta(\theta,\psi)$, and intensity distribution data, $I(\theta,\psi)$. They need be calculated only once for a particular luminaire and visual target combination. They are independent of luminaire or target position in the room. Expressions similar to (12) through (14) can be obtained for the task luminance by simply substituting $\beta_t(\theta_i,\psi_j)$ for $\beta_b(\theta,\psi_j)$ in (14). If the illumination at the task is desired, the *BRDF* data is removed altogether.

We now apply Equation (12) to a typical room condition. First assume the conditions as shown in Fig. 5. A cartesian coordinate system is established with origin at any convenient corner of the room. The x-y plane is at the floor. The target location coordinates are indicated by (X_r,Y_s,Z) where

$$r = 1, \ldots, R$$
$$s = 1, \ldots, S.$$

All target locations are assumed to have the same height, Z. The R possible x-coordinates and the S possible y-coordinates define an R x S rectangular grid of target locations. Edge locations of luminaires are specified by (x_{u1},x_{u2}) and (y_{v1},y_{v2}) where

$$u = 1, \ldots, U$$
$$v = 1, \ldots, V.$$

The U possible pairs of x-coordinates and V possible pairs of y-coordinates define a U x V rectangular grid of luminaire locations. The target is positioned at (X_1, Y_1, Z) and the background luminance due to luminaire *A1* is

$$L_b(1,1) = \sum_{m=0}^{63} \sum_{n=0}^{63} a_{mn} \times \left[\sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{11} - y_1}\right)\right) - \sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{12} - y_1}\right)\right)\right] \times \left[\sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{11} - x_1}\right)\right) - \sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{12} - x_1}\right)\right)\right] \quad (15)$$

The appropriate arctangents have been used to express the angles. The background luminance due to the other luminaires in the *A*-row (*A2,A3,...*) can be calculated similarly. All these luminaires share the same *y*-coordinates; therefore, when the expressions are added together to give the background luminance due to all the luminaires in the *A*-row, the following is obtained:

$$L_b(1,1) = \sum_{m=0}^{63} \sum_{n=0}^{63} a_{mn} \times \left[\sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{11} - y_1}\right)\right) - \sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{12} - y_1}\right)\right)\right] \times \left[\sum_{u=1}^{U} \left[\sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{u1} - x_1}\right)\right) - \sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{u2} - x_1}\right)\right)\right]\right] \quad (16)$$

Similar reasoning will lead to an expression like (16) for the *B*-row luminaires. The only difference is that the *y*-coordinates in the two arctangents in the first set of brackets are those for the *B*-row. The entire quantity in the second set of brackets is exactly as that in (16). If all the resulting "row" equations are added, we obtain

$$L_b(1,1) = \sum_{m=0}^{63} \sum_{n=0}^{63} a_{mn} \left[\sum_{v=1}^{V} \times \left[\sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{v1} - y_1}\right)\right) - \sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{v2} - y_1}\right)\right)\right]\right] \left[\sum_{u=1}^{U} \times \left[\sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{u1} - x_1}\right)\right) - \sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{u2} - x_1}\right)\right)\right]\right] \quad (17)$$

Equation (17) gives the background luminance of a target located at (X_1, Y_1, Z), due to the entire array of luminaires, *U* x *V*. A similar expression results for the task luminance. The target location is changed by substituting the appropriate target coordinates for (X_1, Y_1, Z). As will be seen below, the entire expression need not be re-evaluated for each target location.

The algebraic and computational congestion is relieved by the definition in Equation (18).

$$\left.\begin{aligned} S\theta(m,s) &= \sum_{v=1}^{V} \left[\sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{v1} - y_s}\right)\right) - \sin\left(m \cdot \tan^{-1}\left(\frac{MH - z}{y_{v2} - y_s}\right)\right)\right] \\ & \qquad m = 1, \ldots, 63 \\ S\theta(0,s) &= \sum_{u=1}^{U} \left[\tan^{-1}\left(\frac{MH - z}{y_{v1} - y_s}\right) - \tan^{-1}\left(\frac{MH - z}{y_{v2} - y_s}\right)\right] \\ S\psi(n,r) &= \sum_{v=1}^{V} \left[\sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{u1} - x_1}\right)\right) - \sin\left(n \cdot \tan^{-1}\left(\frac{MH - z}{x_{u2} - x_1}\right)\right)\right] \\ & \qquad n = 1, \ldots, 63 \\ S\psi(0,r) &= \sum_{u=1}^{U} \left[\tan^{-1}\left(\frac{MH - z}{x_{u1} - x_1}\right) - \tan^{-1}\left(\frac{MH - z}{x_{u2} - x_1}\right)\right] \end{aligned}\right\} \quad (18)$$

Using these quantities for target location (X_r, Y_s, Z), Equation (17) becomes Equation (19a).

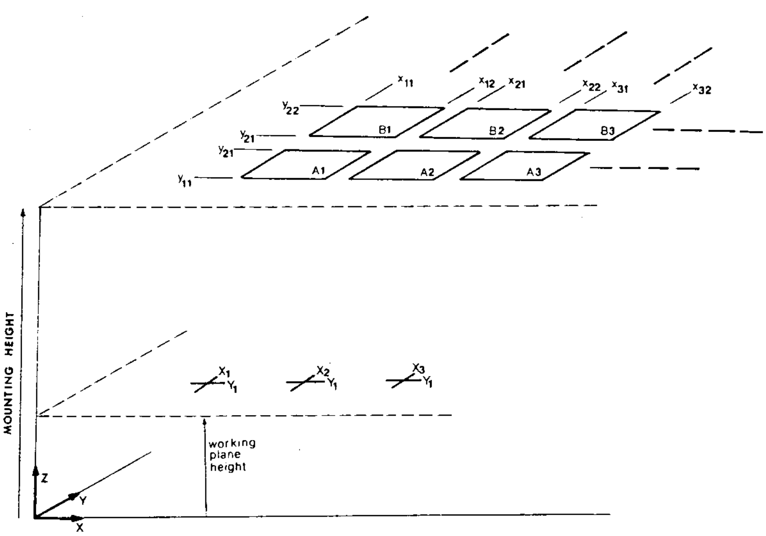


Figure 5. Application of Equation (12) to a typical room condition.

$$L_b(r,s) = \sum_{m=0}^{63} \sum_{n=0}^{63} a_{mn} \cdot S\theta(m,s) \cdot S\psi(n,r) \quad (19a)$$

The task luminance at the same location is given by

$$L_t(r,s) = \sum_{m=0}^{63} \sum_{n=0}^{63} c_{mn} \cdot S\theta(m,s) \cdot S\psi(n,r) \quad (19b)$$

Where the coefficients c_{mn} are calculated using the *BRDF* of the task. Note that the quantity $S\theta(m,s)$ is a function of y-coordinates only, and $S\psi(n,r)$ is a function of x-coordinates only. Full advantage of the separated variables (r,s) in (19a) or (19b) are shown below:

$$L_b(r,s) = \sum_{m=0}^{63} S\theta(m,s) \cdot S\tilde{\psi}(m,r) \qquad \begin{matrix} r = 1, \ldots, R \\ s = 1, \ldots, S \end{matrix}$$

where

$$S\tilde{\psi}(m,r) = \sum_{n=0}^{63} a_{mn} S\psi(n,r) \qquad \begin{matrix} m = 0, \ldots, 63 \\ r = 1, \ldots, R \end{matrix}$$

A similar scheme is used for calculating the $L_t(r,s)$:

The computation of the coefficients of (19a) and (19b) assumed the same specific viewer orientation (north) at each target location. If the luminaire is completely asymmetric, new sets of coefficients must be calculated for each viewer orientation, and Equations (19a) and (19b) re-evaluated. This is because the relationship of the *BRDF* values to candela values of the intensity distribution is changed for each viewing orientation. If the luminaire exhibits symmetry in the four octants of the hemisphere of its distribution, multiple calculation of coefficients for different viewing orientations can be avoided. Since most commercial lighting equipment exhibits this type of symmetry, an explicit development for this case is included.

Under the assumption of octant symmetry, the intensity distribution data has the following property:

$$I\left[\frac{\pi}{32}i, \frac{\pi}{32}j\right] = I\left[\frac{\pi}{32}(32 - i), \frac{\pi}{32}j\right] = I\left[\frac{\pi}{32}i, \frac{\pi}{32}(32 - j)\right] = I\left[\frac{\pi}{32}(32 - i), \frac{\pi}{32}(32 - j)\right] \qquad \begin{matrix} i = 1, \ldots, 16 \\ j = 1, \ldots, 16 \end{matrix} \quad (20)$$

The target exhibits symmetry in its *BRDF* values only about the plane containing the viewer's line of sight. For north viewing, this means that symmetry exists in the ψ coordinate. The *BRDF* data, thus, has the property for north viewing in Equation (21).

$$\beta_b(\theta_i,\psi_j) = \beta_b(\theta_i,\psi_{32-j}) \qquad \begin{matrix} i = 0, \ldots, 32 \\ j = 0, \ldots, 16 \end{matrix}$$

$$\beta_t(\theta_i,\psi_j) = \beta_t(\theta_i,\psi_{32-j}) \qquad \begin{matrix} i = 0, \ldots, 32 \\ j = 0, \ldots, 16 \end{matrix} \quad (21)$$

The $f(i,j)$ of Equation (14) are now defined using the *BRDF* and intensity distribution data which obey relations (20) and (21) respectively. Some tedious algebra will show that

$$a_{mn} = c_{mn} = 0 \qquad \begin{matrix} m = 0, \ldots, 63 \\ n = 1,3,5, \ldots, 63 \end{matrix} \quad (22)$$

The coefficients with odd n have become zero because the symmetry is assumed to be in the ψ variable of the coordinate system.

If the viewer's orientation is rotated 90 degrees, there is an east viewing direction, and in this case the coefficients a'_{mn} and c'_{mn} are such that

$$a'_{mn} = c'_{mn} = 0 \qquad \begin{matrix} m = 1,3,5, \ldots, 63 \\ n = 0, \ldots, 63 \end{matrix} \quad (23)$$

The symmetry is now in the θ variable and the coefficients with odd m are equal to zero. It is not necessary to calculate coefficients for south or west viewing. Figs. 3 and 5 show that the background and task luminances for south viewing can be calculated using (19a) and (19b) with the north viewing coefficients, a_{mn}, and $(\pi - \Theta)$ substituted for the Θ in the calculation of the $S\theta(m,s)$. Since there is symmetry in the ψ-variable, substitution of $(\pi - \psi)$ for ψ does not change the results, so the $S\psi(n,r)$ are left unmodified.

More tedious algebra will show that if $(\pi - \Theta)$ is used for Θ in the $S\theta(m,s)$, those with odd m have a reversed sign, the absolute magnitude is unchanged. The $S\theta(m,s)$ with even m are exactly as before. Thus, for south viewing, (19a) and (19b) are used with the north viewing coefficients, a_{mn}, the same $S\theta(m,s)$ and $S\psi(n,r)$, but multiplied by a factor of $(-1)^m$. Similarly, for west viewing, (19a) and (19b) are used with east viewing coefficients a'_{mn}, the same $S\theta(m,s)$ and $S\psi(n,r)$, multiplied by a factor of $(-1)^n$.

Finally, Equation (24) is obtained.

$$\begin{aligned} L_b(r,s)_{north} &= \sum_{m=0}^{63} \sum_{n=1}^{32} a_{m,2n-2} \cdot S\theta(m,s) \cdot S\psi(2n-2,r) \\ L_b(r,s)_{south} &= \sum_{m=0}^{63} \sum_{n=1}^{32} a_{m,2n-2} \cdot S\theta(m,s) \cdot S\psi(2n-2,r) \cdot (-1)^m \\ L_b(r,s)_{east} &= \sum_{n=0}^{63} \sum_{m=1}^{32} a'_{2m-2,n} \cdot S\theta(2m-2,s) \cdot S\psi(n,r) \\ L_b(r,s)_{west} &= \sum_{n=0}^{63} \sum_{m=1}^{32} a'_{2m-2,n} \cdot S\theta(2m-2,s) \cdot S\psi(n,r) \cdot (-1)^n \end{aligned} \quad (24)$$

Task luminances are obtained by substituting c_{mn} and c'_{mn} for a_{mn} and a'_{mn} in Equations (24). Note that all equations use the same $S\theta(m,s)$ and $S\psi(n,r)$. The algorithm in Appendix A groups the computation of Equations (24) so as to take advantage of the separated variables *(r,s)*.

No assumption concerning uniform spacing of rows or columns of either luminaires or target locations has been made. However, Equations (24) do assume that a luminaire exists at each intersection of the rectangular grid $U \times V$. The luminaire array shown in Fig. 6, taken as a whole, does not have this property. This array can be decomposed into two subarrays: one indicated by cross-hatched luminaires, the other indicated by open luminaires. Each of these subarrays, considered separately, has a luminaire at each intersection of its respective rectangular grid, and fulfills the requirement for use of Equations (24). In this case, Equations (24) are used twice, first using the $S\theta(m,s)$ and $S\psi(n,r)$ calculated for the first subarray, and then using the $S\theta(m,s)$ and $S\psi(n,r)$ calculated for the second subarray. The two resulting values of background luminance at each target location are added to give the total due to the entire luminaire array. The same is done for the task luminances. The coefficients a_{mn}, a'_{mn}, c_{mn}, and c'_{mn} are used in the calculations in *both* subarrays.

If the luminaire layout is comprised of two luminaire types having different intensity distributions, then the layout is decomposed into two sublayouts, each having its own set of coefficients. The computations then proceed as discussed. Two orientations of a totally asymmetric luminaire (as in a perimeter layout) must be considered as two types of luminaires, since the different orientations change the relationship between the candela values and the *BRDF* values. If two orientations of a luminaire with octant symmetry are present, then the north and east viewing coefficients of one sublayout can be used as the east and north viewing coefficients, respectively, for the other sublayout. The extra set of coefficients need not be computed.

Appendix A contains all the equations of this procedure in algorithmic form. Some important computational details have been included. Appendix B contains an algorithm for the conversion of a luminous intensity distribution from spherical coordinates to perpendicular plane angular coordinates.

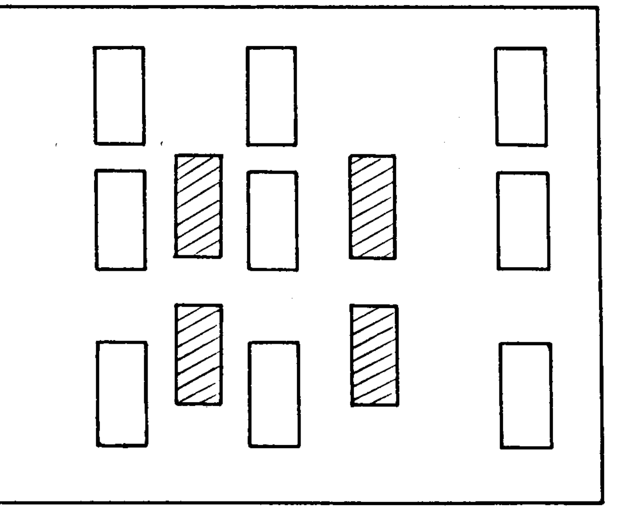


Figure 6. Diagram showing an array of luminaires. The array can be decomposed into two subarrays; one indicated by cross-hatched luminaires, the other by open luminaires.

With this discretization scheme, two opposing walls will have SxT identical elements, the other two opposing walls will have RxT identical elements, and the floor and ceiling will have RxS identical elements.

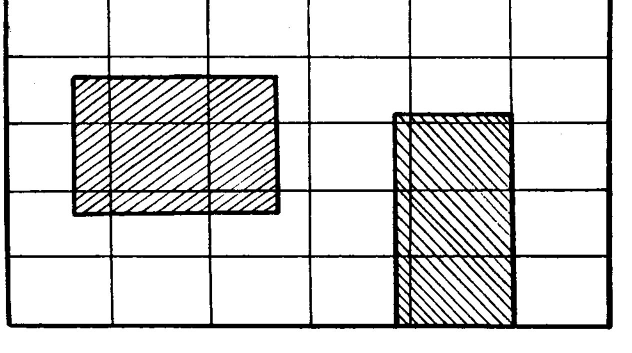
Reflectances are assigned to each element by determining the type of surface contained within the element's borders. Since the discretization in any dimension is uniform, it is possible for an element to contain a section of, say, chalkboard and room wall, each with its own reflectance. A weighted average reflectance, based on area, can be used in such cases to assign a reflectance to the element. All reflectances are assumed lambertian. Fig. 7 shows the discretization of a typical room wall. Within the limits of the lambertian reflectance assumption, the approximate luminance pattern converges to the actual continuous luminance pattern as the elements are made smaller, and, consequently, their number increased.

To calculate the initial illumination at each of the element's centers, an equation similar to (19a) is used. It will yield the illumination at any point in the room from an entire array of luminaires. The coefficients will determine whether the calcu-

Part II—Reflected component computations

This procedure is for the calculation of the reflected component of the task and background luminance of a target positioned anywhere in a room. A finite element method is used to calculate the approximate room surface's luminance pattern due to the inter-reflection of light. The manner of discretization divides the x-dimension of the room, $\bar{X}$, into R equal segments, the y-dimension, $\bar{Y}$, into S equal segments, and the z-dimension, $\bar{Z}$ into T equal segments. The coordinates (x,y,z) are cartesian, with the origin at a corner of the room and the x-y plane coincident with the floor.

Figure 7. Discretization of a typical room wall.



lated illumination is horizontal or vertical. The derivation is completely analogous to that in Part I and need not be repeated. The quantities $S\theta$ and $S\psi$ will determine the point in the room where the illumination is calculated. Their derivation is the same as in Part I and will be omitted; all notation is the same as that in Part I.

All unique $S\theta$ are defined as follows: the coordinate system used for discretization is retained; a rectangular array, $U x V$, of suspended luminaires is assumed (Fig. 8 shows the geometric details). The wall coincident with y-z plane at the coordinate origin is labeled Surface 1; the other walls are numbered consecutively, moving clockwise around the room, when seen in plan view. Thus, Surface 3 is opposite Surface 1, Surface 4 is the x-z plane at the coordinate origin, and Surface 2 is opposite Surface 4. The floor is Surface 5, and the ceiling is Surface 6.

For our present purposes we need the illumination at each element's center. Fig. 8 shows a typical pair of θ angles for an element center on Wall 1 and the first row of luminaires. These angles depend only upon the y- and z-coordinates of the element's centers (y_s, $s = 1, \ldots S$; Z_t, $t = 1, \ldots, T$) and the y-coordinates of the rows of luminaires. Given the V pairs of luminaire row coordinates, (y_{v1}, y_{v2}) $v = 1, \ldots, V$, and the element center coordinates (y_s, z_t), all $S\theta$ can be defined that are used in the calculation of illumination on Wall 1. The same $S\theta$ can be used for Wall 3, since the discretization is identical, and $S\theta$ are not functions of the x-dimension. Now, if the z_0 coordinate is set equal to zero, and the y_s-coordinate is allowed to range from y_1 to y_S, then all the $S\theta$ needed to calculate the illumination on the floor are also defined. With a z_{T+1} coordinate equal to $\bar{Z}$, those $S\theta$ needed for the ceiling computations are included. If the y_0 coordinate is set to zero and the z_t coordinate is allowed to range from z_1 to z_T, the $S\theta$ needed for Wall 4 computations is defined. Setting the y_{S+1} coordinate to $\bar{Y}$ and letting z_t range as before, will include the $S\theta$ needed for Wall 2. All unique $S\theta$ are thus given explicitly by Equation (25).

$$\left.\begin{aligned} S\theta(m,s,t) &= \sum_{v=1}^{V} \left\{ \sin\left[m \cdot \tan^{-1} \left\{ \frac{|MH - z_t|}{y_{v1} - y_s} \right\} \right] - \sin\left[m \cdot \tan^{-1} \left\{ \frac{|MH - z_t|}{y_{v2} - y_s} \right\} \right] \right\}; \; m = 1, \ldots, 63 \\ S\theta(0,s,t) &= \sum_{v=1}^{V} \left\{ \tan^{-1} \left\{ \frac{|MH - z_t|}{y_{v1} - y_s} \right\} - \tan^{-1} \left\{ \frac{MH - z_t|}{y_{v2} - y_s} \right\} \right\} \end{aligned} \right\} \begin{aligned} s &= 0, \ldots, S+1 \\ t &= 0, \ldots, T+1 \end{aligned} \tag{25}$$

where

$$y_s = \frac{\bar{Y}}{2S} + \frac{\bar{Y}}{S}(s-1) \qquad s = 1, \ldots, S$$

$$z_t = \frac{\bar{Z}}{2T} + \frac{\bar{Z}}{T}(t-1) \qquad t = 1, \ldots, T$$

$$y_o = 0; \; y_{S+1} = \bar{Y}$$

$$z_o = 0; \; z_{T+1} = \bar{Z}$$

Similar reasoning leads to the definitions in Equation (25a) for all unique $S\psi$ needed for illumination calculations in the room.

$$\left.\begin{aligned} S\psi(n,r,t) &= \sum_{u=1}^{U} \left\{ \sin\left[n \cdot \tan^{-1} \left\{ \frac{|MH - z_t|}{x_{u1} - x_r} \right\} \right] - \sin\left[n \cdot \tan^{-1} \left\{ \frac{|MH - z_t|}{x_{u2} - x_r} \right\} \right] \right\}; \; n = 1, \ldots, 63 \\ S\psi(0,r,t) &= \sum_{u=1}^{U} \left\{ \tan^{-1} \left\{ \frac{|MH - z_t|}{x_{u1} - x_r} \right\} - \tan^{-1} \left\{ \frac{|MH - z_t|}{x_{u2} - x_r} \right\} \right\} \end{aligned} \right\} \begin{aligned} r &= 0, \ldots, R+1 \\ t &= 0, \ldots, T+1 \end{aligned}$$

where

$$x_r = \frac{\bar{X}}{2R} + \frac{\bar{X}}{R}(r-1) \qquad r = 1, \ldots, R$$

$$z_t = \frac{\bar{Z}}{2T} + \frac{\bar{Z}}{T}(t-1) \qquad t = 1, \ldots, T$$

$$x_o = 0; \; x_{R+1} = \bar{X}$$

$$z_o = 0; \; z_{T+1} = \bar{Z}$$

Three sets of coefficients are needed to calculate the three conditions of illumination present in the room: horizontal, vertical one (as on Walls 1 and 3) and vertical two (as on Walls 2 and 4). These coefficients are indicated H_{mn} and V_{mn} respectively. All three sets are calculated using Equation (13), with the $f(i,j)$ defined for each set in Equation (26)

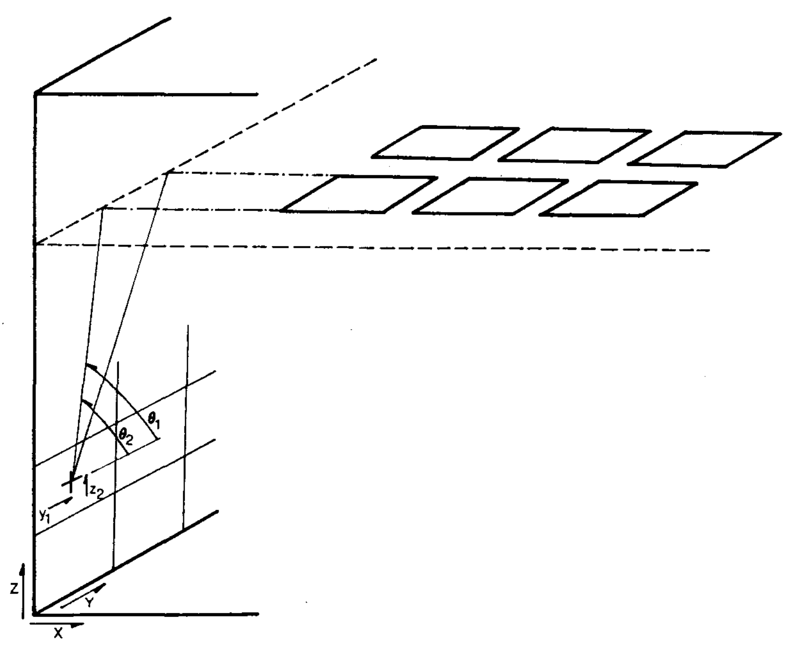


Figure 8. Geometric details for determining $S\theta$ quantities.

$$\text{for } H_{mn} \quad f(i,j) = I_{(\frac{\pi}{32}i,\ \frac{\pi}{32}j)} \cdot \frac{\sin\left(\frac{\pi}{32}i\right) \cdot \sin\left(\frac{\pi}{32}j\right)}{\left(1 - \cos^2\left(\frac{\pi}{32}i\right) \cdot \cos^2\left(\frac{\pi}{32}j\right)\right)^{3/2}} \quad \begin{matrix} j = 1, \ldots, 31 \\ j = 1, \ldots, 31 \end{matrix}$$

$$\text{for } V1_{mn} \quad f(i,j) = I_{(\frac{\pi}{32}i,\ \frac{\pi}{32}j)} \cdot \left| \frac{\sin\left(\frac{\pi}{32}i\right) \cdot \cos\left(\frac{\pi}{32}j\right)}{\left(1 - \cos^2\left(\frac{\pi}{32}i\right) \cos^2\left(\frac{\pi}{32}j\right)\right)^{3/2}} \right| \quad \begin{matrix} i = 1, \ldots, 31 \\ j = 1, \ldots, 31 \end{matrix} \tag{26}$$

$$\text{for } V2_{mn} \quad f(i,j) = I_{(\frac{\pi}{32}i,\ \frac{\pi}{32}j)} \cdot \left| \frac{\cos\left(\frac{\pi}{32}i\right) \cdot \sin\left(\frac{\pi}{32}j\right)}{\left(1 - \cos^2\left(\frac{\pi}{32}i\right) \cos^2\left(\frac{\pi}{32}j\right)\right)^{3/2}} \right| \quad \begin{matrix} i = 1, \ldots, 31 \\ j = 1, \ldots, 31 \end{matrix}$$

In all three cases, $f(i,j) = 0$ if $i = 0, 32$ or $j = 0, 32$. This is equivalent to assuming that the luminaire's distribution is such that there is no luminous intensity when θ or ψ is equal to zero or pi. This is a forced assumption because the coordinate system becomes indeterminate at these angles. The $f(i,j)$ of Equations (26) are derived exactly as those in Part I, Equations (14). Here we are dealing with illumination rather than luminance and so the *BRDF* (Bidirectional Reflectance Distribution Function) and body shadow functions are omitted.

The equations for the initial illumination at all element's centers are

$$\begin{aligned} E(1,s,t) &= \sum_{m=0}^{63} \sum_{n=0}^{63} V1_{mn} \cdot S\theta(m,s,t) \cdot S\psi(n,O,t) \\ E(2,r,t) &= \sum_{m=0}^{63} \sum_{n=0}^{63} V2_{mn} \cdot S\theta(m,S+1,t) \cdot S\psi(n,r,t) \\ E(3,s,t) &= \sum_{m=0}^{63} \sum_{n=0}^{63} V1_{mn} \cdot S\theta(m,s,t) \cdot S\psi(n,R+1,t) \\ E(4,r,t) &= \sum_{m=0}^{63} \sum_{n=0}^{63} V2_{mn} \cdot S\theta(m,O,t) \cdot S\psi(n,r,t) \\ E(5,r,s) &= \sum_{m=0}^{63} \sum_{n=0}^{63} H_{mn} \cdot S\theta(m,s,O) \cdot S\psi(n,r,O) \end{aligned} \tag{27}$$

As in Part I, Equations (27) are the general case, applicable for luminaires with a totally asymmetric intensity distribution. These results can be considerably abbreviated if the luminaire is assumed to have octant symmetry in its hemisphere of distribution. The present case simplifies further than was possible in Part I, since there is no *BRDF* function to introduce any asymmetry. Equations (25) through (27) are restated (in detailed form), making use of assumptions of symmetry [See Equations (28) through (31)]

$$\begin{aligned} E(1,s,t) &= \sum_{m=0}^{31} \sum_{n=0}^{31} V1_{mn} \cdot S\theta(m,s,t) \cdot S\psi(n,O,t) \\ E(2,r,t) &= \sum_{m=0}^{31} \sum_{n=0}^{31} V2_{mn} \cdot S\theta(m,S+1,t) \cdot S\psi(n,r,t) \\ E(3,s,t) &= \sum_{m=0}^{31} \sum_{n=0}^{31} V1_{mn} \cdot S\theta(m,s,t) \cdot S\psi(n,R+1,t) \\ E(4,r,t) &= \sum_{m=0}^{31} \sum_{n=0}^{31} V2_{mn} \cdot S\theta(m,O,t) \cdot S\psi(n,r,t) \\ E(5,r,s) &= \sum_{m=0}^{31} \sum_{n=0}^{31} H_{mn} \cdot S\theta(m,s,O) \cdot S\psi(n,r,O) \\ E(6,r,s) &= \sum_{m=0}^{31} \sum_{n=0}^{31} H_{mn} \cdot S\theta(m,s,T+1) \cdot S\psi(n,r,T+1) \end{aligned} \tag{28}$$

where

$$\left.\begin{aligned} S\theta(m,s,t) &= \sum_{v=1}^{V} \left\{ \sin\left[2m \cdot \tan^{-1}\left\{\frac{|MH - z_t|}{y_{v1} - y_s}\right\}\right] - \sin\left[2m \cdot \tan^{-1}\left\{\frac{|MH - z_t|}{y_{v2} - y_s}\right\}\right]\right\};\ m = 1, \ldots, 31 \\ S\theta(O,s,t) &= \sum_{v=1}^{V} \left\{ \tan^{-1}\left\{\frac{|MH - z_t|}{y_{v1} - y_s}\right\} - \tan^{-1}\left\{\frac{|MH - z_t|}{y_{v2} - y_s}\right\}\right\} \end{aligned}\right\} \begin{matrix} t = 0, \ldots, T+1 \\ s = 0, \ldots, S+1 \end{matrix}$$

$$\left.\begin{aligned} S\psi(n,r,t) &= \sum_{u=1}^{U} \left\{ \sin\left[2n \cdot \tan^{-1}\left\{\frac{|MH - z_t|}{x_{u1} - x_r}\right\}\right] - \sin\left[2n \cdot \tan^{-1}\left\{\frac{|MH - z_t|}{x_{u2} - x_r}\right\}\right]\right\};\ n = 1, \ldots, 31 \\ S\psi(O,r,t) &= \sum_{u=1}^{U} \left\{ \tan^{-1}\left\{\frac{|MH - z_t|}{x_{u1} - x_r}\right\} - \tan^{-1}\left\{\frac{|MH - z_t|}{x_{u2} - x_r}\right\}\right\} \end{aligned}\right\} \begin{matrix} t = 0, \ldots, T+1 \\ r = 0, \ldots, R+1 \end{matrix} \tag{29}$$

$$x_o = 0 \qquad y_o = 0$$

$$x_r = \frac{\bar{X}}{2R} + \frac{\bar{X}}{R}(r-1);\ r = 1, \ldots, R \qquad y_s = \frac{\bar{Y}}{2S} + \frac{\bar{Y}}{S}(s-1);\ s = 1, \ldots, S$$

$$x_{R+1} = \bar{X} \qquad y_{S+1} = \bar{Y}$$

$$z_o = 0$$

$$z_t = \frac{\bar{Z}}{2T} + \frac{\bar{Z}}{T}(t-1);\ t = 1, \ldots, T$$

$$z_{T+1} = \bar{Z}$$

and

$$V1_{mn} = \frac{4}{A}\left[\frac{2(32-m)}{32^2} + \frac{\sin\left(\frac{\pi}{16}m\right)}{(32.5)\pi}\right]\left[\frac{2(32-n)}{32^2} + \frac{\sin\left(\frac{\pi}{16}n\right)}{(32.5)\pi}\right] \times \sum_{i=0}^{16}{}'' \sum_{j=0}^{16}{}'' f(i,j) \cdot \cos\left(\frac{\pi}{16}mi\right) \cdot \cos\left(\frac{\pi}{16}nj\right) \quad \begin{matrix} m = 0, \ldots, 31 \\ n = 0, \ldots, 31 \end{matrix} \tag{30}$$

with

$$\begin{aligned} &V1_{mn} \longleftarrow V1_{mn}/(4 \cdot m \cdot n);\ m = 1, \ldots, 31;\ n = 1, \ldots, 31 \\ &V1_{mo} \longleftarrow V1_{mo}/(4 \cdot m),\ V1_{m,31} \longleftarrow V1_{m,31}/(2 \cdot m \cdot 62); \qquad m = 0, \ldots, 30 \\ &V1_{on} \longleftarrow V1_{on}/(4 \cdot n),\ V1_{31,n} \longleftarrow V1_{31,n}/(2 \cdot n \cdot 62); \qquad n = 1, \ldots, 30 \\ &V1_{oo} \longleftarrow V1_{oo}/4;\ V1_{o,31} \longleftarrow V1_{o,31}/(2 \cdot 62);\ V1_{31,o} \longleftarrow V1_{31,o}/(2 \cdot 62) \\ &V1_{31,31} \longleftarrow V1_{31,31}/(62 \cdot 62) \end{aligned} \tag{31}$$

The $f(i,j)$ for the coefficients $V1_{mn}$ are given in Equation (26). The coefficients $V2_{mn}$ and H_{mn} are calculated in exactly the same fashion, with the appropriate $f(i,j)$ from (26) being used.

Up to this point it has been assumed that the luminaire has no upward component in its intensity distribution (although it has not been assumed that it is recessed or surface mounted). Thus Equations (28) through (31) are used only for values of t such that z_t is less than the luminaire mounting height. The illumination at element's centers with z_t greater than the mounting height is set to zero. If the luminaire does have a downward and upward component, it has proved convenient to proceed as follows. Consider the actual luminaire to be composed of two sub-luminaires: the first with the upward distribution, and the second with the downward distribution. The two sub-luminaires have the same layout, with the first sub-luminaire pointed toward the ceiling and the second pointed toward the floor. Each sub-luminaire has its own sets of coefficients $V1_{mn}$, $V2_{mn}$, and H_{mn}. Which set is used in Equations (28) is determined by whether the z_t coordinate of the element's center is above or below the mounting height of the actual luminaire.

If the luminaire has only an upward distribution, then its set of coefficients is used in Equations (28) for element's centers with z_t coordinate greater than the mounting height. All other points are assumed to have an illumination of zero. Some savings in time can be gained by calculating the $S\theta(m,s,t)$ and $S\psi(n,r,t)$ of Equations (29) only for those values of t that will actually be needed by Equations (28). But savings are miniscule.

As discussed in Part I, if the same luminaire is used in more than one rectangular array, a set of $S\theta(m,s,t)$ and $S\psi(n,r,t)$ is calculated for each array. The same set of coefficients is used in these cases. If more than one luminaire type is used, a set of $S\theta(m,s,t)$, $S\psi(n,r,t)$, and coefficients is calculated for each.

An approximate solution technique for the radiative transfer problem is now defined, which is effective when the number of finite elements is large. Using the notation established in (28) through (31), the following quantities are defined:

$L(i,j,k)$ = final luminance (after inter-reflections) of element (j,k) of room surface i.

$\rho(i,j,k)$ = lambertian reflectance of element (j,k) of room surface i.

$C(i,j,k)(a,b,c)$ = radiative exchange factor from element (j,k) of surface i, to element (b,c) of surface a.

If the Gauss-Seidel iteration scheme is used to solve the radiative transfer equations, then at each iteration the following gives the current estimate of any element's final luminance:

$$L(i,j,k) = \rho(i,j,k)\left[E(i,j,k) + \sum_{\substack{a=1\\a\neq i}}^{6}\sum_{b=1}^{J(a)}\sum_{c=1}^{K(a)} C(i,j,k)(a,b,c)\cdot L(a,b,c)\right] \quad (32)$$

$$i = 1, \ldots, 6 \qquad j = 1, \ldots, J(i) \qquad k = 1, \ldots, K(i)$$

Room surface number a is not allowed to equal room surface number i since it is assumed that elements on the same room surface cannot see one another. The summation limits J and K are indicated as functions of room surface number since the number of elements varies with the surface. Equation (32) defines one complete iteration and is used repeatedly until the change produced in any element's luminance is sufficiently small. It gives the usual solution to the radiative transfer problem in that every element of every surface is related to every element of every surface. It has the drawback that the number of radiative exchange factors used, and the number of computer operations performed, at each iteration is proportional to the square of the number of elements.

A simplification can be derived as follows: the problem is constrained so that each element of each surface is related to each *whole* surface and its average luminance. The required radiative exchange factors $C(i,j,k)(a)$ is defined

$$C(i,j,k)(a) = \sum_{b=1}^{J(a)}\sum_{c=1}^{K(a)} C(i,j,k)(a,b,c).$$

Thus, each element of each surface is related only to every whole surface. Each iteration of the Gauss-Seidel scheme simplifies to

$$L(i,j,k) = \rho(i,j,k) \times \left[E(i,j,k) + \sum_{\substack{a=1\\a\neq i}}^{6} C(i,j,k)(a)\cdot L(a)\right] \quad (33)$$

$$i = 1, \ldots, 6 \qquad j = 1, \ldots, J(i) \qquad k = 1, \ldots, K(i)$$

Where $L(a)$ is the *current* estimate of the average luminance of surface a:

$$L(a) = \frac{1}{J(a)K(a)}\sum_{b=1}^{J(a)}\sum_{c=1}^{K(a)} L(a,b,c).$$

Near the edges of a room surface, Equation (33) is prone to calculate luminance different from those obtained with Equation (32). The differences have not proven to be important. Equation (33) is successful because the variation in luminance on a room surface is usually produced by the variation in the initial illumination, rather than variations in the manner in which light is interreflected. Equation (33) accounts for the varying initial illumination but only approximates for the variation in inter-reflected light.

Calculation of the task and background luminance of a target at any position in the room, due to the luminances of the room surface elements, can be a very clumsy process. Expressed in terms of perpendicular plane angular coordinates, the following rigorous definition is for the target's background luminance:

$$L_b(x,y) = \int_0^{\pi}\int_0^{\pi} \beta_b(\theta,\psi)L(\theta,\psi,x,y,z) \times \frac{\sin^2(\theta)\cdot\sin^2(\psi)}{(1-\cos^2(\theta)\cos^2(\psi))^2}\,d\theta\,d\psi \quad (34)$$

$L(\theta,\psi,x,y,z)$ represents the luminance seen by the target in the direction (θ,ψ), when the target (coordinate origin) is at (x,y,z). This luminance function makes the integrand of (34) a particularly nasty function of target position. A finite summation is not an improvement since an awesome amount of interpolating must be done, even for a moderate number of target positions.

It is possible to couch (34) in a form that reduces to a very simple function of x and y. Start by considering a one dimensional version of the problem. Assume a set of arbitrarily spaced data, $f(\alpha_r)$ $r = 0, \ldots, R$, and a set of equally spaced data $\beta(j)$ $j = 0, \ldots, 2N$. The sum over r of the products $f(\alpha_r)$ and $\beta(\alpha_r)$, are desired with the β-data being interpolated as necessary:

$$\sum_{r=0}^{R} f(\alpha_r)\cdot\beta(\alpha_r);\ 0 \leq \alpha \leq 2N \qquad (35)$$

A schematic representation of the relationship between $f(\alpha_r)$ and $\beta(\alpha_r)$ is shown in Fig. 9. If the f-data were spaced similarly to the β-data, a simple lagged product would give the desired result of (35). The following construction procedure is used to give the results in lagged product form. The first datum $f(\alpha_0)$ of the f-data is considered separately. A new set, f_0, is formed with $f(\alpha_0)$ as its first datum, the other data are set equal to zero and spaced like the β-data, thus

$$f_o = f(\alpha_o), 0,0,0,0, \ldots$$

for a total of $2N + 1$ values. Sum of products is:

$$\sum_{k=0}^{2N} f_o(k)\cdot\beta(k + \alpha_o) \qquad (36)$$

This is a conventional lagged product, thus:

$$(2N + 1)\sum_{n=0}^{2N} F_o(2N + 1 - n)\cdot B(n)\cdot \exp\left(i\frac{2\pi n\alpha_o}{2N + 1}\right);\ \text{where } i = (-1)^{1/2} \qquad (37)$$

$F_0(n)$ and $B(n)$ are the complex fourier coefficients of the f_0-data and β-data, respectively. The complex form is used to avoid a jungle of sines and cosines. The symbol W^j_{2N+1} will be used to indicate $\exp\ (i2\pi j/2N+1)$; $i = (-1)^{1/2}$. Equation (37) gives the product of $f(\alpha_0)$ times an interpolated value of β, $\beta(\alpha_0)$. This is true since only the first datum of the f_0 set is nonzero.

Another set is formed, f_1, with $f(\alpha_1)$ as its first datum, and the rest filled with zeros as before. The sum of products

$$\sum_{k=0}^{2N} f_1(k)\cdot\beta(k + \alpha_1)$$

has the equivalent form

$$(2N + 1)\sum_{n=0}^{2N} F_1(2N + 1 - n)\cdot B(n)\cdot W_{2N+1}{}^{n\alpha_1} \qquad (38)$$

$F_1(n)$ are the complex fourier coefficients of the f_1 data. Equation (38) gives the product $f(\alpha_1)\cdot\beta(\alpha_1)$. Proceeding similarly for the other data in the original f-data, and summing each of the results, gives an alternate form for (35):

$$\frac{1}{2N + 1}\sum_{r=0}^{R} f(\alpha_r)\cdot\beta(\alpha_r) = \sum_{n=0}^{2N} F_o(2N + 1 - n)\cdot B(n)\cdot W^{n\alpha_0} + \ldots + F_R(2N + 1 - n)\cdot B(n)\cdot W_{2N+1}{}^{n\alpha_R} = \sum_{n=0}^{2N} B(n)\cdot \left[\sum_{r=0}^{R} F_r(2N + 1 - n)\cdot W_{2N+1}{}^{n\alpha_r}\right] \qquad (39)$$

Note that the $B(n)$ are constants, independent of the data being operated upon. Now the coefficients, $F_r(n)$, are defined in the usual way

$$F_r(n) = \frac{1}{2N + 1}\sum_{k=0}^{2N} f_r(k)\cdot W_{2N+1}{}^{-kn};\ n = 0, \ldots, 2N$$

where, as above $f_r(0) = f(\alpha_r)$ and all other values are zero. Because of the zero values, this reduces to

$$F_r(n) = \frac{1}{2N + 1}\cdot f_r(O)\cdot W_{2N+1}{}^{-0} = \frac{1}{2N + 1}\cdot f_r(O) = \frac{1}{2N + 1}\cdot f(\alpha_r);\ n = 0, \ldots, 2N$$

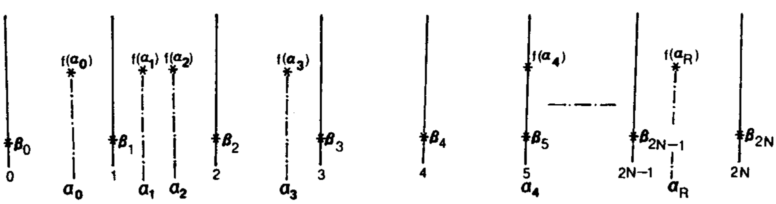


Figure 9. Schematic representation of the relationship between $f(\alpha_r)$ and $\beta(a_r)$.

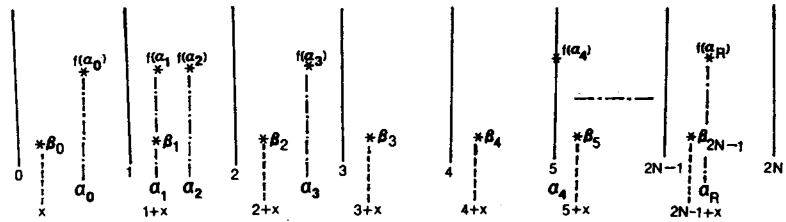


Figure 10. Relationship between *f*-data and shifted β-data.

The $F_r(n)$ are actually independent of n. The quantity in brackets in equation (39) is now grouped to define $\bar{F}(n)$.

$$\bar{F}(n) = \sum_{r=0}^{R} f(\alpha_r)\cdot W_{2N+1}{}^{n\alpha_r}. \quad (40)$$

and (39) becomes

$$\sum_{r=0}^{R} f(\alpha_r)\cdot\beta(\alpha_r) = \sum_{n=0}^{2N} B(n)\cdot\bar{F}(n)$$

If the position of the first datum of the β-data is changed from position zero to position x, this is modified to

$$\sum_{r=0}^{R} f(\alpha_r)\cdot\beta(x + \alpha_r) = \sum_{n=0}^{2N} B(n)\cdot\bar{F}(n)\cdot W_{2N+1}{}^{nx} \quad (41)$$

The relationship between the f-data and the β-data in this case is shown in Fig. 10.

Equation (40) shows that the f-data coefficients, $F(n)$ can be prepared for use in the lagged product, independent of the β-data values, or the position of the β-data with respect to the f-data. Equation (41) shows that the β-data coefficients, $B(n)$, are permanent data, independent of the f-data, or the shift value x, in the lagged product.

These results are extended to three dimensions in the following way. The f-data, $f(\alpha_r, \beta_s, \gamma_t)$ is defined in three-space along with the β-data, $\beta(l,m,n)$, where

$$r = 0, \ldots, R \qquad \ell = 0, \ldots, 2L$$
$$s = 0, \ldots, S \qquad m = 0, \ldots, 2M$$
$$t = 0, \ldots, T \qquad n = 0, \ldots, 2N$$

and

$$0 \leq \alpha_r \leq 2L;\ 0 \leq \beta_s \leq 2M;\ 0 \leq \gamma_t \leq 2N$$

The triple sum (over three-space) of the products of the f-data is desired with appropriately interpolated values of the β-data. With reasoning analogous to the one dimension case, we obtain

$$\sum_{r=0}^{R}\sum_{s=0}^{S}\sum_{t=0}^{T} f(\alpha_r,\beta_s,\gamma_t)\cdot\beta(x + \alpha_r, y + \beta_s, z + \gamma_t) = \sum_{l=0}^{2L}\sum_{m=0}^{2M}\sum_{n=0}^{2N} B(l,m,n)\cdot\bar{F}(l,m,n)\cdot W_{2L+1}{}^{lx}\cdot W_{2M+1}{}^{my}\cdot W_{2N+1}{}^{nz} \quad (42)$$

where

$$\bar{F}(l,m,n) = \sum_{r=0}^{R}\sum_{s=0}^{S}\sum_{t=0}^{T} f(\alpha_r,\beta_s,\gamma_t)\cdot W_{2L+1}{}^{l\alpha_r}\cdot W_{2M+1}{}^{m\beta_s}\cdot W_{2N+1}{}^{n\gamma_t} \qquad l = 0, \ldots, 2L;\ m = 0, \ldots, 2M;\ n = 0, \ldots, 2N \quad (43)$$

As before, the $B(l,m,n)$ are permanent data given by

$$B(l,m,n) = \frac{1}{(2L + 1)\cdot(2M + 1)\cdot(2N + 1)} \sum_{a=0}^{2L}\sum_{b=0}^{2M}\sum_{c=0}^{2N} \times \beta(a,b,c)\cdot W_{2L+1}{}^{-la}\cdot W_{2M+1}{}^{-mb}\cdot W_{2N+1}{}^{-nc} \qquad l = 0, \ldots, 2L;\ m = 0, \ldots, 2M;\ n = 0, \ldots, 2N \quad (44)$$

These results are applied to the approximate solution of equation (34) by letting the luminances of the room surfaces assume the role of the f-data and the *BRDF* of the task and background assume the role of the β-data. We assume a 21 × 21 × 11 array of *BRDF* values of the target's background.

$$\beta_b \equiv \beta_b(x,y,z) \qquad x = 0, \ldots, 20;\ y = 0, \ldots, 20;\ z = 0, \ldots, 10$$

The values are arranged so that with the target positioned at (10, 10, 0), the (x,y,z) define incident directions to the target.

Fig. 11 shows the direction of view in the y-z plane, and the *BRDF* array configuration. A permanent set of coefficients, $B_{yz}(l,m,n)$, is formed with the *BRDF* values and values of solid angle appropriate for luminances on surfaces which are parallel to the y-z plane.

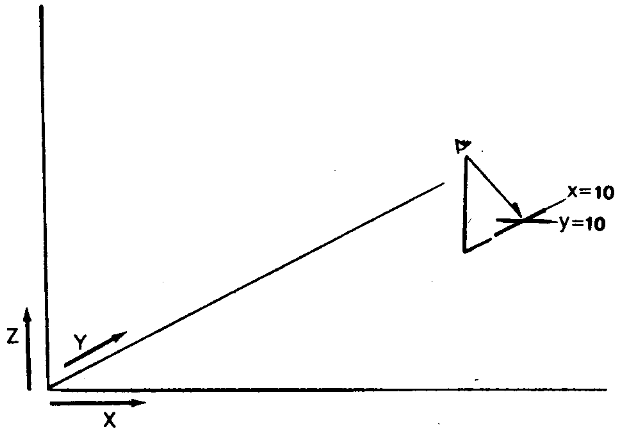


Figure 11. Diagram showing the direction of view in the *y-z* plane, and the *BRDF* array configuration.

$$\beta_{yx}(l,m,n) = \frac{1}{21\cdot21\cdot11}\sum_{x=0}^{20}\sum_{y=0}^{20}\sum_{z=0}^{10} \times$$
$$\beta_b(x,y,z)\frac{xz}{(x^2+y^2+z^2)^2} W_{21}{}^{-lx}W_{21}{}^{-my}W_{11}{}^{-nz}$$
$$l = 0, \ldots, 20 \quad m = 0, \ldots, 20 \quad n = 0, \ldots, 10 \qquad (45)$$

To complete the definition of solid angle, (45) should include the increments Δ_y, Δ_z. These will be included in the computation of the coefficients using the luminances. The $W_{21}{}^{ab}$ appearing in (45) is defined as

$$W_{21}{}^{ab} = \exp\left(\frac{2\pi ab}{21}i\right)$$

where

$$i = (-1)^{1/2}.$$

Similarly for the $W_{11}{}^{ab}$;

$$W_{11}{}^{ab} = \exp\left(\frac{2\pi ab}{11}i\right)$$

where

$$i = (-1)^{1/2}$$

Two other sets of coefficients, $B_{xz}(l,m,n)$ and $B_{xy}(l,m,n)$, are calculated in a similar way. $B_{xz}(l,m,n)$ is used for luminances on surfaces parallel to the x-z plane and $B_{xy}(l,m,n)$ used for luminances of surfaces parallel to the x-y plane. They are given by Equation (46).

$$B_{xz}(l,m,n) = \frac{1}{21\cdot21\cdot11}\sum_{x=0}^{20}\sum_{y=0}^{20}\sum_{z=0}^{10} \beta_b(x,y,z)\frac{y\times z}{(x^2+y^2+z^2)^2} W_{21}{}^{-lx}W_{21}{}^{-my}W_{11}{}^{-nz}$$
$$B_{xy}(l,m,n) = \frac{1}{21\cdot21\cdot11}\sum_{x=0}^{20}\sum_{y=0}^{20}\sum_{z=0}^{10} \beta_b(x,y,z)\frac{z^2}{(x^2+y^2+z^2)^2} W_{21}{}^{-lx}W_{21}{}^{-my}W_{11}{}^{-nz}$$
$$l = 0, \ldots, 20 \quad m = 0, \ldots, 20 \quad n = 0, \ldots, 10 \qquad (46)$$

In order for the lagged products to be properly formed, the dimensions of the room must be properly scaled. The largest of the three principle room dimension $\bar{X},\bar{Y},\bar{Z}$, designated D, is assigned the value 10, which is one half of the x or y-dimension in the *BRDF* array. Any room dimension, d, is scaled by $10/D$ before it is used in any of the equations.

Three sets of coefficients are formed using luminances on surfaces of the room which are parallel to the three principle planar orientations y-z, x-z, and x-y. The coefficients $\bar{F}_{yz}(l,m,n)$ are formed with the luminances of Walls 1 and 3. The $\bar{F}_{xz}(l,m,n)$ are formed with the luminances of Walls 2 and 4, and the $\bar{F}_{xy}(l,m,n)$ are formed with the ceiling luminances. It is assumed that the target is not illuminated by the floor. See Equation (47).

$$\bar{F}_{yz}(l,m,n) = \frac{\bar{Y}\bar{Z}}{ST}\frac{100}{D^2}\sum_{s=1}^{S}\sum_{t=t_1}^{T}[L(1,s,t) + L(3,s,t)W_{21}{}^{lx_{R+1}10/D}]W_{21}{}^{my_s 10/D}W_{11}{}^{nz_t 10/D} \quad l = 0, \ldots, 20;\ m = 0, \ldots, 20;\ n = 0, \ldots, 10$$
$$\bar{F}_{xz}(l,m,n) = \frac{\bar{X}\bar{Z}}{RT}\frac{100}{D^2}\sum_{r=1}^{R}\sum_{t=t_1}^{T}[L(4,r,t) + L(2,r,t)W_{21}{}^{my_{S+1}10/D}]W_{21}{}^{lx_r 10/D}W_{11}{}^{nz_t 10/D} \quad l = 0, \ldots, 20;\ m = 0, \ldots, 20;\ n = 0, \ldots, 10$$
$$\bar{F}_{xy}(l,m,n) = \frac{\bar{X}\bar{Y}}{RS}\frac{100}{D^2}\sum_{r=1}^{R}\sum_{s=1}^{S}[L(6,r,s)W_{11}{}^{nz_t 10/D}]W_{21}{}^{lx_r 10/D}W_{21}{}^{my_s 10/D} \quad l = 0, \ldots, 20;\ m = 0, \ldots, 20;\ n = 0, \ldots, 10 \qquad (47)$$

The coordinates x_r, y_s, and z_t are those of the room element's centers. The arrays $L(1,s,t)$, $L(2,r,t)$, $L(3,s,t)$, $L(4,r,t)$, and $L(6,r,s,)$ are those obtained with either Equation (32) or (34). The index t_1 is chosen so that z_{t1} is greater than the working plane in the room. That is, only the luminances of those elements which have centers above the plane of the task are considered in the results. The three factors $(\bar{Y}\bar{Z}/ST)(100/D^2)$, $(\bar{X}\bar{Z}/RT)(100/D^2)$, and $(\bar{X}\bar{Y}/RS)(100/D^2)$, express the increment size needed to complete the definition of the solid angle. The fact that the solid angle is defined differently for the three principle planar orientations is the reason that three sets of coefficients must be used.

Now three expressions can be formed similar to (42) and added together to give the total background luminance due to the luminous room surface. However, since the target is at a constant height throughout the room, we can eliminate the functional dependence on target height, z_{wp}, and form the combined coefficients all in one step,

$$BF(l,m) = \sum_{n=0}^{10}[B_{yz}(l,m,n)\cdot F_{yz}(l,m,n) + B_{xz}(l,m,n)\cdot F_{xz}(l,m,n) + B_{xy}(l,m,n)\cdot F_{xy}(l,m,n)]\,W^{nz_{wp}10/D}$$
$$l = 0, \ldots, 20 \quad m = 0, \ldots, 20 \qquad (48)$$

Finally, the target's background luminance at any position (X_r, Y_s) in the room is given by

$$L_b(r,s) = \sum_{l=0}^{20} \sum_{m=0}^{20} BF(l,m) \cdot W_{21}^{l(10-X_r 10/D)} W_{21}^{m(10-Y_s 10/D)} \qquad \begin{array}{l} r = 1, \ldots, R \\ s = 1, \ldots, S \end{array} \tag{49}$$

The exponential quantities in (49) have a form determined by the fact that the target is assumed located at (10, 10, 0) in the *BRDF* array of values. The only data preparation necessary to use (49) is that given by equations (47) and (48).

As can be seen from Fig. 10 and from the fact that the coordinate directions in the *BRDF* data and the room coincide, Equation (49) gives the background luminance for north viewing. Other viewing directions are obtained by using other sets of coefficients like those in (45) through (46), calculated on the basis of different viewing directions. They need be calculated only once and form a permanent data base for the procedure. Although this means a large data base, it leaves the coefficients in (47) unaltered and only the computations indicated in equations (48) and (49) need be repeated for the different viewing directions. Slightly more elaborate schemes have been devised to obtain the other viewing directions without the need of extra coefficients.

An equation for the task luminance is obtained by substituting the coefficients for the task into (48). The coefficients of (47), calculated with the room luminances, are unchanged.

At this point in the overall computation, both direct and reflected components of the target's background and task luminance are known. Values of ESI at each of the target locations (X_r, Y_s) $r = 1, \ldots, R$; $s = 1 \ldots, S$ follows from the total values of task and background luminance. The following equation for the RCS function of luminance reproduces the tabulated values of RCS within one per cent and simplifies this final step.

$$\text{RCS} = 10^{\left[\frac{2.195721274}{1 + \frac{1}{2.25(L_b)^{.2}}}\right]}; \quad 10 \le L_b \le 1600$$

The values of luminance must be in footlamberts. The inverse luminance function of RCS is

$$L_b = \left[\frac{.4444444}{\frac{2.195721274}{\text{Log(RCS)}} - 1} \right]^5$$

The resulting values of luminance are in footlamberts.

Conclusion

Two techniques based on finite fourier series have been introduced in Parts I and II. The first simplifies the computation of the direct component of illumination (or luminance produced by it) at an array of points, due to rectangular arrays of luminaires. Although this technique has been applied to the computational problem of ESI, it has much broader applications. For incandescent or high-intensity discharge luminaires without lenses, integration over a luminaire opening does not have direct physical significance. In these cases, the luminaire is considered a point source and the integration which lead to the quantities $S\theta(m,s)$ and $S\psi(n,r)$ is eliminated, and they become sums of cosines rather than sums of the difference of sines. These new $S\theta(m,s)$ and $S\psi(n,r)$ are used exactly as in Equation (19a) and (19b).

The coefficients, as used typically in Equation (19a), can be calculated to yield illumination in any required plane, so called scalar illumination, or (as was done in Part I) luminances based on a set of *BRDF* data; all at a large array of points. The computational work involved being proportional to the square root of the number of points.

The first technique's most powerful extension will probably be in the area of design synthesis. The basic equations are so simple that it is practical to drive them with an optimizing algorithm. A design requirement would be established, such as an average illumination level, an average ESI level, an array of required ESI levels, or an array of required illumination levels. The optimizer would find either the luminaire layout (given the intensity distribution) or the intensity distribution (given the layout) which best meets the design requirements. Research in this area is currently under way.

The second technique simplifies the computation of the reflected component of illumination (or luminance produced by it) at an array of points, due to the luminous interior surface of a parallelepiped. As with the direct component technique, this can be used to determine other quantities of engineering interest. The second technique has several merits. The overall computational work is only slightly dependent upon the number of discrete luminances used to approximate the continuous pattern in the room. No explicit interpolating is necessary, it is contained in the lagged products. The final result is a simple function of the two position-variables in the room. Such is the power of fourier analysis. The most useful extension will probably be an application to daylight calculations. The BRDF data for the standard pencil target, in equal increments of the perpendicular-plane angular coordinates, are available from the author.

APPENDIX A

The following shows a complete algorithm for the calculations of the direct components of background luminance (L_b) and task luminance (L_t). Luminaires with octant symmetry are assumed.

1. Establish an $R \times S$ rectangular grid of target locations:

(X_r, Y_s) $r = 1, \ldots, R$
$s = 1, \ldots, S$
Z assumed constant

2. For each luminaire type, do Steps 2.1 through 2.2.

2.1 Calculate coefficients a_{mn}, a'_{mn}, c_{mn}, and c'_{mn} using Steps 2.1.1 through 2.1.5.

2.1.1 Calculate $f_a(i,j)$, $f_{a'}(i,j)$, $f_c(i,j)$, $f_{c'}(i,j)$ using Steps 2.1.1.1 through 2.1.1.5.

2.1.1.1

$$\tilde{I}(i,j) = \frac{I\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) \sin\left(\frac{\pi}{32}i\right) \sin\left(\frac{\pi}{32}j\right)}{\left[1 - \cos^2\left(\frac{\pi}{32}i\right) \cos^2\left(\frac{\pi}{32}j\right)\right]^{3/2}} \quad \begin{matrix} i = 1, \ldots, 16 \\ j = 1, \ldots, 16 \end{matrix}$$

2.1.1.2

$$f_a(i,j) = \beta_b\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) S\left(\frac{\pi}{32}i, \frac{\pi}{32}i\right) \tilde{I}(i,j) \quad \begin{matrix} i = 1, \ldots, 16 \\ j = 1, \ldots, 16 \end{matrix}$$

$$f_a(i,j) = \beta_b\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) S\left(\frac{\pi}{32}i, \frac{\pi}{32}j\right) \times \tilde{I}(32 - i, j) \quad \begin{matrix} i = 17, \ldots, 31 \\ j = 1, \ldots, 16 \end{matrix}$$

$$f_a(i,j) = f_a(i, 32 - j) \quad \begin{matrix} i = 1, \ldots, 31 \\ j = 17, \ldots, 31 \end{matrix}$$

$$f_a(i,j) = 0 \quad \begin{matrix} i = 0, 32 \text{ or} \\ j = 0, 32 \end{matrix}$$

2.1.1.3

$$f_{a'}(i,j) = \beta_b\left(\frac{\pi}{32}j, \frac{\pi}{32}i\right) S\left(\frac{\pi}{32}j, \frac{\pi}{32}i\right) \tilde{I}(i,j) \quad \begin{matrix} i = 1, \ldots, 16 \\ j = 1, \ldots, 16 \end{matrix}$$

$$f_{a'}(i,j) = \beta_b\left(\frac{\pi}{32}j, \frac{\pi}{32}i\right) S\left(\frac{\pi}{32}j, \frac{\pi}{32}i\right) \times I(i, 32 - j) \quad \begin{matrix} i = 1, \ldots, 16 \\ j = 17, \ldots, 31 \end{matrix}$$

$$f_{a'}(i,j) = f_{a'}(32 - i, j) \quad \begin{matrix} i = 17, \ldots, 31 \\ j = 1, \ldots, 31 \end{matrix}$$

$$f_{a'}(i,j) = 0 \quad \begin{matrix} i = 0, 32 \text{ or} \\ j = 0, 32 \end{matrix}$$

2.1.1.4 $f_c(i,j)$ calculated as in Step 2.1.1.2 with β_t substituted for β_b.

2.1.1.5 $f_c(i,j)$ calculated as in Step 2.1.1.3 with β_t substituted for β_b.

2.1.2 Calculate the intermediate quantities $A(k,l)$, $A'(k,l)$, $C(k,l)$, and $C'(k,l)$ using Steps 2.1.2.1 through 2.1.2.4.

2.1.2.1

$$\bar{A}(k,j) = \sum_{i=0}^{32}{}'' f_a(i,j) \cos\left(\frac{\pi}{32}ik\right) \quad \begin{matrix} k = 0, \ldots, 32 \\ j = 0, \ldots, 32 \end{matrix}$$

then

$$A(k,l) = \sum_{j=0}^{32}{}'' \bar{A}(k,j) \cos\left(\frac{\pi}{32}j2l\right) \quad \begin{matrix} k = 0, \ldots, 32 \\ l = 0, \ldots, 31 \end{matrix}$$

2.1.2.2

$$\bar{A}'(k,j) = \sum_{i=0}^{32}{}'' f_{a'}(i,j) \cos\left(\frac{\pi}{32}k2i\right) \quad \begin{matrix} k = 0, \ldots, 31 \\ j = 0, \ldots, 32 \end{matrix}$$

then

$$A'(k,l) = \sum_{j=0}^{32}{}'' \bar{A}(k,j) \cos\left(\frac{\pi}{32}jl\right) \quad \begin{matrix} k = 0, \ldots, 31 \\ l = 0, \ldots, 32 \end{matrix}$$

The recursion $\cos(n\theta) = 2 \cdot \cos(\theta) \cdot \cos((n-1)\theta) - \cos((n-2)\theta)$ should be used in Steps 2.1.2.1 through 2.1.2.2.

2.1.2.3 $C(k,l)$ calculated as in Step 2.1.2.1, using $f_c(i,j)$.

2.1.2.4 $C'(k,l)$ calculated as in Step 2.1.2.2, using $f_{c'}(i,j)$.

2.1.3 Calculate the intermediate quantities $\Theta(i)$ using Step 2.1.3.1.

2.1.3.1

$$\Theta(i) = 2\left[\frac{2(64 - i)}{64^2} + \frac{\sin\left(\frac{\pi}{32}i\right)}{65\pi}\right] \quad i = 0, \ldots, 63$$

2.1.4 Calculate coefficients using Steps 2.1.4.1 through 2.1.4.4.

2.1.4.1

$$a(m,n) = \Theta(m) \cdot \Theta(2n) \cdot A(m,n) \cdot 1/\alpha \quad \begin{matrix} m = 0, \ldots, 32 \\ n = 0, \ldots, 31 \end{matrix}$$

$a(m,n) = \Theta(m) \cdot \Theta(2n) \cdot A(64 - m, n) \cdot 1/\alpha \quad m = 33, \ldots, 63$
$n = 0, \ldots, 31$
where α = area of luminaire

2.1.4.2 $a'(m,n) = \Theta(2m) \cdot \Theta(n) \cdot A'(m,n) \cdot 1/\alpha \quad m = 0, \ldots, 31$
$n = 0, \ldots, 32$

$a'(m,n) = \Theta(2m) \cdot \Theta(n) \cdot A'(m, 64 - n) \cdot 1/\alpha \quad m = 0, \ldots, 31$
$n = 33, \ldots, 63$

2.1.4.3 $c(m,n)$ calculated as in Step 2.1.4.1 using $C(m,n)$.

2.1.4.4 $c'(m,n)$ calculated as in Step 2.1.4.2 using $C'(m,n)$.

2.1.5 Modify all coefficient using Steps 2.1.5.1 through 2.1.5.4.

2.1.5.1 $a(m,O) \longleftarrow a(m,O)/(2 \cdot m) \quad m = 1, \ldots, 63$
$a(O,n) \longleftarrow a(O,n)/(4 \cdot n) \quad n = 1, \ldots, 31$

$a(O,O) \longleftarrow a(O,O)/4$
$a(m,n) \longleftarrow a(m,n)/(2 \cdot m \cdot n)$
$m = 1, \ldots, 63; n = 1, \ldots, 31$

2.1.5.2 $a'(m,O) \longleftarrow a'(m,O)/(4 \cdot m) \quad m = 1, \ldots, 31$
$a'(O,n) \longleftarrow a'(O,n)/(2 \cdot n) \quad n = 1, \ldots, 63$
$a'(O,O) \longleftarrow a'(O,O)/4$
$a'(m,n) \longleftarrow a'(m,n)/(2 \cdot m \cdot n)$
$n = 1, \ldots, 63; m = 1, \ldots, 31$

2.1.5.3 $c(m,n)$ modified as in Step 2.1.5.1.

2.1.5.4 $c'(m,n)$ modified as in Step 2.1.5.2.

2.2 For each sublayout of luminaire, do Steps 2.2.1 through 2.2.4.

2.2.1 Calculate $S\theta(m,s)$ and $S\psi(n,r)$ using Steps 2.2.1.1 through 2.2.1.3.

2.2.1.1 Establish the $U \times V$ rectangular grid of luminaire locations $(x_{u2}, x_{u1}), (y_{v2}, y_{v1})$

$y_{v2} = y_v + y_l/2$
$y_{v1} = y_v - y_l/2$
$x_{u2} = x_u + x_l/2$
$x_{u1} = x_u - x_l/2$

where y_v = y-coordinate of center of luminaires in vth row
y_l = y-dimension of luminaire
x_u = x-coordinate of center of luminaires in uth column
x_l = x-dimension of luminaire.

2.2.1.2 Calculate intermediate quantities $ATV2(v,s)$, $ATV1(v,s)$, $ATU2(u,r)$, and $ATU1(u,r)$ using Steps 2.2.1.2.1 through 2.2.1.2.2.

2.2.1.2.1

$$\left.\begin{aligned} ATV2(v,s) &= \tan^{-1}\left(\frac{MH - z}{y_{v1} - y_s}\right) \\ ATV1(v,s) &= \tan^{-1}\left(\frac{MH - z}{y_{v2} - y_s}\right) \end{aligned}\right\} \quad \begin{aligned} v &= 1, \ldots, V \\ s &= 1, \ldots, S \end{aligned}$$

2.2.1.2.2

$$\left.\begin{aligned} ATU2(u,r) &= \tan^{-1}\left(\frac{MH - z}{x_{u1} - x_r}\right) \\ ATU1(u,r) &= \tan^{-1}\left(\frac{MH - z}{x_{u2} - x_r}\right) \end{aligned}\right\} \quad \begin{aligned} u &= 1, \ldots, U \\ r &= 1, \ldots, R \end{aligned}$$

MH is the mounting height of the luminaires. Arctangents must be in the quadrant determined by the quotient of the arguments.

2.2.1.3 Calculate $S\theta(m,s)$ and $S\psi(n,r)$ using Steps 2.2.1.3.1 through 2.2.1.3.2

2.2.1.3.1
$$S\theta(m,s) = \sum_{v=1}^{V}\{\sin(m \cdot ATV2(v,s)) - \sin(m \cdot ATV1(v,s))\} \quad \begin{aligned} m &= 1, \ldots, 63 \\ s &= 1, \ldots, S \end{aligned}$$

$$S\theta(O,s) = \sum_{v=1}^{V}\{ATV2(v,s) - ATV1(v,s)\} \quad s = 1, \ldots, S$$

2.2.1.3.2 $S\psi(n,r) = \sum_{u=1}^{U} \{\sin(n\cdot ATU2(u,r)) - \sin(n\cdot ATU1(u,r))\} \quad \begin{matrix} n = 1, \ldots, 63 \\ r = 1, \ldots, R \end{matrix}$

$$S\psi(O,r) = \sum_{u=1}^{U} \{ATU2(u,r) - ATU1(u,r)\} \quad r = 1, \ldots, R$$

The recursion $\sin(n\theta) = 2\cdot\cos(\theta)\cdot \sin((n-1)\theta) - \sin((n-2)\theta)$ should be used in Steps 2.2.1.3.1 through 2.2.1.3.2.

2.2.2 Calculate L_b at all target locations in four viewing direction: east, north, south, and west.

2.2.2.1 Calculate intermediate quantities $SA\theta_e(n,s)$, $SA\theta_o(n,s)$, $SA\psi_o(m,r)$ and $SA\psi_e(m,r)$

2.2.2.1.1 $SA\theta_o(n,s) = \sum_{m=1}^{32} a(2m-1,n)\cdot S\theta(2m-1,s)$

$$SA\theta_e(n,s) = \sum_{m=1}^{32} a(2m-2,n)\cdot S\theta(2m-2,s)$$

$$\begin{cases} n = 0, \ldots, 31 \\ s = 1, \ldots, S \end{cases}$$

2.2.2.1.2 $SA\psi_o(m,r) = \sum_{n=1}^{32} a'(m, 2n-1)\cdot S\psi(2n-1,r)$

$$SA\psi_e(m,r) = \sum_{n=1}^{32} a'(m, 2n-2)\cdot S\psi(2n-2,r)$$

$$\begin{cases} m = 0, \ldots, 31 \\ r = 1, \ldots, R \end{cases}$$

2.2.2.2 Calculate background luminances

$$L_b(r,s)_{\text{north}} = \sum_{n=0}^{31} [SA\theta_e(n,s) + SA\theta_o(n,s)]S\psi(2n,r)$$

$$L_b(r,s)_{\text{south}} = \sum_{n=0}^{31} [SA\theta_e(n,s) - SA\theta_o(n,s)]S\psi(2n,r)$$

$$L_b(r,s)_{\text{east}} = \sum_{m=0}^{31} [SA\psi_e(m,r) + SA\psi_o(m,r)]S\theta(2m,s)$$

$$L_b(r,s)_{\text{west}} = \sum_{m=0}^{31} [SA\psi_e(m,r) - SA\psi_o(m,r)]S\theta(2m,s)$$

The addition and subtraction of the odd and even components in these equations has the same effect as the $(-1)^n$ and $(-1)^m$ factors in Equations (24) in the text.

2.2.3 Calculate task luminances, L_t, at all target locations in four viewing directions.

2.2.3.1 Same as Step 2.2.2.1 with $c(m,n)$ and $c'(m,n)$ used in place of $a(m,n)$ and $a'(m,n)$.

2.2.3.3 Exactly as Step 2.2.2.2.

2.2.4 Add values of $L_b(r,s)$ and $L_t(r,s)$ to those previously calculated (if any). This step results if there is more than one sublayout for a type of luminaire, or if there is more than one type of luminaire.

APPENDIX B

The following is an algorithm for conversion of an octant symmetric luminous intensity distribution from spherical coordinates to perpendicular plane angular coordinates.

1. Establish interpolating function $\vartheta(\phi,\xi)$, in spherical coordinates of the discrete values of $I(\phi,\xi)$. Data assumed to be in five planes

$(\phi = \frac{\pi}{8}t;\ t = 0, \ldots, 4)$, 19 values in each

$(\xi = \frac{\pi}{36}u;\ u = 0, \ldots, 18)$.

1.1 Calculate coefficients $b(j,k)$ of function $\vartheta(\phi,\xi)$ using Steps 1.1.1 through 1.1.4.

1.1.1 Calculate intermediate quantities $M(j,k)$

$$M(j,k) = 4\left[\frac{2(9-j)}{9^2} + \frac{\sin\left(\frac{2\pi}{9}j\right)}{10\pi}\right] \times$$

$$\left[\frac{2(37-k)}{37^2} + \frac{\sin\left(\frac{2\pi}{37}k\right)}{38\pi}\right] \quad \begin{matrix} j = 0, \ldots, 8 \\ k = 0, \ldots, 36 \end{matrix}$$

1.1.2 Calculate intermediate quantities $A(m,n)$

$$\widetilde{A}(m,n) = \sum_{t=0}^{4}{}' I\left(\frac{\pi}{8}t, \frac{\pi}{36}u\right) \times \cos\left(\frac{2\pi}{9}mt\right) \quad \begin{matrix} u = 0, \ldots, 18 \\ m = 0, \ldots, 4 \end{matrix}$$

then $$A(m,n) = \sum_{u=0}^{18}{}' \widetilde{A}(m,u) \times \cos\left(\frac{2\pi}{37}nu\right) \quad \begin{matrix} m = 0, \ldots, 4 \\ n = 0, \ldots, 18 \end{matrix}$$

The tick marks indicate halving of the first summend.

1.1.3 Calculate coefficients

$$b(j,k) = M(j,k) \cdot A(j,k) \quad \begin{matrix} j = 0, \ldots, 4 \\ k = 0, \ldots, 18 \end{matrix}$$

$$b(j,k) = M(j,k) \cdot A(9-j,k) \quad \begin{matrix} j = 5, \ldots, 8 \\ k = 0, \ldots, 18 \end{matrix}$$

$$b(j,k) = M(j,k) \cdot A(j,37-k) \quad \begin{matrix} j = 0, \ldots, 4 \\ k = 19, \ldots, 36 \end{matrix}$$

$$b(j,k) = M(j,k) \cdot A(9-j,37-k) \quad \begin{matrix} j = 5, \ldots, 8 \\ k = 19, \ldots, 36 \end{matrix}$$

1.1.4 Modify coefficients

$$b(O,k) \leftarrow b(O,k)/2 \quad k = 1, \ldots, 36$$

$$b(j,O) \leftarrow b(j,O)/2 \quad j = 1, \ldots, 8$$

$$b(O,O) \leftarrow b(O,O)/4$$

2. Interpolate values of luminous intensity at equal increments of the variables in the perpendicular plane angular coordinate system, using Steps 2.1 through 2.2.

2.1 Express values of spherical coordinates (ϕ,ξ) in terms of equal increments of perpendicular plane angular coordinates, (θ_m, ψ_n). The $\phi = 0$ plane is assumed coincident with plane in which θ is measured.

2.1.1 $$\phi(m,n) = \tan^{-1}\left[\cot\left(\frac{\pi}{32}n\right) \Big/ \cot\left(\frac{\pi}{32}m\right)\right] \quad \begin{matrix} n = 1, \ldots, 31 \\ m = 1, \ldots, 31 \end{matrix}$$

2.1.2 $$\xi(m,n) = \tan^{-1}\left[\left\{\cot^2\left(\frac{\pi}{32}m\right) + \cot^2\left(\frac{\pi}{32}n\right)\right\}^{1/2}\right] \quad \begin{matrix} n = 1, \ldots, 31 \\ m = 1, \ldots, 31 \end{matrix}$$

2.2 Interpolate values of candlepower, $I(\theta_i, \psi_j)$, using

$$I(\theta_m,\psi_n) = \sum_{j=0}^{8} \sum_{k=0}^{36} b(j,k) \times \cos\left(\frac{16}{9}j\phi(m,n)\right) \cos\left(\frac{72}{37}k\xi(m,n)\right)$$

2.2.1 Calculate intermediate quantities $\widetilde{I}(j,m,n)$

$$\widetilde{I}(j,m,n) = \sum_{k=0}^{36} b(j,k) \cos\left(\frac{72}{37}k\xi(m,n)\right) \quad \begin{matrix} j = 0, \ldots, 8 \\ m = 1, \ldots, 31 \\ n = 1, \ldots, 31 \end{matrix}$$

2.2.2 Calculate values of luminous intensity $I(\theta_m,\psi_n)$

$$I(\theta_m,\psi_n) = \sum_{j=0}^{8} \widetilde{I}(j,m,n) \cos\left(\frac{16}{9}j\phi(m,n)\right) \quad \begin{matrix} m = 1, \ldots, 31 \\ n = 1, \ldots, 31 \end{matrix}$$

$$I(\theta_m,\psi_n) = 0; \; m = 0, 32 \text{ or } n = 0, 32$$

DISCUSSION

T. L. BALLMAN:* The inductive mathematical reasoning evidenced has made a breakthrough into the computation of lighting quantities. There have been papers in the past dealing with the inaccuracies of various intensity interpolation procedures which have been fairly well solved with the application of the finite Fourier series and its polynomial modifiers. The algorithm in the Appendicies works successfully; however, the sheer size of the core requirements causes a problem in itself.

The author has stated that the reduced run-time obtained by integration of the surface of radiation using the series and trignometric recursion in evaluating the series and its coefficients,

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has made computation of ESI practical for general use. The run-time has been drastically reduced, but the very nature of the hardware being used to perform the computation can seriously effect the run-time, for example, to complete the integration portion of the program, the equipment run-time required three times the total run-time of the complete program on the author's computer.

Because of core limitations (34K), it was necessary to structure the program as follows: (I) integrate the surface of radiation and compute the A, A', C, C', and E, E' coefficients; (2) compute the direct component; (3) compute the reflected component; and (4) compute the ESI at each point and print out the results. This is perhaps oversimplifying the process. In each of the first three steps, the results were written into files and then accessed by the succeeding step. As a result, the program is limited by I/O time.

Appendices A and B are valuable, but must be followed implicitly to make the whole thing work. Part II, unhappily, does not provide the assistance of the appendices. After fighting through the procedures, it is not that difficult to program using the mathematical relationships established. The author omits the expression for the determination of the radiative exchange factors. Would he provide the expression for both parallel and perpendicular surfaces?

It seems valid to relate each surface element to the whole surface which it sees, but would it also be valid, in the case of floor ceiling interchange, to treat the floor elements as seeing to total ceiling area less the total luminaire area? This would eliminate the need for dealing with a quantity of interchanges (number of elements times number of luminaires) and help keep the run-time within bounds.

The author has developed a powerful tool. From the data obtained, some simplified application techniques might be developed that will allow a more effective utilization of available energy. Once the relationship is established between total point illumination and visibility, contrast (or what other names might be given to visual performance potential) then the differences may be more easily demonstrable.

J. M. CRAMER:* The mathematical elegance of the author's method, and the use of techniques particularly well suited to the digital computer do much to recommend the method. There is a loss of generality, however, which weighs against the method in certain types of application.

The only exception of note to be taken with the method of the algorithms is the lack of a trap to handle the necessary mathematical forcing of the intensity to zero at the indeterminate points of the coordinate system.

With regard to increase in computation speed, the limiting factor seems to be in the calculation of the initial incident illumination on the sections of the room surfaces. The author's comments on the increase in speed obtained here would be appreciated.

A severe loss of generality is observed in dealing with markedly asymmetric luminaires and unusual dispositions. If a luminaire of marked asymmetry is used in many orientations within a room, the numbers of sets of $a(m,n)$, $c(m,n)$, . . . , which must be generated and handled becomes large. In the event of unusual dispositions (such as alignment of the luminaires parallel to an inclined ceiling), certain of the algorithms assumptions fail. The equations must then be rederived. For totally irregular dispositions about the plane, no difficulty is seen except for the nuisance of establishing a large number of *1x1* grids. It is assumed that such grids are not forbidden.

Even in those situations where the algorithms are not suitable for final calculations, the intermediate quantities $a(m,n)$, $c(m,n)$, . . . , are of value. Assessed for selected orientations of asymmetric luminaires and normally for symmetric luminaires, the coefficients represent the potential of the luminaire to produce task illuminance, ESI, etc, when applied in a system. Thus, in the most general case, the intermediate quantities may prove to be of greater value than the specific algorithms of solution.

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P. NGAI:* In *Part I*, trigonometrical polynomial interpolation ensures an exact fit on the finite numbers of data points and its derivatives. Hence, it is a well behaved interpolating function. The computation time is greatly reduced by the integrability of the working function and the separation of variables—thanks to the author's Perpendicular Plane Angular Coordinate.

The author is forced to make the assumption that

$$I_{dA}(\theta,\psi) = I(\theta,\psi)\cdot dA/A$$

In most cases it is not so. Perhaps the author would comment on this.

In theory, the process of calculation of direct component can be eliminated by the following technique. Assuming that the intensity distribution of the luminaire is the resultant contribution of M lambertian emitters of different orientations, let $\hat{p}_i$ be the unit directional vector normal to the surface of the emitter and $\hat{q}_j$ be the unit directional vector of the intensity at the q_j direction. Further, assume the area of these emitters are constant, A

$$I(q_j) = L_1\cdot A\cdot\hat{q}_j{}^*\hat{p}_1 + L_2\cdot A\cdot\hat{q}_j{}^*\hat{p}_2 \dots\dots\dots + L_M\cdot A\cdot\hat{q}_j{}^*\hat{p}_M = A\cdot\left[\sum_{i=1}^{M} L_i\cdot\hat{q}_j{}^*\hat{p}_i\right]$$

Or, in matrix form,

$$\frac{1}{A}\cdot\begin{bmatrix} I(q_1) \\ I(q_2) \\ \\ I(q_N) \end{bmatrix} = \begin{bmatrix} \hat{q}_1{}^\dagger\hat{p}_1 + \hat{q}_1{}^\dagger\hat{p}_2 + \dots\dots\dots + \hat{q}_1{}^\dagger\hat{p}_M \\ \hat{q}_2{}^\dagger\hat{p}_1 + \hat{q}_2{}^\dagger\hat{p}_2 + \dots\dots\dots + \hat{q}_2{}^\dagger\hat{p}_M \\ \\ \hat{q}_N{}^\dagger\hat{p}_1 + \hat{q}_N{}^\dagger\hat{p}_2 + \dots\dots\dots \quad \hat{q}_N{}^\dagger p_M \end{bmatrix}\cdot\begin{bmatrix} L_1 \\ L_2 \\ \\ L_M \end{bmatrix}$$

$L_1 \dots\dots L_M$ can be solved easily (by Gauss-Sidel). Then, the whole calculation can be treated as many lambertian flux transfer–inter-reflectance problems (Part II of the author's paper). Of course, more research must be done before this can be put into practice, for example, the question of the existence of the solution and the uniqueness of the solution. The interest is in the minimum number of L_i's required for a given set of NI's. As an example, in the case of the luminaire distribution lambertian, only one emmitter is needed. As a first approximation, the time required for the computation will be linearly proportional to the number of L_i's required.

In *Part II*, The discussor is very interested in the optimizer. It seems that the author is using some form of analytical technique. The discussor, himself, abandoned this approach after some fruitless search. While analytical technique can ensure that one solution is better than some others, the difficulty is that one never knows how far it is from the optimal solution. The discussor did find some success in the discrete search technique; however, the multiple point problem, as the discussor calls it, is still unsolvable. It is his sincere hope that the author can shed some light on this important problem.

R. N. HELMS:‡ With these two papers, anyone with a background in computer programming and an understanding of the basics of mathematics can write an ESI program. The program may appear long and involved because of the larger number of variables necessary in assessing illumination at a point. The

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† Dot product operation.

‡ Associate Professor of Illuminating Engineering, University of Colorado, Boulder, Colo.

complexity of the mathematics also allows for the most efficient utilization of computer time to minimize computer costs. Mr. DiLaura has indeed kept his word by developing a computerized technique of calculating ESI which can analyze 100 points in a room for less than $10 of computer cost. The program gives not only the ESI values on the work surface but the luminance patterns on the surfaces. With the addition of the VCP program presented at the 1968 Boston Computer Symposium, the engineer can get a complete picture of the effects of the luminous environment in terms of ESI, VCP, and luminance ratios. This is the kind of complete service the engineer needs to provide his client. However, there are points that need clarification.

In *Part I:* (1) Does the position point p indicate the luminaire position or task location in Fig. 2? (2) Is $\bar{\theta}\bar{\psi}$) measured relative to nadir at the differential element dA, that is, are $(\bar{\theta}\bar{\psi})$ and $(\theta\psi)$ complementary angles? (3) In Fig. 5, the x, y coordinate of the luminaires are relative to the corners of the luminaires. When treating the luminaire as a whole shouldn't the coordinate points be at the center of each luminaire?

In *Part II:* (1) what effect does the presence of the luminaire have on the ceiling discretization? (2) In dealing with an indirect unit is the blocking effect of the luminaires (relative to the task location) taken into account?

The IES should move forward in the measurement of additional bi-directional reflectance factors. It is essential that computational techniques and physical measurements be cross-checked or validated for indirect lighting systems. These checks have not been made to date. These validation experiments will allow one to evaluate the magnitude of variations that may be caused by the presence of the luminaires between the ceiling and the task locations. These measurements and computations may give additional insight into the unresolved problems of obstacles in the work-plane area such as furniture. The University of Colorado will be working on this validation procedure, utilizing the computer program (Lumen II) donated by Smith, Hinchman, and Grylls for educational use. It is hoped that validation of indirect systems, as well as practical ESI measurement techniques, can be developed for presentation next year.

I. LEWIN:* Rapid strides in the subject of ESI have been made in recent years. With the publication of "RQQ Report No. 5," a method of calculating ESI from luminous intensity data was established. This latest contribution by the author is one of the most significant advances made since that time.

I am concerned about the emphasis placed upon the four viewing directions, termed by the author North, South, East and West. As we learn more about the application and design of lighting systems for ESI improvement, I am convinced we will see a greater emphasis on other viewing directions. An observer does not view along or across the rows of luminaires only, but does so at various orientations depending on how he is seated and the nature of his task. We recently have extended the ERL program to compute ESI for any viewer orientation, and some significant facts are emerging regarding luminaire design, which we hope to present in a future paper. It would be better if we could all express viewing direction in terms of, for instance, the orientation angle of the plane of the viewer's line of sight from the forward Y-direction. In this way, we might avoid having to become experts in nautical terminology to express viewer orientations other than the four covered in the paper. This will also eliminate possible confusion between arbitrarily defined North for the computation and true North shown on building plans.

Illumination design is moving in the direction of non-uniform lighting. The present methods of ESI computation do not suffer from increased computational time under this type of situation. However, the author's method requires splitting the non-uniform lighting into luminaire sub-arrays, each sub-array requiring its own solution. Could the author tell us what amount of increased computer time is needed in such situations?

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* Director, Environmental Research Laboratories, Scottsdale, Ariz.

The computation method forces the assumption that the luminous intensity emitted by a luminaire at a 90-degree vertical angle is zero. In the case of luminaires with luminous sidewalls, this is not the case. Could the author comment as to whether significant errors could arise in the case of ESI values calculated for this type of luminaire?

On the equation relating RCS to luminance, is it the author's experience that its use achieves increased computational speed over table look-up and interpolation, or is its chief advantage in the reduction of computer core requirements made possible by the elimination of the block data tables?

I have one main concern over the use of this fine computational work by the author. Most lighting designers are still overwhelmed by ESI and the plethora of data and tables that the subject now involves. Having seen some of the print-outs from the new program, I question whether tables of 200, 300 or 400 ESI computations for a given room can be readily analysed, or whether they are really necessary. There is the danger that the designer may be confused by the amount of data that he is asked to assimilate, and the efforts by the author and others could suffer as a result. There are various approaches to this problem, and the RQQ committee is considering the ESI Rating system, which condenses ESI data into a meaningful form.

In cases where the designer is satisfied by the calculation of, for instance, 10 ESI points, it would seem that the efficiency of the new system is reduced, as the solution of the various equations, which presumably involves much of the cost, still is required. If this is so, would there not be a substantial increase in the cost per ESI point calculated in such cases, versus that for rooms where several hundred points are computed?

Further, we must be careful to realize that the cost of ESI computations does not consist solely of computer time. In fact, most of the cost involved in obtaining such computations may well consist of the amortization of the programming dollars, plus operator time. A program which runs twice as fast as another can in no way half the cost.

Lastly, Mr. DiLaura mentioned the necessity for interpolation of luminous intensity data from photometric curves. The RQQ committee is of the belief that the normal type of photometric data available, which is presented in 10-degree vertical stips, will give reduced accuracy in such interpolations. We feel that 5-degree photometry is important, and that 2½-degree photometry is highly desirable. Would Mr. DiLaura please comment as to whether he feels 10-degree photometry is adequate for ESI computations, or whether he would prefer to see improved photometry based on 5- or 2½-degree steps.

AUTHOR: Mr. Ballman correctly states that the actual run-times will be a function of the particular computer hardware being used. Although this is true, processing time on slower machines is almost always less costly than that on faster machines; the overall cost remains roughly constant. This is especially true of procedures which are processing-time limited, rather than input/output time limited. If core limitation proves to be a problem, as Mr. Ballman indicates, reduced problem size, or program modularity would offer a solution to this difficulty. The expressions for the radiative exchange factors requested by Mr. Ballman can be found in one of the many texts on radiative heat transfer.

Regarding to Mr. Cramer's question concerning the calculation time for the initial illuminances on the room surfaces, the author has found this calculation time small when compared to other sections of the procedure, especially the determination of the reflected components of L_b and L_t. Mr. Cramer correctly indicates that for luminaires with totally asymmetric intensity distributions in several orientations, the number of sets of coefficients can become large. However, a technique is available to reduce the number of coefficients necessary in each set. It is almost always the case that the value (and hence the numerical importance) of the coefficients deminishes rapidly as the harmonic number is increased. Beyond a certain harmonic, there is little accuracy to be gained by adding more coefficients to an evaluation. Finding this cut-off point is an easy matter. A modified Parseval Relation gives the total mean squared error (MSE) for the entire data range, as a function of the number of coefficients being used. The required equation for the coefficients discussed in Part I is:

$$\mathrm{MSE} = \frac{1}{64^2} \sum_{j=0}^{63} \sum_{k=0}^{63} f^2(j,k) +$$

$$\frac{1}{4} \left\{ {}'\sum_{m=0}^{M} {}'\sum_{n=0}^{N} a^2{}_{mn} + 2 \sum_{\substack{m=32 \\ M \geq 32}}^{M}{}' \; {}'\sum_{n=0}^{N} a_{mn} a_{64-m,\,n} + \right.$$

$$2\,{}'\sum_{m=0}^{M} \sum_{\substack{n=32 \\ N \geq 32}}^{N}{}' a_{mn} a_{m,\,64-n} 4 \sum_{\substack{m=32 \\ M \geq 32}}^{M}{}' \sum_{\substack{n=32 \\ N \geq 32}}^{N}{}' \times$$

$$\left. a_{mn} a_{64-m,\,64-n} - \frac{32}{64^2} \sum_{m=0}^{M} \sum_{n=0}^{N} a_{mn} \frac{a_{mn}}{\Theta(m)\Theta(n)} \right\}$$

A tick mark preceding a summation indicates doubling the first term, and a tick mark after a summation indicates halving the first term. *M* and *N* are proposed limits on the number of coefficients to be used. Beginning with the lowest order, *M* and *N* are increased until the MSE is below an acceptable level. Five per cent of the square of the average value of the data has shown to be a reasonable criterion. The number of coefficients is typically one quarter of the total number available. The consequent savings in space and execution time is not small.

Mr. Cramer mentions the difficulties in applying this procedure to non-rectangular rooms. Calculating in a non-parallelepiped room will demand a different coordinate system and geometric assumptions. The author feels that a similar procedure could be developed for this condition.

Mr. Ngai correctly pinpoints a fundamental assumption, namely that the surface of the luminaire is of constant luminance and distributive characteristics. Although not accurate (or even roughly descriptive) for some luminaires, it is necessary to assume this if one is to conveniently integrate over the luminous surface of the luminaire. A more accurate statement would require photometric information far beyond that which is commonly available. Mr. Ngai offers an interesting alternate procedure for the direct component calculation. Although his result replaces the actual luminaire with a collection of lambertian emitters, their effect on task and background luminance must still be calculated explicitly. Since their emittances will be high and localized, treating them in a manner as outlined in *Part II* of the paper will not provide sufficient resolution. Evidently, a scheme such as that in *Part I* is still necessary.

The answers to the questions raised by Dr. Helms are as follows. Point *P* of Fig. 2 indicates the location of an arbitrary point with respect to two origins of two perpendicular-plane angular coordinate system. A specific application of the idea is shown in Fig. 3. The angles used to give the direction of intensity from a differential emitter *dA* are not measured from its normal but are measured exactly as those which indicate the incident direction to the target. Thus, they are not complementary angles. The coordinates indicated in Fig. 5 determine the limits of the integrals over the luminaires. As such, they must indicate the edges of the luminaires rather then their centers. For the purpose of calculating the radiative transfer in the room, the presence of luminaires is ignored. The ceiling is discretized as any of the other room surfaces. The luminaires' reflective properties can be taken into account by specifying their reflectances as different from the ceiling. Where appropriate, a discretized element's reflectance will assume the value assigned to a luminaire. The obfuscating effect of suspended luminaire presents formidable geometric difficulties. No efficient manner has been devised to account for this.

Dr. Lewin points out the problems with the assumption that the luminaire is to have zero candelas emitted at 90 degrees from its normal. It should be emphasized that this does not effect the calculation of L_b and L_t since the *BRDF's* are zero at 90 degrees from the target normal. This assumption will affect the calculation of the initial illuminances only at room surface elements located 90 degrees from the luminaire's normal. This will obviously affect the resulting final luminances at these elements, but the overall effect on the values of L_b and L_t is likely to be small. For recessed luminaires, this assumption poses no problem.

Dr. Lewin is doubtful of the applicability of the author's procedure when the number of ESI calculations needed is small. The author is not convinced that a small number of points is ever sufficient for designing purposes. A contour plot of iso-ESI lines is an excellent way to assess a lighting system's performance. The generation of such contours demands a large number of points. Even if the ESI is needed at only a few location, it seems unwise to consider only single points, since local ESI gradients with respect to observer position may be very high. Gradients of ESI are as important as the values of ESI themselves. Reliable calculation of gradients requires numerous and dense points of computation.

Dr. Lewin questions the economics involved in programming and calculating ESI. Having produced two very different programs for the calculation of ESI, the author feels that the programming dollars necessary to produce an ESI program are essentially independent of the particular computational technique being programmed. However, the cost per run for the different techniques can easily be an order of magnitude different. Frequent use of an expensive program can easily generate more cost than the production of the program. Or worse, an expensive program may prompt infrequent use. It is estimated that 600 man-hours are needed to take the contents of this paper and produce a very complete ESI program. The amortization cost, even over a few years, of such an investment can easily be overshadowed by the execution costs of a slow program.

With regard to photometrics, it is the author's opinion that 10-degree increments in photometric measurements is usually insufficient. It is important to accurately assess the shape of the luminaire's intensity distribution. Photometric measurements in five planes and 5-degree increments is essential to make such an assessment. For luminaires with highly controlled distributions, 2½-degree steps are necessary. This is especially important when gradients of ESI and luminaire comparisons are being calculated.