Specifcation of Color-Rendering Properties Of Fluorescent Lamps

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IN RECENT years, increasing attention has been paid to the color-rendering properties of light sources; that is, how objects of various colors appear under them. Prior to the advent of the fluorescent lamp, the measure of the color of a light source also, to a large extent, specified its color-rendering properties. Such lamps were incandescent and as such had spectral energy distributions (*SED*'s) similar to black-body radiators at relatively low color temperatures as modified by any filters which might be used in conjunction therewith. Therefore, the establishment of the color of a light source also established its spectral energy distribution, and thereby the appearance of colors under it.

This is not the case with fluorescent lighting. These lamps make possible relatively high levels of illumination of almost any desired color and the large selection of usable phosphors also makes possible considerably different spectral energy distributions for any given color. This latter variable may give rise to a considerable change in the subjective color of an object in going from an area lighted by one type of lamp to one lighted by another. The most common example of this change is found in going from a room lighted by incandescent lamps to one lighted by warm-white fluorescent. These lamps are a reasonably good color match but most deep reds will appear darker and grayer under the fluorescent lamp due to the deficiency of the latter in far-red emission. By the same token, some greens and blues may appear lighter and more saturated by this change due to higher density of radiant flux from the fluorescent lamp in these regions. It is this obvious change in colors, coupled with the advanced state of development of the sciences of photometry and colorimetry, which has been largely responsible for the increased interest in color-rendition measurements.

The present study was undertaken by the authors as members of a sub-committee on color of the American Standards Association's C-78 Committee investigating this subject. The results reported herein are by no means the complete story but merely represent a start in the amassing of the large amount of data which will be necessary for the final solution of this problem. It is hoped that these data will serve to indicate the intricacies of this problem, and serve as a springboard for future investigations.

As stated generically above, color-rendition of a light source is determined by the appearance of colors under it. This suggests that color rendition can be estimated by viewing objects of various colors under several light sources and judging which source gives the best color-rendition. This method is unsatisfactory for four reasons:

1. A random choice of test objects will usually not include those required for a critical test.
2. No criterion is set up for the correct color of the test objects.
3. The comparison leads to a qualitative answer only; that is, no strictly quantitative measure can be made.
4. It is subject to personal preferences as to the appearance of colors. These preferences are different for each individual, and may even be different for the same individual at different times.

Therefore, the first problem in this study is the establishment of a criterion for judging colors; in other words, setting up a standard light source under which the true colors of samples can be determined. Then the color-rendering properties of a test lamp can be judged on the basis of how nearly alike in color the same sample looks under it and under the standard.

Fluorescent lamps were first developed to have the same visual color as a black-body radiator at a particular temperature. This temperature became an integral part of the color nomenclature of these lamps, for example, 6500K, Daylight; 4500K White (Cool White); 3500K White; 3000K White (Warm White). Since these various "white" lamps differ widely in color, any object color will look different under one than under another. Consequently, a standard light source should be set up for each "white" fluorescent lamp.

In the rendition of colors, energy is emitted by the illuminating source and is reflected or absorbed in varying proportions by the test object depending

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This paper is an extension of the data presented under the same title (and as Preprint No. 4) before the 1950 National Technical Conference of I.E.S. at Pasadena, Calif.

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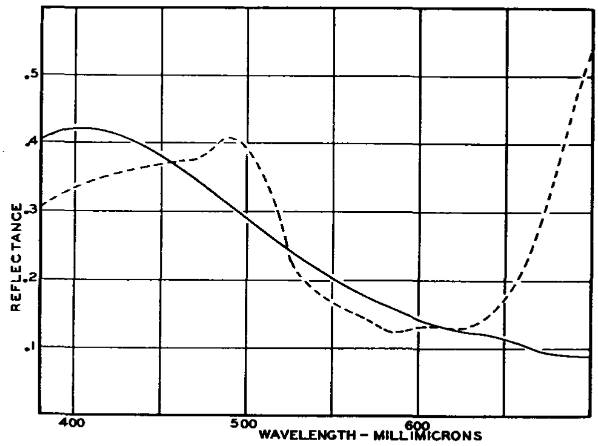


Figure 1a. Spectral reflectance of a metameric pair of blue colors.

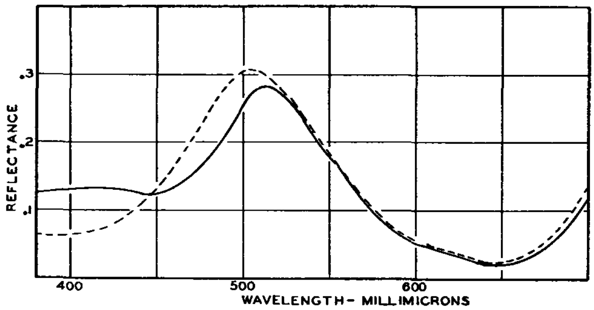


Figure 1b. Spectral reflectance of a metameric pair of green colors.

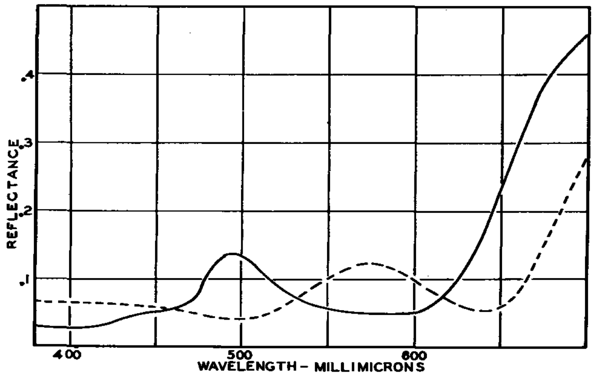


Figure 1c. Spectral reflectance of a metameric pair of olive colors.

TABLE I.—Spectral Reflectances of Three Pairs of Specimens Used To Test the Color-Rendering Of One Light Source Relative to Another.

Wavelength (mu)	Blue Specimens		Green Specimens		Olive Specimens	
400	0.419	0.333	0.130	0.066	0.029	0.066
410	.420	.343	.131	.068	.030	.066
420	.414	.351	.130	.073	.032	.065
430	.406	.358	.128	.086	.039	.063
440	.393	.363	.124	.106	.046	.061
450	.380	.368	.125	.132	.051	.058
460	.362	.371	.132	.163	.054	.055
470	.346	.376	.151	.200	.065	.050
480	.328	.386	.180	.242	.101	.046
490	.309	.408	.215	.281	.132	.042
500	.289	.391	.256	.303	.131	.040
510	.270	.348	.279	.301	.117	.043
520	.251	.277	.274	.282	.093	.052
530	.233	.221	.247	.252	.075	.068
540	.217	.183	.211	.213	.065	.085
550	.201	.167	.177	.182	.058	.101
560	.188	.153	.145	.148	.054	.114
570	.174	.141	.110	.116	.053	.123
580	.163	.129	.083	.087	.051	.124
590	.153	.126	.064	.068	.050	.115
600	.142	.132	.052	.056	.050	.100
610	.134	.132	.044	.048	.057	.083
620	.128	.130	.036	.039	.079	.070
630	.122	.131	.029	.032	.116	.058
640	.120	.144	.024	.026	.166	.053
650	.116	.173	.022	.024	.229	.055
660	.105	.217	.026	.030	.295	.078
670	.098	.279	.036	.043	.358	.118
680	.093	.367	.054	.064	.405	.166
690	.091	.462	.079	.096	.435	.220
700	.090	.546	.116	.140	.460	.280

on the wavelength of the energy and spectral selectivity of the object. Then, for the color of the object to be properly rendered, the source must emit energy within every one of a set of 5 to 10 wave bands in the visible spectrum in order that the selectivity of the test object in this band may properly alter the incident energy to give rise to the proper color sensation. Broad bands are specified here because if very narrow bands were used, certain discontinuous sources that have been found to give good color rendition would be disqualified. The best example of this is probably daylight itself wherein the Fraunhofer lines make this a discontinuous source but do not interfere with the rendition of object colors. The CO_2 gaseous discharge tube is another example of a discontinuous source that gives very satisfactory color rendition.

A blackbody radiator fills the requirement of a suitable standard source; that of having energy in every broad wavelength band of the visible spectrum. Its spectral energy distribution is also well defined. Therefore, it seems logical to take a blackbody radiator at the same nominal color temperature as the lamps to be investigated as the standard for that color.

The establishment of a standard source overcomes the objection (2) of having no criterion for the correct color. If a standard observer is used to compute color specifications of the test objects for the standard and test sources, quantitative values may be obtained which provide a basis for overcoming objection (3). If, further, the object-color differences caused by passing from standard to test source are expressed in terms of number of just perceptible differences, objection (4) is overcome provided the test objects chosen are critical (objection 1).

Such a procedure has been used by the National Bureau of Standards[1] to test the degree of duplication of natural daylight by various artificial light sources. This procedure takes the form of computations of chromaticity coordinates, x, y, in the C.I.E. system[2] for three metameric pairs of colors that are so chosen that one pair (green) serves to detect deviations in the shortwave end of the spectrum between the source being investigated and the standard source; one pair (blue) similarly tests for the longwave and the third pair (olive) tests the middle of the spectrum. The spectral reflectances of these specimens are listed in Table I and are shown in Fig. 1. The chromaticity coordinates are then expressed in terms of a uniform-chromaticity-scale diagram, such as that of Breckenridge and Schaub,[3] so that equal distances anywhere in the diagram will represent approximately equally perceptible chromaticity differences, and the coordinate differences ($\Delta_1 x''$ and $\Delta_1 y''$) between the members of the pairs are determined. This is done for the standard and each source to be compared to it and the change in these differences (second differences, $\Delta_2 x''$ and $\Delta_2 y''$) in going from the standard to the test source is determined. That is, $\Delta_2 x'' = \Delta_1 x''_{\text{(Std.)}} - \Delta_1 x''_{\text{(Fluorescent Lamp)}}$ with a similar expression for $\Delta_2 y''$.

These data are reduced to a single deviation, D, for each test source from the standard by averaging the values of $\sqrt{\Delta_2 x''^2 + \Delta_2 y''^2}$ for the three metameric pairs. D in turn is converted to a "Duplication Index," I, by the transformation:

$$I = \frac{1 - 10D}{1 + 10D} \times 100.$$

This latter transformation is employed to arrive at an easily recognized scale of values. For example, by this transformation, an index of 100 represents exact duplication; an index of 50 is approximately the degree of duplication of gas-filled incandescent lamp light for natural daylight; and the index for monochromic light of wavelength 700 millimicrons for natural daylight is approximately zero.

This method of assessing the degree to which a test source duplicates the color rendition of the standard source is strictly valid for these three pairs of test objects only. It has general validity only to the degree that these test objects adequately represent objects to be viewed under the test sources. Furthermore, even with the number of test objects reduced to six, the computations are laborious and the method is quite unsuited for any quick routine check such as might be employed with a lamp factory. However, for all of the considerable number of sources of artificial daylight to which it has been applied, this method has yielded results in good agreement with the qualitative results of the subjective method; so it is proposed in this paper to use this method as a yardstick of the efficacy of shorter methods devised for the same purpose. Accordingly, by this method, the duplication indices, I, of four spectral distributions of cool white (4500K) for black-body radiation of 4500K, and four spectral energy distributions of warm white for blackbody radiation of 2854K (C.I.E. source A) have been computed with the following results.

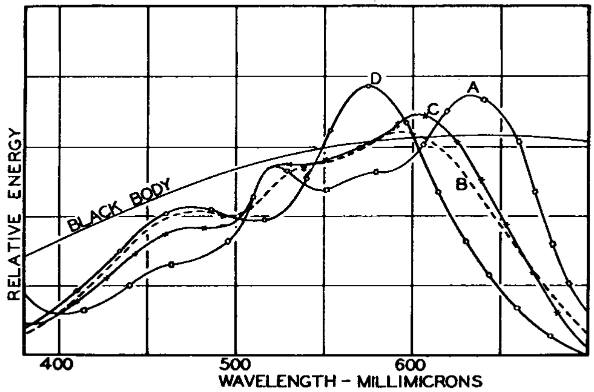


Figure 2. Spectral energy distributions — cool whites.

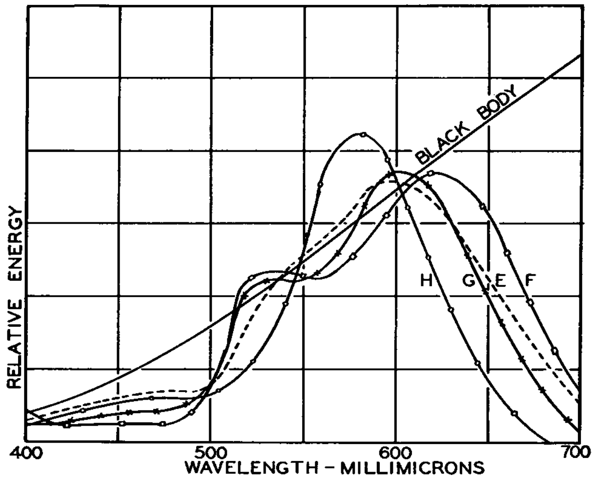


Figure 3. Spectral energy distributions — warm whites

Cool	**Whites**	**Warm**	**Whites**
	I		*I*
A	69	E	62
B	75	F	69
C	76	G	54
D	58	H	45

The spectral energy distributions of these light sources without the mercury-lines are shown in Figs. 2 and 3 and are given with the mercury-lines in Table II.

In all the computations in this paper, the weighted-ordinate method of integration is used. The effect of the mercury lines energy of the fluorescent lamps is added to that of the phosphor emis-

TABLE II.—Spectral Energy Distributions.

	COOL WHITES					WARM WHITES				
λ	Std. (4500K)	A	B	C	D	(2854K)	E	F	G	H
400	54.8	4.0	7.6	5.8	8.7	7.9	3.9	4.0	2.0	3.1
*405.0	——	14.5	21.0	16.8	17.3	——	16.0	19.8	19.8	18.3
410	58.8	3.9	10.1	7.2	11.2	9.5	4.4	2.8	2.2	3.8
420	62.7	4.1	12.6	8.9	13.3	11.3	5.1	2.1	2.8	4.8
430	66.6	5.0	16.0	10.7	16.0	13.3	5.9	2.1	3.2	5.8
*435.8	——	37.5	51.5	40.0	44.3	——	38.5	48.3	46.5	43.5
440	70.4	6.0	18.8	12.5	18.8	15.5	6.6	2.0	3.6	6.5
450	73.9	7.0	21.9	14.7	21.4	17.8	7.5	1.9	4.0	7.3
460	77.2	7.7	24.0	16.1	23.3	20.4	7.9	1.9	4.4	7.7
470	80.4	7.9	24.7	16.4	24.2	23.1	8.0	2.1	4.4	7.9
480	83.3	8.4	25.0	17.0	24.3	26.0	8.2	2.2	4.6	7.9
490	86.1	9.2	24.7	17.2	24.0	29.1	8.6	3.6	5.8	8.2
500	88.6	10.4	24.4	18.2	22.7	32.3	9.5	6.8	8.2	8.6
510	91.0	13.5	26.0	21.5	22.3	35.6	13.5	12.4	14.4	10.5
520	93.3	16.2	29.0	25.2	22.7	39.1	20.0	20.0	21.4	13.7
530	95.2	15.6	31.8	25.5	24.7	42.7	25.0	21.0	22.4	17.0
540	97.0	14.9	33.2	24.8	29.2	46.4	28.5	20.8	22.4	24.2
*546.1	——	21.8	26.5	22.0	22.0	——	20.3	25.0	26.8	22.5
550	98.6	14.1	33.6	24.0	35.0	50.1	31.0	20.0	22.4	33.0
560	100.0	14.6	34.6	24.2	40.4	53.9	34.0	20.2	24.2	46.6
570	101.1	15.0	36.2	26.2	44.2	57.8	37.3	21.4	27.2	54.5
*578.0	——	4.1	6.2	5.0	5.2	——	4.8	4.8	5.2	4.4
580	102.2	15.5	37.8	28.3	44.1	61.7	40.6	23.4	31.2	57.9
590	103.0	15.9	38.8	30.5	40.8	65.6	43.4	26.4	35.8	54.5
600	103.6	16.9	38.8	32.4	36.0	69.6	44.0	29.2	38.0	48.5
610	104.1	18.8	37.1	31.8	29.0	73.5	42.4	31.8	37.2	39.0
620	104.6	20.7	35.0	30.2	23.4	77.3	40.3	33.0	35.0	30.4
630	104.8	22.2	32.0	27.5	18.5	81.3	36.5	32.8	31.0	23.4
640	104.9	21.6	26.8	23.0	14.6	85.1	32.4	30.6	26.0	16.8
650	104.9	20.9	22.2	18.5	10.6	89.0	27.2	27.2	20.6	10.8
660	104.8	18.2	18.9	14.8	8.0	92.6	23.5	23.4	15.6	6.8
670	104.4	14.0	14.5	10.5	5.2	96.3	18.8	18.2	10.4	3.0
680	104.1	10.3	10.9	7.5	3.0	100.0	14.8	14.0	7.4	0.8
690	103.6	6.3	7.0	3.4	1.2	103.3	10.2	10.0	4.4	——
700	103.1	3.8	4.0	1.3	——	106.9	6.5	6.6	0.6	——

*Mercury lines. Energy values are those in excess of energy per 10 $m\mu$ interval of the continuum.

sion after multiplying their measured energy by the factor: effective slit width of monochromator divided by the wavelength interval used in the weighted-ordinate integration as described elsewhere by one of us.[4]

As brought out in the above discussion, the spectral energy distribution of a light source is the determining factor in its color-rendering properties. This suggests the *SED* curve as a specification for this property. Two lamps having identical *SED* curves will have identical color-rendering properties and also identical color. Therefore, the establishment of a standard *SED* curve suffices to specify the color-rendering properties. However, there are practical limitations to such a specification. Lamps from different manufacturers may use similar blends of phosphors but may differ considerably in color. Also, there is a variation in color from lamp to lamp of a single manufacturer, the amount of the variation depending on the strictness of quality control measures employed. Consequently, concurrently with the establishment of a standard *SED* curve, tolerances on either side of it must also be established. These tolerances must be made loose enough so that maximum deviation from the standard at any one point will be allowed. When this is done for all points, the tolerances tend to lose significance since they would seem to allow maximum deviation at several, or all, points, whereas such an occurrence is obviously unsuitable. This objection could be overcome by making a single figure of merit similar to the "duplication index" described above, which would average the deviations from the standard, together with suitable tolerances thereon, as a supplementary part of the specification.

Another drawback in the use of *SED* curves as a complete measure of color-rendition is the difficulty in their interpretation, especially when lamps of considerably different *SED*'s are to be compared. This is indicated by the curves in Figs. 2 and 3. The comparison of any two such curves can lead only to qualitative estimates of the rendition of specific colors. For quantitative results recourse must be made to laborious computations similar to those used in the determination of the "duplication indices" above. What is most desired, then, is some simpler specification.

The *SED* curve implies that measurements of the energy in a number of narrow bands have been made. These bands are narrow and of sufficient number so that a continuous curve can be drawn. For example, the radiometer on which the *SED* curves of Figs. 2 and 3 were determined used bands 5 millimicrons wide throughout. Therefore, one simplification possible is to measure the energy in a smaller number of wider bands. This is the basis of a measure of color rendition of fluorescent lamps developed in Europe.[5, 6, 7] In their procedure, the per cent luminous energy (radiant energy weighted by the spectral luminous efficiency of the normal eye) in eight spectral bands, and tolerable limits to deviations therefrom are specified. The bands chosen as of equal importance in color rendition, and the tolerances set to accord with fluorescent-lamp control attainable in England in 1947 are:

Band No. 1	from 380 to 420 mu	±20%
Band No. 2	from 420 to 440 mu	±10%
Band No. 3	from 440 to 460 mu	±10%
Band No. 4	from 450 to 510 mu	±10%
Band No. 5	from 510 to 560 mu	± 5%
Band No. 6	from 560 to 610 mu	± 5%
Band No. 7	from 610 to 660 mu	±10%
Band No. 8	from 660 to 760 mu	±20%

These band limits are shown superimposed on a spectral energy distribution curve of a typical fluorescent lamp in Fig. 4.

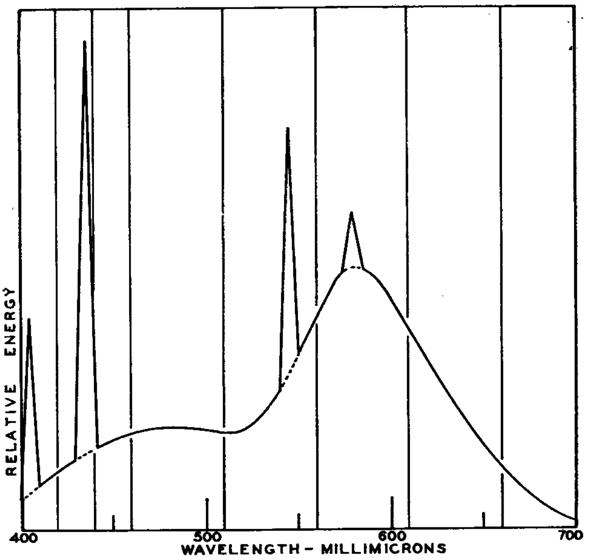


Figure 4. Spectral band limits in British specification.

Table III shows the distributions of luminous energy in per cent for the two standards (4500K and 2854K) and for the eight fluorescent lamps (see Table II and Figs. 2 and 3). The deviations from standard are also shown for the eight lamps.

A sample computation sheet for one of the lamps is shown as Table IV.

It is evident from the deviations from the standard given in Table III and the tolerances used by the British that a fluorescent lamp must be the standard used in the latter specification. This may be convenient for routine checks when such a standard has been established, but it seems to us desirable to try to establish a standard which will represent the best color-rendering light source available. This is one of the factors influencing our choice of blackbody radiation for this purpose. The greatest apparent drawback to the British procedure is the specification of eight different tolerances and the resultant difficulty in properly evaluating the over-all effect of the deviations.

We have, therefore, tried averaging the deviations in order to arrive at a single numerical index which would be a measure of the color-rendering properties of the lamp in question and which could

TABLE III.—Luminous Energies and Deviations from Standard in Eight Spectral Bands.

COOL WHITES

	4500K	Lamp A		Lamp B		Lamp C		Lamp D	
Band	L%	L%	ΔL	L%	ΔL	L%	ΔL	L%	ΔL
1	.02	.01	.01	.01	.01	.01	.01	.01	.01
2	.16	.42	.26	.35	.19	.33	.17	.33	.17
3	.56	.30	.26	.46	.10	.40	.16	.47	.09
4	8.63	5.68	2.95	6.75	1.88	6.41	2.22	6.61	2.02
5	39.17	44.92	5.75	42.27	3.10	42.21	3.04	40.26	1.09
6	38.50	34.81	3.69	39.76	1.26	39.23	.73	45.34	6.84
7	12.05	13.20	1.15	10.06	1.99	11.09	.96	6.84	5.21
8	.91	.66	.25	.34	.57	.32	.59	.13	.78
		ΔL:	1.79		1.14		.99		2.03

WARM WHITES

	2854K	Lamp E		Lamp F		Lamp G		Lamp H	
Band	L%	L%	ΔL	L%	ΔL	L%	ΔL	L%	ΔL
1	.01	.01	—	.01	—	.01	—	.01	—
2	.06	.24	.18	.35	.29	.31	.25	.24	.18
3	.25	.17	.08	.06	.19	.11	.14	.15	.10
4	5.47	2.91	2.56	2.56	2.91	2.89	2.58	2.40	3.07
5	33.45	37.67	4.22	42.43	8.98	40.15	6.70	33.56	.11
6	42.59	45.89	3.30	39.19	3.40	43.47	.88	55.21	12.62
7	16.64	12.64	4.00	14.77	1.87	12.74	3.90	8.35	8.29
8	1.53	.47	1.06	.63	.90	.32	1.21	.08	1.45
		ΔL:	1.93		2.32		1.96		3.23

TABLE IV.—Computation of Radiant and Luminous Energy Distribution.

Lamp: B

Band System: British Specification

NOTE: The energies of the mercury lines are separately recorded at 405.0, 435.8, 546.1, and 578.0 millimicrons.

Wavelength	E	y	L = Ey
400 mu	7.6	.0004	.0030
405.0	21.0	.0006	.0126
410	10.1	.0012	.0121
420	12.6	.0040	.0504
430	16.0	.0116	.1856
435.8	51.5	.0178	.9167
440	18.8	.0230	.4324
450	21.9	.038	.8322
460	24.0	.060	1.4400
470	24.7	.091	2.2477
480	25.0	.139	3.4650
490	24.7	.208	5.1376
500	24.4	.323	7.8812
510	26.0	.503	13.0780
520	29.0	.710	20.5900
530	31.8	.862	27.4116
540	33.2	.954	31.6728
546.1	26.5	.984	26.0760
550	33.6	.995	33.4320
560	34.6	.995	34.4270
570	36.2	.952	34.4624
578.0	6.2	.889	5.5118
580	37.8	.870	32.8860
590	38.8	.757	29.3716
600	38.8	.631	24.4828
610	37.1	.503	18.6613
620	35.0	.381	13.3350
630	32.0	.265	8.4800
640	26.8	.175	4.6900
650	22.2	.107	2.3754
660	18.9	.061	1.1529
670	14.5	.032	.4640
680	10.9	.017	.1853
690	7.0	.0082	.0574
700	4.0	.0041	.0164
Totals:	863.2		385.4462

Band No.	Wavelength Limits	E E%	L L%
1	380-	45.0	.0529
	420	5.2%	.01%
2	420-	83.2	1.3437
	440	9.6%	.35%
3	440-	43.3	1.7684
	460	5.0%	.46%
4	460-	123.8	26.0005
	510	14.4%	6.75%
5	510-	184.4	162.9349
	560	21.4%	42.27%
6	560-	193.6	153.2587
	610	22.4%	39.76%
7	610-	144.0	38.7875
	660	16.7%	10.06%
8	660-	45.9	1.2996
	760	5.3%	.34%
		863.2	385.4462
		100.0%	100.00%

then be compared to its duplication index, *I*. For example, the magnitude of the deviations given in Table III have been averaged with the results as indicated. The correlation between these averages and the computed duplication indices is shown in Fig. 5. These data are shown in another way in Fig. 6 wherein the correlation between computed indices (*I*) and average differences between per cent luminous energy of blackbody and per cent luminous energy of the fluorescent lamp in each of the bands expressed as per cent of that of the standard is shown. This correlation is not as good as that shown in Fig. 5.

A further disadvantage of the British specification is the extremely low luminous energy in the

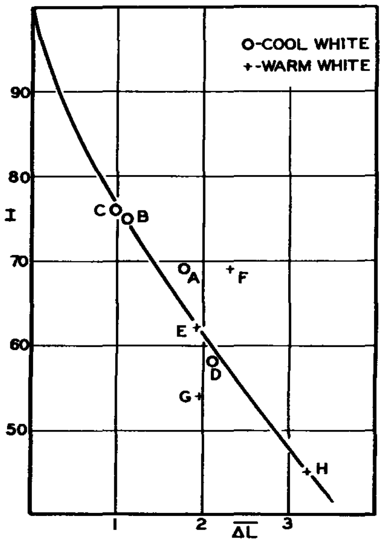
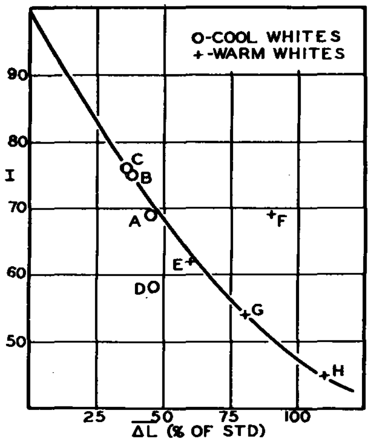


Figure 5. (left) Average luminous energy differences in British bands vs. duplication indices.

Figure 6. (right) Average luminous energy differences (per cent of standard) in British bands vs. duplication indices.



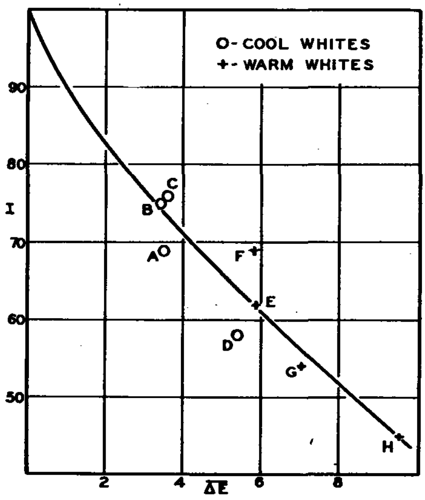


Figure 7. (left) Average radiant energy differences in British bands vs. duplication indices.

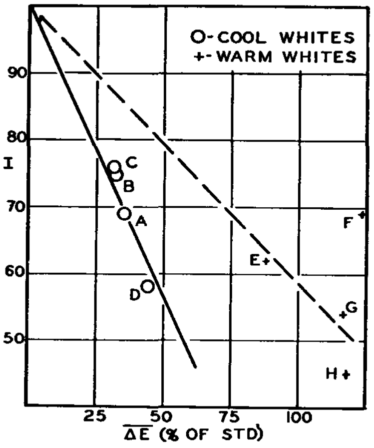


Figure 8. (right) Average radiant energy differences (per cent of standard) in British bands vs. duplication indices.

bands at the end of the spectrum due to the low sensitivity of the eye in these regions. Since the fundamental controlling factor in color rendition is the radiant energy emitted by the light source, a survey was made of this property of the test lamps in the spectral bands of the British specification. The results are given in Table V and shown in Figs. 7 and 8. In Fig. 7, the abscissa is the average difference between per cent radiant energy of the test lamp and per cent radiant energy of the standard. Fig. 8 shows the correlation between *I* and the average deviation from the standard per cent radiant energy in each band, the deviations being expressed as percentages of the per cent radiant energy of the standard in that band.

Fig. 8 indicates a possible correlation within each white although in each case there are deviations which are nearly as large as when a single correlation is assumed. Fig. 7 shows a much better correlation between duplication indices and average radiant energy differences.

Figs. 5, 6, 7, and 8 point up an observation which has been corroborated in the other studies reported herein. That is, the average of the actual differences in the magnitude of either luminous energy or radiant energy in a number of bands leads to a better correlation with the duplication indices than when these differences are expressed as a percentage of the standard values in each band and then averaged. Therefore, in the other results reported, it is the actual magnitude of the differences which are considered.

In view of the disadvantage of low luminous energies in the British bands at the ends of the spectrum another set of 10 bands was set up such that for a light source of equal spectral energy

TABLE V.—Radiant Energies and Deviations from Standard in Eight Spectral Bands.

COOL WHITES

Band	4500K	Lamp A		Lamp B		Lamp C		Lamp D	
	E%	E%	ΔE	E%	ΔE	E%	ΔE	E%	ΔE
1	5.1	5.3	.2	5.2	.1	5.2	.1	5.8	.7
2	4.7	10.3	5.6	9.6	4.9	9.3	4.6	10.2	5.5
3	5.2	3.0	2.2	5.0	.2	4.4	.8	5.7	.5
4	15.0	10.1	4.9	14.4	.6	13.3	1.7	15.7	.7
5	16.9	21.0	4.1	21.4	4.5	21.9	5.0	22.0	5.1
6	18.1	18.3	.2	22.4	4.3	22.8	4.7	27.4	9.3
7	18.5	22.6	4.1	16.7	1.8	18.6	.1	11.4	7.1
8	16.5	9.4	7.1	5.3	11.2	4.5	12.0	1.8	14.7
		ΔE:	3.55		3.45		3.63		5.45

WARM WHITES

Band	2854K	Lamp E		Lamp F		Lamp G		Lamp H	
	E%	E%	ΔE	E%	ΔE	E%	ΔE	E%	ΔE
1	1.4	3.7	2.3	4.8	3.4	4.3	2.9	4.2	2.8
2	1.6	6.9	5.3	9.2	7.6	8.9	7.3	8.4	6.8
3	2.2	2.0	.2	.7	1.5	1.4	.8	2.2	—
4	8.5	6.2	2.3	3.8	4.7	5.5	3.0	6.4	2.1
5	13.7	20.5	6.8	21.5	7.8	22.8	9.1	21.4	7.7
6	19.5	28.7	9.2	23.0	3.5	28.4	8.9	40.3	20.8
7	25.4	23.4	2.0	26.4	1.0	23.5	1.9	16.0	9.4
8	27.7	8.6	19.1	10.6	17.1	5.2	22.5	1.1	26.6
		ΔE:	5.90		5.81		7.05		9.53

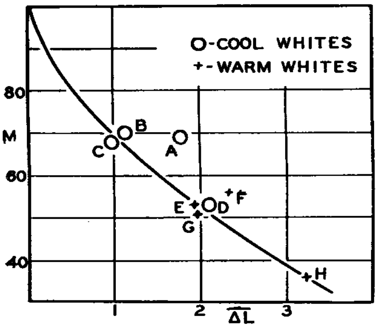
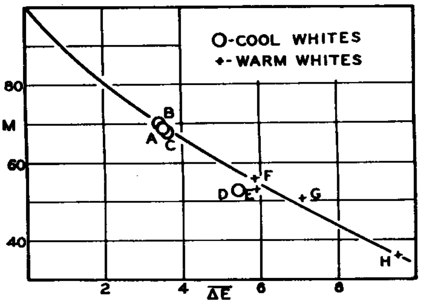


Figure 9. (left) Average luminous energy differences in British bands vs. modified indices.

Figure 10. (right) Average radiant energy differences in British bands vs. modified indices.



distribution approximately equal luminous energy would be obtained in each. The band limits are:

Band No. 1	Below 497.5 mu
Band No. 2	from 497.5 to 517.5
Band No. 3	from 517.5 to 532.5
Band No. 4	from 532.5 to 547.5
Band No. 5	from 547.5 to 557.5
Band No. 6	from 557.5 to 567.5
Band No. 7	from 567.5 to 582.5
Band No. 8	from 582.5 to 597.5
Band No. 9	from 597.5 to 617.5
Band No. 10	Above 617.5

Since other than established band systems are being considered, still another set of bands has been investigated. These are of equal width, each 50 millimicrons wide (400-450, 450-500, 500-550, 550-600, 600-650, and 650-700 millimicrons). The average deviations from the standard in each of the sets of bands tested of the luminous and radiant energies of our test lamps are listed in Table V.

These data would seem to indicate no particular advantage of one band system over another. The British band system has been used for several years in Europe and although there have been rumors of dissatisfaction with the system, considerable data and experience with it have been obtained and special photometers built for simplified determination of the luminous energies in the separate bands. For these reasons there has been a great reluctance to change of the system and until a band system that can show decided advantages over the old system can be developed, there will be no point in making a change.

Up to this point, the discussion has been predicated on the duplication indices derived from the use of metameric pairs as explained earlier in this paper. This index has obvious advantages for the specific application for which it was developed, that is, as a measure of the efficacy of light sources to be used for color matching of filters, painted surfaces, and so on. In that application, the occurrence of metameric pairs of colors can be commonplace. In the more general application for overall illumination, the appearance of individual colors is more important than the comparison of pairs of colors. This has led some investigators[8] to study the color changes evinced by individual objects in going from one kind of illumination to another. It has also been suggested that commonplace objects should be used for this determination, such as butter, lettuce, coffee, and meat to mention a few. In view of these suggestions, a modified duplication index (M) has been developed and the correlation of our radiant and luminous energy data for the several band systems investigated above correlated thereto.

The modified index is derived similarly to the duplication index, I, using the same six test objects

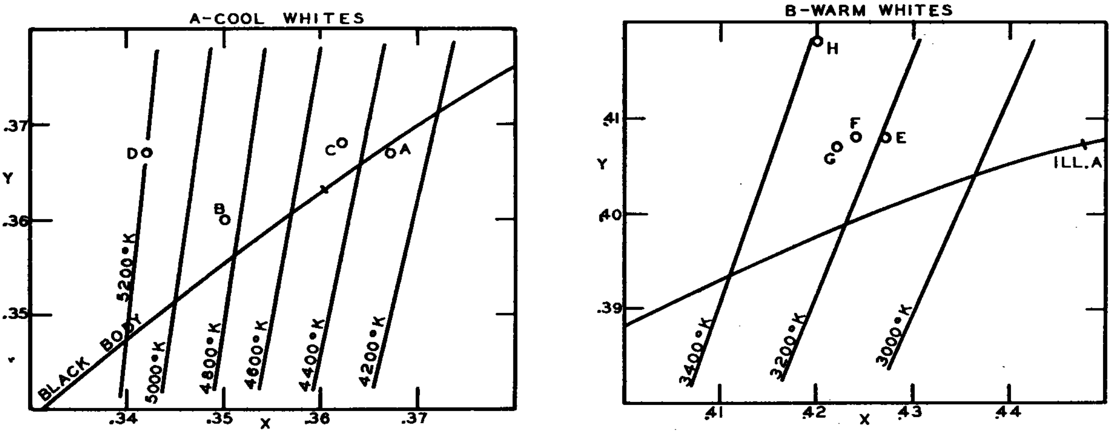


Figure 11. Chromaticities of lamps A to H on the C.I.E. color mixture diagram.

TABLE VI.—Average Luminous and Radiant Energy Differences in Three Sets of Band Systems.

SED	I	M	British Spec.		10 Bands		6 Bands of 50 mu	
			L	E	L	E	L	E
A	69	69	1.79	3.55	2.02	1.10	2.57	3.97
B	75	70	1.14	3.45	1.48	2.70	1.23	4.08
C	76	68	.99	3.63	1.54	2.48	1.26	4.68
D	58	53	2.03	5.45	2.49	4.42	2.77	6.77
E	62	53	1.93	5.90	1.82	5.16	2.21	7.15
F	69	56	2.32	5.81	2.12	3.32	3.18	7.27
G	54	51	1.96	7.05	2.89	5.34	2.42	8.80
H	45	36	3.23	9.53	3.59	7.50	4.86	11.33

comprising the metameric pairs. However, in the modification the changes in the color parameters in the rectangular uniform chromaticity scale are used instead of the second differences ($\Delta_2 x''$ and $\Delta_2 y''$). That is:

$$\Delta x'' = x''_{(\text{Std.})} - x''_{(\text{fluorescent lamps})}$$

and $$\Delta y'' = y''_{(\text{Std.})} - y''_{(\text{fluorescent lamps})}$$

Then, as with *I*, a single deviation, *D*, for each illuminant is obtained by averaging the six values of $\sqrt{\Delta x''^2 + \Delta y''^2}$. *M* is then derived from *D* by the transformation:

$$M = \frac{1 - 10D}{1 + 40D} \times 100.$$

The modified index, *M*, for our eight fluorescent lamps for their respective blackbody standards are computed to be:

Cool Whites		Warm Whites	
SED	*M*	*SED*	*M*
A	69	E	53
B	70	F	56
C	68	G	51
D	53	H	36

The correlation between these indices and the luminosity and energy analyses of the several band systems show that the correlation in the British band system is fully as good as that in the others. These relationships are shown in Figs. 10 and 11.

Before closing, it should be mentioned that the fluorescent lamps used in this study were chosen at random from several different production lots and from different manufacturers. Consequently, they are not all of the same color, nor do they match the color of the blackbody radiators chosen as the standards. The position of the lamps on the C.I.E. Color Mixture Diagram is shown in Fig. 11. The duplication indices of these lamps (both *I* and *M*) might be raised somewhat if the standard chosen in each case were the blackbody radiator of the nearest color temperature.

In summary, the conclusions arising from the present study are:

1. The spectral energy distribution of a light source is the only complete specification of its color-rendering properties.

2. A single index is required to supplement the *SED* curve to give an overall quantitative measure of the color rendition. When two *SED* curves coincide throughout, there is no need for the index since the color rendition of the two light sources will also be identical. The need for the single index arises to evaluate the relative merits of two lamps whose *SED* curves do not coincide.

3. Equipment for the precise determination of *SED* curves is too complicated and expensive for universal use. Therefore, a simpler procedure is required.

4. The measurement of relative energies in a finite number of spectral bands fulfills the requirements of number 3 above.

5. The use of the eight wavelength bands proposed by the British is recommended until such time as a selection of bands showing superior utility is developed.

6. The use of radiant energies in the spectral bands is recommended rather than the use of luminous energies. Radiant energy is the fundamental parameter in color rendition and the data contained herein show better correlation between duplication indices and energy relationships.

7. The single index suggested for color rendition of a lamp is the average difference between lamp and standard in per cent radiant energy within the eight British bands. Indices based on differences expressed as per cents of energies of the standard within the same wavelength band show poorer correlation with the duplication indices.

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