DAYLIGHT ILLUMINATION ON HORIZONTAL, VERTICAL AND SLOPING SURFACES*

REPORT OF THE COMMITTEE ON SKY BRIGHTNESS,

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SYNOPSIS: The report summarizes sky-brightness and daylight-illumination measurements made during the year ending April 6, 1922. For ten months the measurements were made in a suburb of Washington that is comparatively free from city smoke. During the other two months, one in summer and one in winter, the measurements were made in the smoky atmosphere of the city of Chicago.

The measurements were made as nearly as possible with the sun at altitudes above the horizon of 0°, 20°, 40°, 60°, and 70°. From the sky-brightness measurements the resulting illumination on vertical surfaces differently oriented with respect to the sun, and on surfaces sloping at different angles and in different directions, has been computed. These computed values have been utilized, in connection with daylight-illumination measurements, to construct charts showing for latitude 42° north, illumination intensities for each hour of each day of the year as follows:

(1) On a vertical and on a horizontal surface, from a cloudy sky.

(2) On a horizontal surface and on vertical surfaces facing the eight principal points of the compass, from a clear sky.

(3) On a horizontal surface and on vertical surfaces facing the eight principal points of the compass, from the sun and clear sky combined.

The illumination on sloping surfaces from skylight and from solar and skylight combined has been summarized in tables.

The application of these data to the lighting of working space in a building through saw-tooth roof construction is show. It is pointed out that with a clear sky the larger proportion of the illumination should result from the reflection of light from the outside roof surface through the window opening, rather than by the direct transmission of skylight through the window.

With a cloudy sky the illumination on a horizontal surface is considerably more than twice that on a vertical surface, due to the fact that the region of maximum brightness is in or near the zenith.

With high sun, as at midday in summer, the illumination from a cloudy sky averages higher than the illumination from a clear sky, except on a vertical surface facing the sun. This is not the case with low sun.

The maximum illumination from a clear sky on vertical surfaces is a little in excess of 1,400 foot-candles, and occurs on surfaces facing the sun from early June to early September, between the hours of 8.30 A. M., and 3:30 P. M.

The minimum illumination from skylight is on a vertical surface facing away from the sun. At Chicago in the smoky Loop District the illumination from a cloudless sky on such a surface averages about two-thirds the illumination at Washington on a similar surface from a clear sky.

*A report presented at the annual convention of the Illuminating Engineering Society, Swampscott, Mass., September 28, 1922; later revised and extended.

The total (solar + sky) illumination generally increases on surfaces sloping toward the south until the angle of slope reaches 20°, except with low sun during the summer months. The maximum is about 11,000 foot-candles at noon in midsummer.

At Washington the illumination from a clear sky on both horizontal and vertical surfaces varies between 150 and 60 per cent of the average values; from a cloudy sky, between 200 and 30 per cent; from a sky partly covered with white clouds, on a horizontal surface three to four times, and on a vertical surface two to three times that from a clear sky; with rain falling, about half that from a cloudy sky.

SKY-BRIGHTNESS AND DAYLIGHT-ILLUMINATION MEASUREMENTS

In a report of this committee[1] presented at the Rochester meeting in September, 1921, the program of sky-brightness measurements was outlined, and some preliminary results were given. The program of a full year of sky-brightness measurements was completed on April 6, 1922. During four weeks ending with August 13, 1921, and a second four weeks ending with February 2, 1922, the measurements were made in the city of Chicago. During the remainder of the year they were made in a suburb of the city of Washington that is practically free from city smoke.

As was explained in the previous report, at Washington the photometer was mounted on a stand inside a small shelter that was painted white on the outside and flat black on the inside. The upper edge of the sides of the house is on a level with the center of the elbow tube of the photometer when the latter is horizontal. This exposure permits measurements of the illumination from skylight on both horizontal and vertical surfaces. With a clear sky, however, illumination measurements on vertical surfaces have been confined to surfaces facing in azimuth 0° and 45° from the sun, as at greater azimuths the blackened inside walls of the shelter reflect too much sunlight to the photometer.

It may be well to recall to mind some details of the sky-brightness measurements. Figure 1 is a stereographic projection of the half of the sky on either side of the sun's vertical. The sun's position is indicated by the letter X. The horizontal straight line represents the horizon, and above it are lines of equal altitude 10° apart. Extending from the zenith to the horizon are azimuth circles also 10° apart.

[1] TRANSACTIONS, Illum. Eng. Soc., Oct., 1921. Vol. XVI. pp. 255-275. *Mo. Weather Rev.*, Sept., 1921, 49, pp. 481-488.

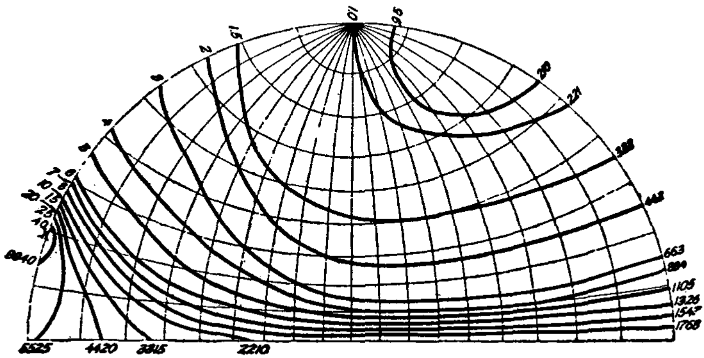

Fig. 1.—Sky brightness in millilamberts. Sun's position indicated by X. Washington, D. C., cloudless sky, ground covered with snow.

A complete series of sky-brightness measurements consists of three photometric readings on each of the points at 2°, 15°, 30°, 45°, 60°, 75°, and 90° above the horizon, and on azimuth circles 0°, 45°, 90°, 135°, and 180° from the sun, covering half the sky only. Unless the sky is cloudy the point that falls nearest the sun on azimuth circle 0° is usually too bright to measure with the screens at our disposal. There are therefore 102 photometric readings in each series. In addition, with a clear sky the intensity of the total illumination from the sun and sky is measured on a horizontal surface, and on a surface normal to the direct solar rays, and the illumination from skylight alone is measured on these two surfaces and also on vertical surfaces facing 0° and 45° in azimuth from the sun. If the sky is cloudy the illumination is measured on a horizontal surface, and on vertical surfaces facing 0°, 45°, 90°, 135°, and 180° in azimuth from the sun. There are therefore eighteen photometric readings (six sets of three readings each) in each series of illumination measurements. It is usually a matter of chance whether the readings are on azimuth circles to the right or to the left of the sun. Unless there is inequality in the cloud or haze distribution, or in the character of the earth's surface on the two sides of the sun's vertical, the sky brightness on the two sides should be symmetrical.

TABLE I.—NUMBER OF SERIES OF SKY-BRIGHTNESS MEASUREMENTS
WASHINGTON, D. C.

Solar altitude	0°	20°	40°	60°	70°	Total
Clear sky	9	62	122	23	11	227
Thin clouds	2	34	36	16	......	88
Partly cloudy sky	1	21	57	24	17	120
Cloudy sky	6	30	26	21	4	87
Rain or snow	......	3	6	8	......	17
Total	18	150	247	92	32	539

CHICAGO, ILL., FEDERAL BUILDING

Solar altitude	0°	20°	26°	40°	60°	Total
Clear sky	6	13	3	8	3	33
Thin clouds	......	8	5	2	6	21
Partly cloudy sky	5	9	......	7	9	30
Cloudy sky	4	6	3	1	1	15
Total	15	36	11	18	19	99

UNIVERSITY OF CHICAGO

Solar altitude	0°	20°	29°	40°	60°	Total
Clear sky	2	11	3	9	10	35
Thin clouds	.....	9	5	2	2	18
Partly cloudy sky	6	4	......	5	6	21
Cloudy sky	2	7	2	1	1	13
Total	10	31	10	17	19	87

The attempt is made to obtain complete series of sky-brightness and illumination measurements when the sun is 0°, 20°, 40°, 60°, and 70° above the horizon. On account of the length of the day the readings at 0° and 20° solar altitude are generally omitted in midsummer, and in winter the sun does not reach an altitude much in excess of 40°. In fact, at the Federal Building, Chicago, in January, the average altitude of the sun at the time of making the noon readings was 26°, and at the University of Chicago, at the end of January and early in February it was 29°. When rain was falling, only sky-brightness measurements up to an altitude of 60° could be made by pointing the photometer out of a window.

Table I gives the number of series of sky-brightness measurements obtained at Washington and Chicago with the sun at the altitudes indicated and with the different types of sky. In all

TABLE II.—AVERAGES OF SKY-BRIGHTNESS MEASUREMENES EXPRESSED IN TERMS OF ZENITH BRIGHTNESS. WASHINGTON, D. C.

Solar altitude	Azimuth from sun	Point in sky where brightness was measured: Altitude 2°	15°	30°	45°	60°	75°	90°	Zenith brightness mL.
		CLEAR SKY, WINTER							
0°.......	0	12.66	9.61	4.33	2.40	1.44	1.15	1.00	27.1
	45	4.40	4.76	2.83	1.98	1.36	1.15		
	90	2.43	2.76	2.08	1.47	1.20	1.10		
	135	2.99	3.43	2.41	1.58	1.17	1.03		
	180	3.83	4.20	2.74	1.77	1.25	1.08		
20°......	0	23.52	......	9.78	4.27	2.26	1.42	1.00	281
	45	8.91	5.54	3.76	2.57	1.75	1.27		
	90	3.95	2.60	1.69	1.30	1.10	1.06		
	135	3.92	2.34	1.42	1.01	0.87	0.88		
	180	4.38	2.55	1.42	0.99	0.81	0.79		
40°......	0	8.29	6.81	9.44	......	2.72	1.53	1.00	544
	45	4.56	3.45	2.77	2.21	1.76	1.36		
	90	2.62	1.66	1.12	1.00	0.89	0.95		
	135	2.51	1.33	0.82	0.68	0.66	0.75		
	180	2.76	1.44	0.81	0.60	0.58	0.69		
		CLEAR SKY, SUMMER							
20°......	0	21.10	......	10.85	4.06	2.34	1.40	1.00	400
	45	7.72	5.90	3.91	2.74	1.94	1.31		
	90	3.42	2.81	1.78	1.35	1.07	1.14		
	135	2.48	1.86	1.23	0.83	0.72	0.82		
	180	2.85	2.13	1.19	0.85	0.74	0.76		
40°......	0	7.35	6.19	8.43	......	2.75	1.58	1.00	803
	45	4.15	3.13	2.69	2.45	1.88	1.42		
	90	2.13	1.41	1.13	0.97	0.99	1.04		
	135	1.74	1.13	0.74	0.60	0.62	0.74		
	180	1.83	1.11	0.68	0.54	0.53	0.68		
60°......	0	2.08	1.74	1.84	2.51	......	1.65	1.00	1,650
	45	1.74	1.53	1.30	1.53	1.67	1.41		
	90	1.16	0.88	0.72	0.74	0.88	0.99		
	135	0.95	0.61	0.47	0.50	0.58	0.75		
	180	1.00	0.64	0.47	0.50	0.54	0.72		
70°......	0	1.45	1.29	1.13	1.60	3.30	......	1.00	2,300
	45	1.25	0.90	0.89	1.07	1.33	1.53		
	90	0.77	0.65	0.56	0.54	0.62	0.89		
	135	0.62	0.42	0.37	0.38	0.44	0.66		
	180	0.58	0.39	0.31	0.31	0.41	0.61		
		CLOUDY SKY, WINTER							
20°......	0	0.39	0.59	0.80	0.93	1.03	1.04	1.00	989
	45	0.40	0.55	0.67	0.84	0.97	0.97		
	90	0.31	0.56	0.68	0.81	0.90	0.92	Max.=2,211	
	135	0.31	0.49	0.66	0.80	0.86	0.94		
	180	0.32	0.54	0.67	0.83	0.94	0.98	Min.=245	

there are about 55,000 photometric readings of sky brightness and 9,000 photometric readings of illumination intensity at Washington and 19,000 and 2,200, respectively, at Chicago.

In Tables II and III are summarized the sky-brightness measurements made at Washington and Chicago, respectively, during

TABLE III.—AVERAGE OF SKY-BRIGHTNESS MEASUREMENTS EXPRESSED IN TERMS OF ZENITH BRIGHTNESS. CHICAGO, ILL.

Place	Solar altitude	Point in sky where brightness was measured.								Zenith brightness
		Azimuth from sun	Altitude							
			2°	15°	30°	45°	60°	75°	90°	mL.
		CLEAR SKY, WINTER.								
University	0°	0	9.80	8.04	3.67	2.03	1.38	1.08	1.00	19.4
		45	3.04	4.06	2.14	1.50	1.13	0.87		
		90	1.03	2.36	1.79	1.46	1.29	1.07		
		135	1.31	2.46	1.84	1.47	1.18	0.78		
		180	0.80	2.84	3.34	1.64	1.22	0.96		
Federal Building	0°	0	1.80	4.41	4.00	1.21	1.48	1.13	1.00	19.7
		45	0.85	2.11	2.04	1.54	1.36	1.10		
		90	0.80	1.29	1.27	1.24	1.03	0.99		
		135	0.88	1.88	1.00	1.43	1.05	1.08		
		180	1.11	2.12	1.85	1.66	1.25	1.09		
University	20°	0	16.38	. . .	9.18	4.25	2.21	1.22	1.00	345
		45	6.38	5.00	3.59	2.58	1.93	1.33		
		90	2.51	2.22	1.59	1.30	1.40	1.05		
		135	2.06	1.50	1.25	0.98	0.95	0.85		
		180	2.03	1.75	1.14	0.89	0.72	0.77		
Federal Building	20°	0	13.56	. . .	9.86	4.37	2.34	1.55	1.00	340
		45	5.30	4.87	3.37	2.59	1.96	1.16		
		90	1.98	1.93	1.70	1.35	1.31	1.00		
		135	1.24	1.27	1.01	0.75	0.66	0.72		
		180	1.32	1.45	1.08	0.79	0.72	0.82		
University	29°	0	13.65	18.80	. . .	3.72	2.61	1.37	1.00	433
		45	5.32	4.11	3.50	2.59	1.92	1.32		
		90	2.59	1.96	1.49	1.07	1.02	1.03		
		135	1.89	1.65	1.01	0.82	0.71	0.78		
		180	2.13	1.78	1.05	0.82	0.77	0.71		
Federal Building	26°	0	7.81	15.73	12.14	4.96	2.44	1.46	1.00	511
		45	3.84	4.12	4.02	2.85	2.04	0.94		
		90	0.95	1.20	1.39	0.93	0.79	1.00		
		135	0.73	0.72	0.73	0.73	0.73	0.89		
		180	0.65	0.80	0.70	0.58	0.63	0.75		
		CLOUDY SKY, WINTER.								
University	20°	0	0.29	0.52	0.86	1.06	1.07	1.08	1.00	614
		45	0.30	0.47	0.73	0.89	0.96	0.99		
		90	0.29	0.41	0.63	0.86	0.91	0.97		Max.=1,074
		135	0.30	0.42	0.58	0.71	0.86	0.88		
		180	0.25	0.37	0.56	0.73	0.94	0.91		Min.=84
Federal Building	20°	0	0.38	0.56	0.83	1.02	1.23	1.30	1.00	500
		45	0.33	0.61	0.56	0.78	0.93	1.10		
		90	0.42	0.64	0.74	0.92	1.09	1.10		Max.=1.034
		135	0.34	0.43	0.60	0.98	1.13	1.08		
		180	0.33	0.56	0.80	0.84	0.70	0.93		Min.=132

TABLE III. (CONTINUED)—AVERAGE OF SKY-BRIGHTNESS MEASUREMENTS EXPRESSED IN TERMS OF ZENITH BRIGHTNESS
CHICAGO, ILL.

Place.	Solar altitude	Point in sky where brightness was measured: Azimuth from sun	Altitude 2°	15°	30°	45°	60°	75°	90°	Zenith brightness mL.
			CLEAR SKY, SUMMER							
University	8°	0	12.29	. . .	5.64	2.37	1.18	1.19	1.00	231
		45	5.46	5.09	2.65	1.83	1.17	0.90		
		90	3.43	2.37	1.35	1.07	0.95	0.76		
		135	2.62	2.18	1.36	0.96	0.72	0.84		
		180	2.73	2.69	1.64	0.91	0.79	0.86		
Do	20°	0	14.35	. . .	11.44	5.07	2.71	1.62	1.00	528
		45	5.91	5.04	3.88	2.62	1.86	1.32		
		90	2.77	2.26	1.61	1.25	1.09	1.04		
		135	1.69	1.60	1.04	0.73	0.68	0.72		
		180	1.58	1.78	1.04	0.71	0.64	0.70		
Federal Building . .	20°	0	15.08	13.73	13.58	6.19	2.79	2.20	1.00	419
		45	4.76	4.81	4.54	3.27	1.97	1.37		
		90	1.65	1.56	1.32	1.06	0.91	0.90		
		135	1.50	1.51	0.91	0.73	0.71	0.80		
		180	1.20	1.13	0.76	0.54	0.55	0.59		
University	40°	0	6.29	5.67	7.28	. . .	2.66	1.63	1.00	810
		45	3.72	3.07	2.70	2.57	2.16	1.49		
		90	1.68	1.41	1.15	1.09	0.96	0.98		
		135	1.73	1.05	0.77	0.73	0.72	0.83		
		180	1.26	0.97	0.61	0.48	0.50	0.62		
Federal Building . .	40°	0	4.50	5.56	6.35	. . .	3.23	1.76	1.00	900
		45	1.51	3.09	3.38	2.69	2.17	1.37		
		90	1.16	1.19	0.99	. . .	0.95	. . .		
		135	0.67	0.57	0.49	0.43	0.49	0.70		
		180	0.97	0.78	0.51	0.41	0.47	0.68		
University	60°	0	1.75	1.88	2.06	2.44	. . .	1.67	1.00	1920
		45	1.32	1.23	1.25	1.38	1.52	1.30		
		90	0.84	0.72	0.64	0.82	0.88	0.88		
		135	0.80	0.69	0.52	0.52	0.63	0.82		
		180	0.65	0.55	0.45	0.50	0.50	0.66		
Federal Building . .	60°	0	1.63	1.88	2.19	3.24	. . .	1.83	1.00	1360
		45	1.36	1.33	1.11	1.54	1.36	1.40		
		90	0.94	0.67	0.61	0.58	0.72	1.09		
		135	0.96	0.69	0.56	. . .	0.63	. . .		
		180	1.00	0.63	0.39	0.38	0.43	0.60		

summer and winter months with a cloudless sky, and during winter months with the sun 20° above the horizon and the sky covered with clouds. It will be noted from Table I that at Chicago most of the sky-brightness measurements with a cloudy sky were obtained when the sun was at an altitude of 20°.

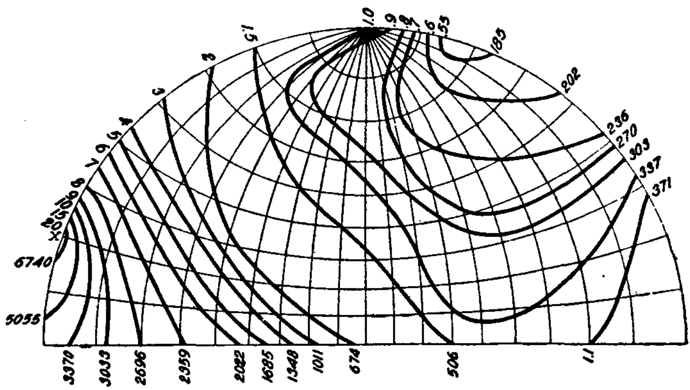

Fig. 2.—Sky brightness in millilamberts. Sun's position indicated by X. Federal Building, Chicago, Ill., cloudless sky with dense smoke.

Referring again to Figure 1, the irregularly curved lines are lines of equal brightness that have been drawn to represent the brightness of the sky at Washington on the morning of February 17, 1922, with the sun at an altitude of 20°, and the ground covered with newly fallen crusted snow. The figures on the left above the sun represent the brightness of the sky with reference to the zenith brightness; the figures on the right and at the bottom of the figure, the brightness of the sky in millilamberts. The sky 90° from the sun and in his vertical was a deep blue and unusually dark. Near the horizon it was unusually bright on account of the reflection of light to the atmosphere from the snow surface, and especially beyond 90° in azimuth from the sun.

Figure 2 shows the brightness of the sky as measured from the top of the dome on the Federal Building, Chicago, Ill., on the morning of January 16, 1922, with no clouds in the sky, but heavy smoke in the lower atmosphere. The sun was at altitude 20°, and the ground was covered with snow, as was the case at Washington on February 17, but the snow was not clean. Compared with Figure 1, Figure 2 gives a brighter zenith, a point of minimum that is less bright, and a horizon beyond azimuth 90° from the sun only about one-fourth as bright.

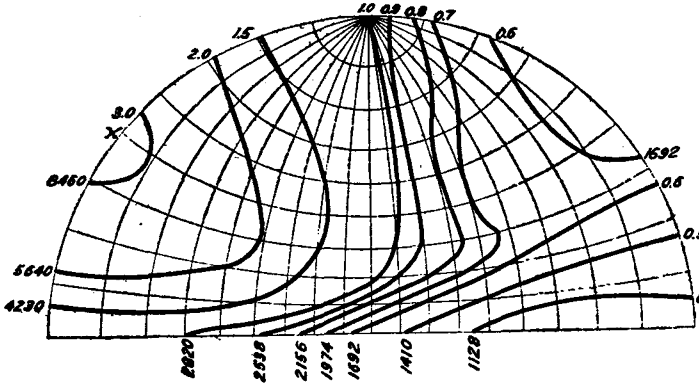

Fig. 3.—Sky brightness in millilamberts. Sun's position indicated by X. Washington, D. C., sky covered with dense haze.

Figure 3 represents the sky brightness at Washington on the morning of July 5, 1921, with the sky covered with dense haze, but without clouds, and the sun 40° above the horizon. The sky is much brighter than a clear blue sky, except near the horizon opposite the sun.

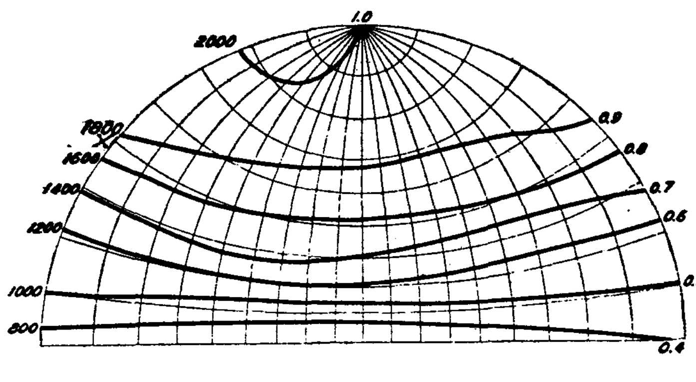

Fig. 4.—Sky brightness in millilamberts. Sun's position indicated by X. Washington, D. C., sky covered with dense clouds.

Figure 4 represents the mean of all the sky-brightness measurements at Washington with the sun 40° above the horizon, and the clouds so dense that neither blue sky nor the sun could be seen. The brightest point is near the zenith, and there is little variation in brightness with azimuth. The zenith and in general the sky opposite the sun is brighter when covered with clouds than when clear, but near the horizon and in the vicinity of the sun the clear sky is much the brighter. Thin clouds, and clouds that partly cover the sky, increase its brightness much the same as does haze.

ILLUMINATION FROM SKYLIGHT ON HORIZONTAL AND VERTICAL SURFACES

During the past year the energies of the committee, aside from the observational work, have been directed principally to computing from the clear-sky-brightness measurements, as summarized in Tables II and III, the resulting illumination on vertical surfaces facing in azimuth 70°, 90°, 135°, and 180° from the sun. The process is a simple one. As explained in the 1921 report,[3]

TABLE IV.—Illumination from Skylight, Washington, D. C.

Solar altitude	On horizontal surface	On vertical surface: Azimuth between normal to surface and sun's azimuth 0°	45°	70°	90°	135°	180°	Mean	Zenith brightness
	Foot-candles								mL.
	CLOUDY SKY								
0°	15.2	5.6	5.8	. . . .	6.4	6.7	7.1	6.3	15.8
20.2°	726	298	280	. . . .	273	273	272	279	989
41.0°	1,505	614	608	. . . .	615	622	606	613	2,000
61.4°	2,150	881	941	. . . .	977	932	929	932	3,600
71.4°	2,950	1,142	1,103	. . . .	1,118	1,122	1,203	1,138	4,840
	CLEAR SKY, SUMMER								
20°	840	1,252	1.028	803	526	316	293	. . . .	400
40°	1,340	1,454	1,325	932	686	417	358	. . . .	803
60°	1,600	1,420	1,255	923	751	559	486	. . . .	1,650
70°	1,600	1,291	1,074	903	754	542	475	. . . .	2,300
	CLEAR SKY, WINTER								
0°	67.8	64.6	63.7	. .	30.6	30.2	31.5	. . . .	27.1
20°	683	1,042	873	562	393	265	257	. . . .	281
40°	977	1,121	936	690	505	325	295	. . . .	544

[3] Trans., Illum. Eng. Soc., Vol. XVI, p. 267; *Mo. Weather Rev.*, Sept., 1921, 49, p. 485

the sky is divided into zones of equal angular width about a point on the horizon 90° in azimuth from the illuminated surface, and the horizontal component of the illumination from each zone is determined. The sum of the illumination from all the zones gives the total skylight illumination on the vertical surface.

Table IV summarizes the results of these computations from Washington measurements, and also the illumination measurements, for both clear and cloudy skies. The cloudy-sky measurements have been confined to skies with so dense a cloud layer that the position of the sun could not be seen. No seasonal variation in the illumination intensity is apparent. With clear skies the computations have been made for both midsummer (June to August) and midwinter (December to February) conditions.

These data have been plotted on Figure 5 (summer conditions) and Figure 6 (winter conditions) with the solar altitude as abscissas and illumination intensities as ordinates. By interpolating between the curves it is possible to determine the illumination intensity for both summer and winter conditions on a vertical surface facing at any desired azimuth from the sun, and with the sun at any desired altitude. For spring and fall months a straight-line interpolation has been made between winter and summer values.

Measurements with the sun on the horizon were made during the winter months only, and these measurements have been used for summer as well. With the sun at altitudes 20° and 40° it will be noted that the zenith brightness in winter is approximately 70 per cent of the corresponding brightness in summer. The percentage of winter to summer illumination is somewhat greater than this, since the brightness of the sky near the horizon in terms of the zenith brightness is greater in winter than in summer.

In the *Monthly Weather Review* for November, 1919, **47**, pp. 770-771, are given the altitude and azimuth of the sun for the 21st day of each month and for even-hour angles of the sun from the meridian, for latitudes 30°, 36°, 42°, and 48° north. Using the azimuths and altitudes for latitude 42° N., in connection with

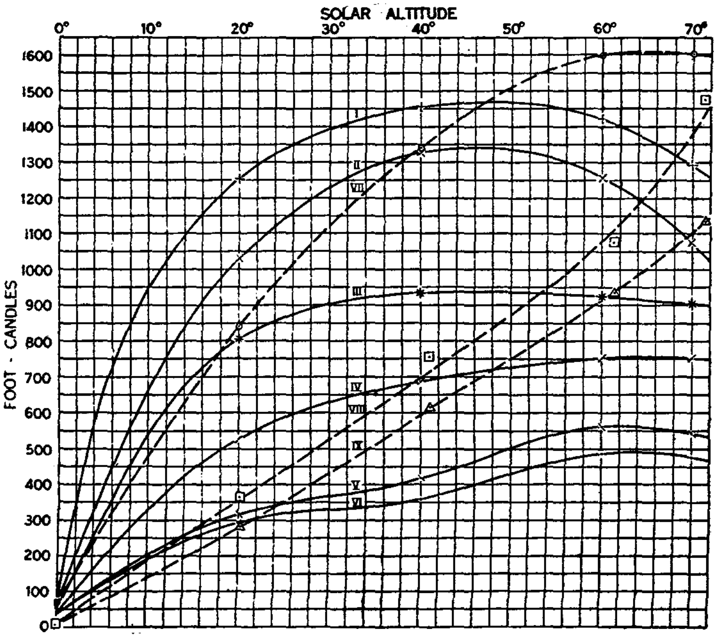

Fig. 5.—Curves of summer skylight illumination intensity on different surfaces.
Curve I. Clear sky. Vertical surface facing 0° in azimuth from sun.
Curve II. Clear sky. Vertical surface facing 45° in azimuth from sun.
Curve III. Clear sky. Vertical surface facing 70° in azimuth from sun.
Curve IV. Clear sky. Vertical surface facing 90° in azimuth from sun.
Curve V. Clear sky. Vertical surface facing 135° in azimuth from sun.
Curve VI. Clear sky. Vertical surface facing 180° in azimuth from sun.
Curve VII. Clear sky. Horizontal surface.
Curve VIII. Cloudy sky. Horizontal surface. (Note: Double the intensity scale).
Curve IX. Cloudy sky. Vertical surface.

the illumination intensity curves of Figures 5 and 6, Figures 7 to 18 have been drawn. Latitude 42° N. was selected because many important industrial districts are near this latitude, and also because measurements made at Washington, latitude 38° 56′ N., and Chicago, Ill., latitude 41° 53′ N., represent fairly well the sky brightness at this latitude east of the Mississippi River. Farther west the clear sky is generally a deeper blue and not so bright. Probably clear skies average brighter in low than in high latitudes, especially in winter. This conclusion is supported by Little's measurements made near Key

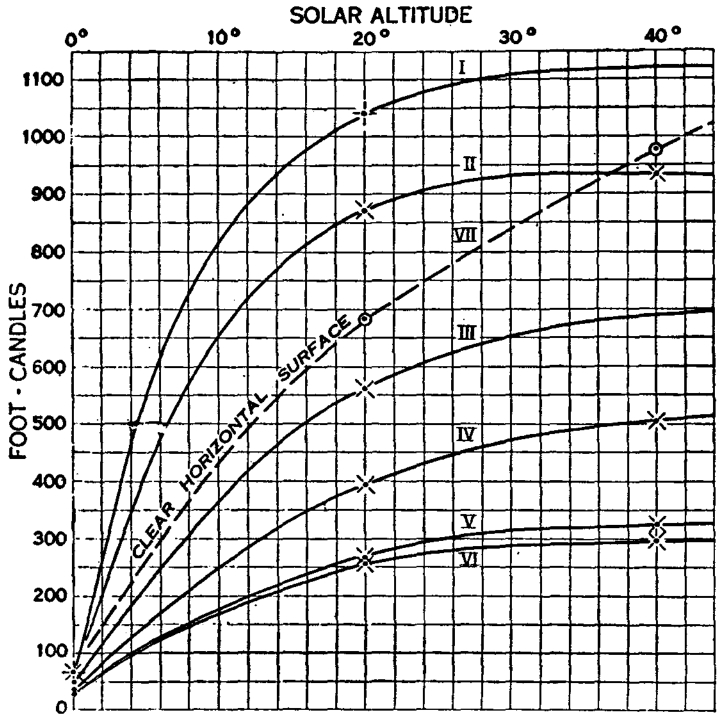

Fig. 6.—Curves of winter skylight illumination intensity on different surfaces.
Curve I. Clear sky. Vertical surface facing 0° in azimuth from sun.
Curve II. Clear sky. Vertical surface facing 45° in azimuth from sun.
Curve III. Clear sky. Vertical surface facing 70° in azimuth from sun.
Curve IV. Clear sky. Vertical surface facing 90° in azimuth from sun.
Curve V. Clear sky. Vertical surface facing 135° in azimuth from sun.
Curve VI. Clear sky. Vertical surface facing 180° in azimuth from sun.
Curve VII. Clear sky. Horizontal surface.

West, Fla., in February, 1918, which give for the zenith sky brightness with the sun at altitudes averaging 22.8°, 42°, and 53°, 390, 780, and 1,150 millilamberts, respectively; while measusements made by him from a ship off Long Island, N. Y., in October, 1917, give for the sky brightness with solar altitudes averaging 20.8°, and 41.2°, 296 and 495 millilamberts, respectively. The latter are somewhat lower readings than those obtained at Washington in winter with similar solar altitudes, while the Key West measurements are considerably higher.

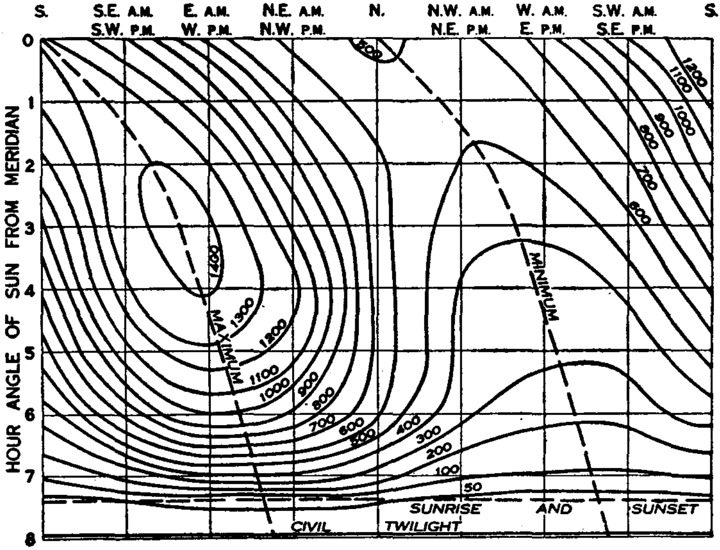

Fig. 7.—Variations in skylight illumination on vertical surfaces differently oriented at latitude 42° north on July 21. Cloudless sky. Foot-candles.

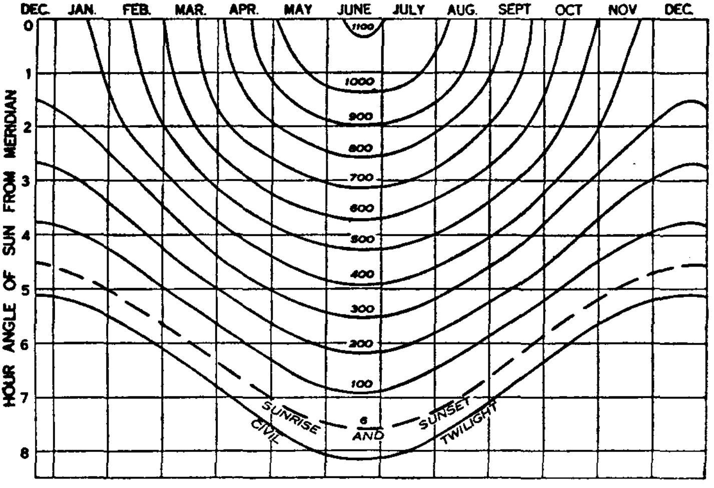

Fig. 8.—Illumination from a cloudy sky on a vertical surface at latitude 42° north. Foot-candles.

Figure 7 shows the variations with the hour of the day in skylight illumination on vertical surfaces, such as the walls of buildings, differently oriented, on July 21, at latitude 42° N. with a clear sky. The maximum illumination is, of course, on a vertical surface facing the sun. It faces about east-northeast at sunrise, east at about 7:30 A. M., south at noon, west at about 4:30 P. M., and about west-northwest at sunset. The minimum illumination is on a vertical surface facing 180° in azimuth from the sun, or about west-southwest at sunrise, west at about 7:30 A. M., north at noon, east at about 4:30 P. M., and about east-southeast at sunset. Taking into consideration the hours between 7 A. M. and 5 P. M., which cover the usual working day, in the morning vertical surfaces facing northwest are most unfavorably oriented for illumination from a clear sky, and in the afternoon vertical surfaces facing northeast. On the other hand, surfaces facing northwest are favorably oriented for skylight illumination in the afternoon, and those facing northeast, in the morning.

From Table IV and Figures 8, 9 and 10 we derive the following:

(1) With a cloudy sky the illumination on a vertical surface is practically independent of the orientation of that surface.

(2) With a cloudy sky the illumination on a horizontal surface is considerably more than twice that on a vertical surface, due to the fact that the point of maximum brightness of a cloudy sky is in or near the zenith.

(3) With high sun, as at midday in summer, the illumination from a cloudy sky exceeds that from a clear sky except on vertical surfaces facing the sun. This is not true with low sun, however.

The eight figures, 11 to 18, inclusive, give the illumination from clear skies on vertical surfaces oriented as indicated. Were it not for the fact that clear skies in July, August, September, October, and November, are on the average whiter and therefore brighter than clear skies in May, April, March, February, and January, respectively, the lines of equal illumination intensity would be nearly symmetrical on each side of a vertical line representing June 21.

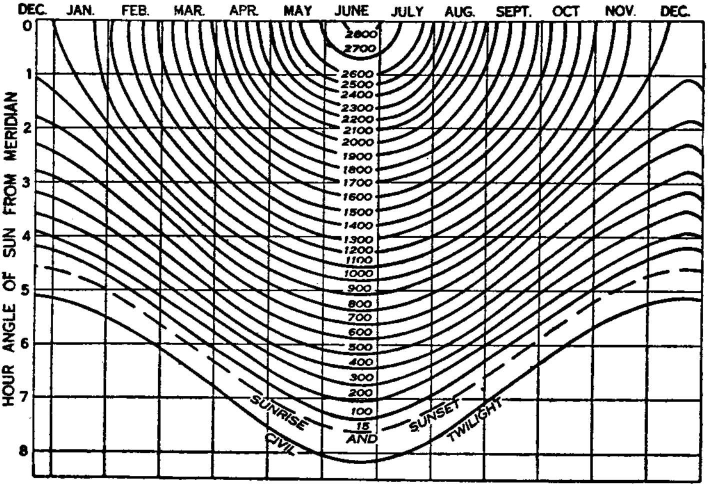

Fig. 9.—Illumination from a cloudy sky on a horizontal surface at latitude 42° north. Foot-candles.

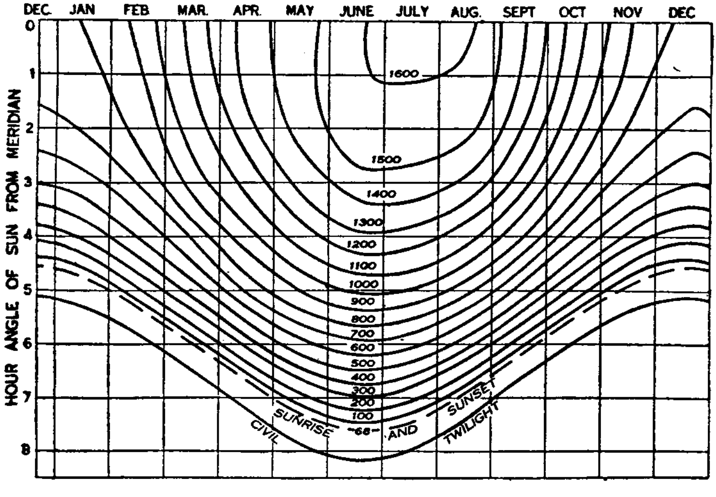

Fig. 10.—Illumination from a cloudless sky on a horizontal surface at latitude 42° north. Foot-candles.

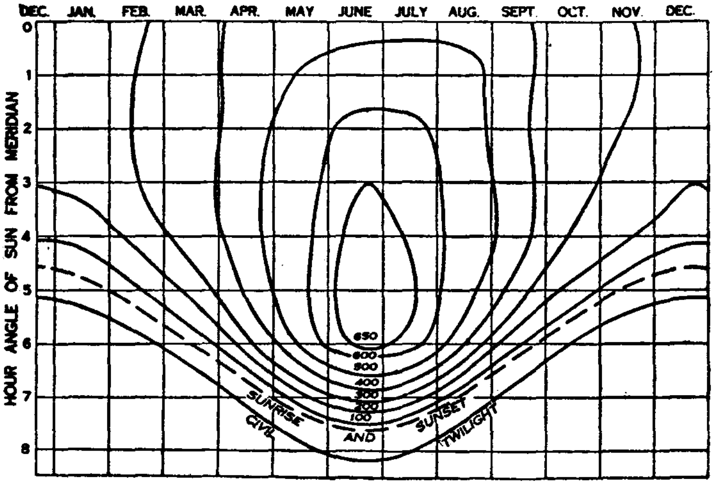

Fig. 11.—Illumination from a cloudless sky on a vertical surface facing north at latitude 42° north. Foot-candles.

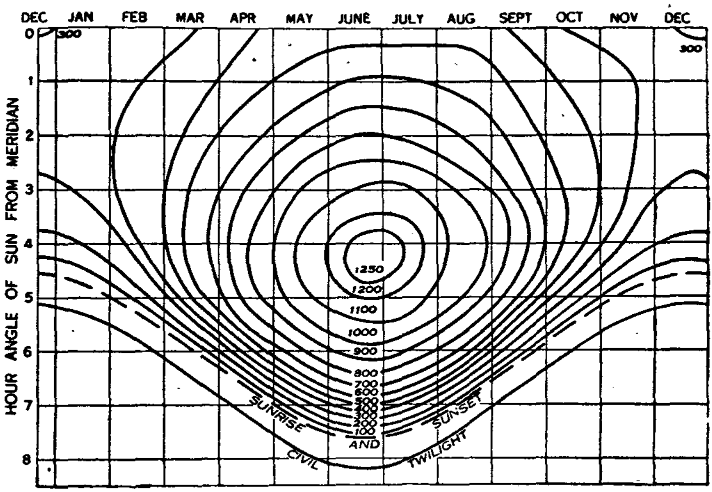

Fig. 12.—Illumination from a cloudless sky on a vertical surface facing northeast, A. M., or northwest, P. M., at latitude 42° north. Foot-candles.

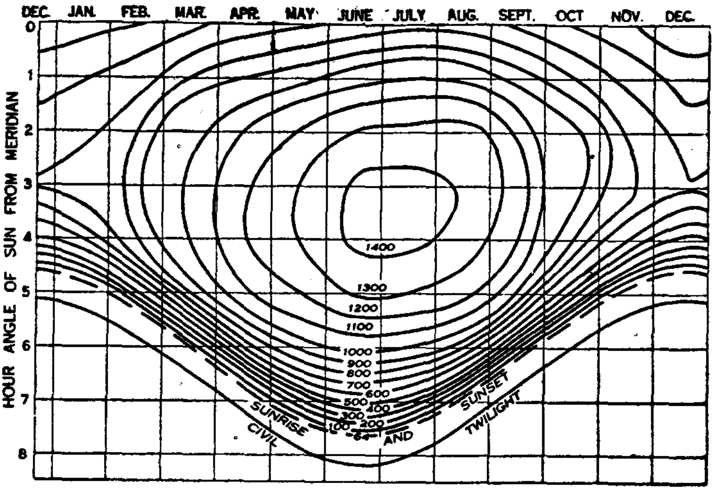

Fig. 13.—Illumination from a cloudless sky on a vertical surface facing east, A. M., or west, P. M., at latitude 42° north. Foot-candles

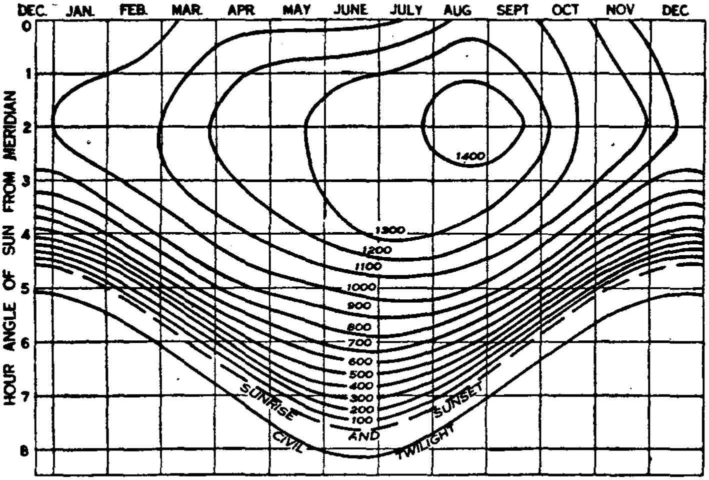

Fig. 14.—Illumination from a cloudless sky on a vertical surface facing southeast, A. M. or southwest, P. M., at latitude 42° north. Foot-candles.

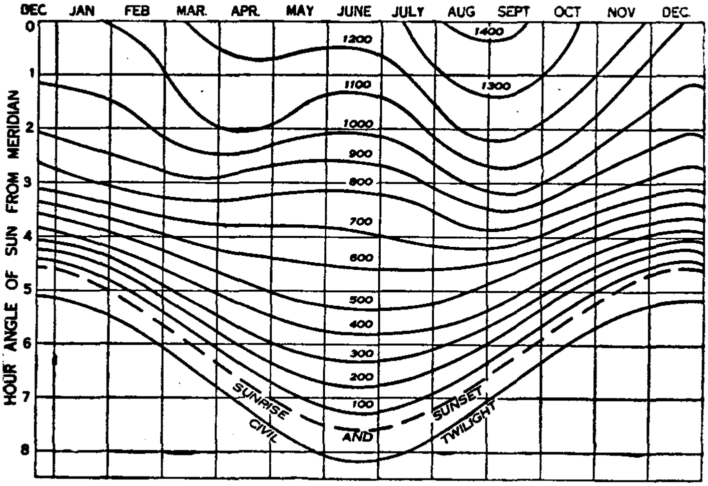

Fig. 15.—Illumination from a cloudless sky on a vertical surface facing south at latitude 42° north. Foot-candles.

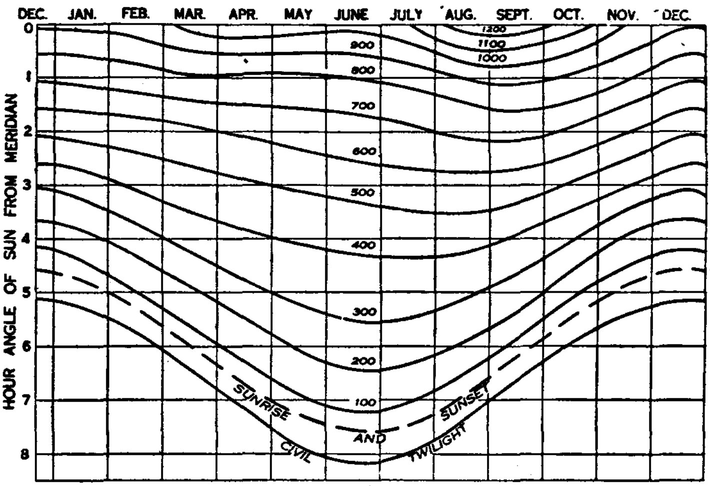

Fig. 16.—Illumination from a cloudless sky on a vertical surface facing southwest, A. M., or southeast, P. M., at latitude 42° north. Foot-candles.

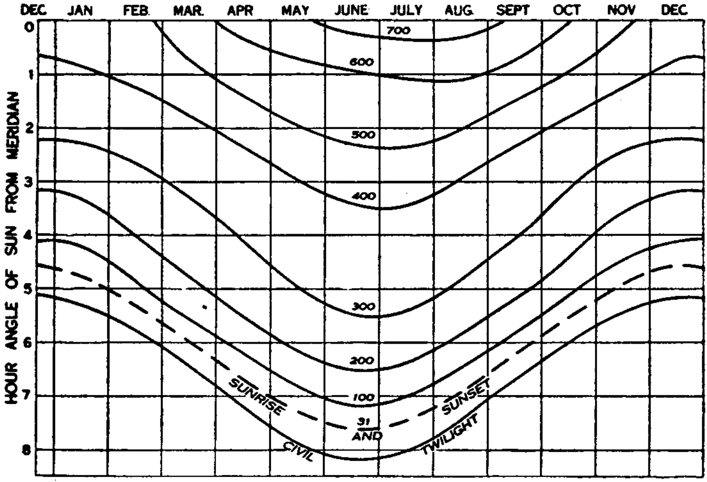

Fig. 17.—Illumination from a cloudless sky on a vertical surface facing west, A. M., or east, P. M., at latitude 42° north. Foot-candles.

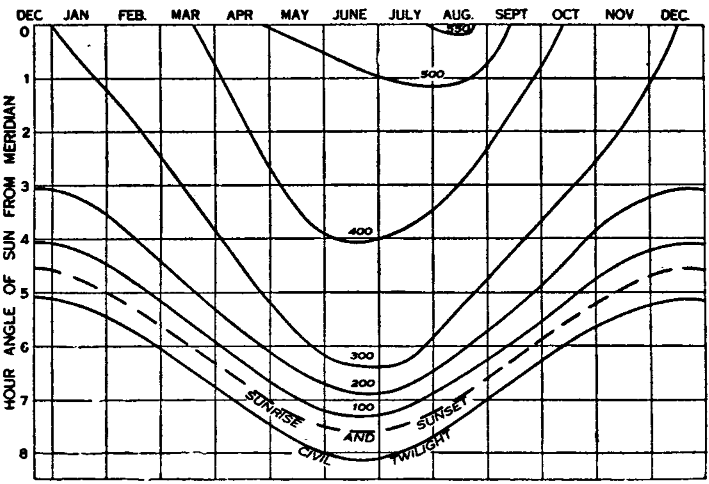

Fig. 18.—Illumination from a cloudless sky on a vertical surface facing northwest, A. M., or northeast, P. M., at latitude 42° north. Foot-candles.

These figures, like Figure 7, show the low intensity of clear-sky illumination during the morning hours on a vertical surface facing northwest and during the afternoon hours on a vertical surface facing northeast. The maximum illumination, slightly in excess of 1,400 foot-candles, occurs late in August and in early September at midday, on a vertical surface facing south; from the end of July to the latter part of September, on a vertical surface facing southeast at 10 A. M., and southwest at 2 P. M., and from early in June to the middle of August, on a vertical surface facing east at 8:30 A. M., and west at 3:30 P. M. Illumination on a vertical surface facing north is good throughout the day in summer but poor in winter.

In the report for 1921[4] it was shown that at the Federal Building, Chicago, which is in the smoky Loop District, the illumination on a vertical surface facing 180° from the sun is only about two-thirds as intense as at Washington, while at the University of Chicago the sky-brightness and the illumination measurements differ but little from the corresponding Washington measurements. Table V gives a summary of comparisons between Washington and Chicago illumination measurements on a horizontal surface, and on a vertical facing 180° from the sun, under winter conditions.

TABLE V.—RATIO, CHICAGO/WASHINGTON ILLUMINATION FROM WINTER SKIES

ON HORIZONTAL SURFACE					
Federal Building			University of Chicago		
Solar altitude	Clear sky	Cloudy sky	Solar altitude	Clear sky	Cloudy sky
0°	0.49	0.90	0°	0.48	0.69
20°	1.05	0.77	20°	1.03	0.56
26°	0.84	0.82	29°	1.14	0.76
ON VERTICAL SURFACE FACING 180° FROM THE SUN					
0°	0.42	0.85	0°	0.46	0.55
20°	0.69	0.54	20°	0.84	0.56
26°	0.62	0.88	29°	0.97	0.98

[4] *Mo. Weather Rev.*, Sept., 1921, 49, p. 482 and p. 486.

The darkening effect of the smoke is rather more pronounced in winter than in summer on a vertical surface facing away from the sun. The effect is slight at both seasons of the year on surfaces facing the sun when no clouds are present, except when the sun is near the horizon. The effect is closely related to the velocity of the wind. With light wind, and especially when the sky is covered with clouds, the smoke sometimes forms a cover or blanket of great thickness which cuts off practically all the daylight. A dark day results, and artificial lighting is necessary outdoors as well as in. No such days are included in the sky-brightness measurements for Chicago here considered, although on January 4, with the sun 20° above the horizon, the zenith brightness was only 150 millilamberts, and 2° above the horizon it averaged only 37 millilamberts, while a measurement of the illumination on a horizontal surface gave only 34 foot-candles. A comparison with the corresponding data of Table IV shows that the zenith brightness was 15 per cent and the illumination on a horizontal surface 5 per cent that for Washington with average cloudy conditions and the sun 20° above the horizon. The measurements show that the smoke cloud varied greatly in intensity during the period of observation.

With a cloudless sky, and solar altitude 20°, in winter the intensity of direct solar illumination at normal incidence at Chicago averages about half the intensity at Washington; in summer, in the Loop District, with solar altitudes 20° and 40°, about three-fourths as intense.

At Washington, with a clear sky, the illumination measurements on both a horizontal and on a vertical surface vary between 150 per cent and 60 per cent of the values given in Table IV. With a cloudy sky the variation is between 200 and 30 per cent. When rain is falling, the illumination is about half as great as the average for cloudy skies; with a sky partly covered with clouds, the illumination on a horizontal surface may be from three to four times as intense, and on a vertical surface two to three times as intense, as the corresponding illumination from a clear sky given in Table IV.

From seasonal averages of sky brightness for Davos Platz, Switzerland, given by Dorno,[5] it appears that when expressed in terms of the zenith brightness the sky at Davos Platz opposite the sun is brighter than at Washington. The zenith brightness in winter averages more than 50 per cent brighter, and in summer a few per cent less bright at Davos Platz than at Washington. On the whole, Davos Platz skies when free from clouds are brighter in winter and less bright in summer than at Washington. Probably the increased brightness in winter is due in part to reflection of light from the snow-covered surface.

TOTAL SOLAR AND SKY ILLUMINATION

In the *Monthly Weather Review* for November, 1919, **47**, p. 785, Table XIV, are given the illumination equivalents of solar energy expressed in heat units, with the sun at different altitudes. These equivalents were derived from simultaneous readings made at Mount Weather, Va., in 1913-14, with a pyrheliometer and a photometer. The photometer had its uncompensated test plate exposed horizontally, and the error, due to the oblique angle at which the sun's rays were received, was unknown.

TABLE VI.—ILLUMINATION EQUIVALENT OF 1 GRAM-CALORY PER MINUTE PER SQUARE CENTIMETER OF SOLAR ENERGY WITH THE SUN AT DIFFERENT ALTITUDES

Air mass. . . .	1.06	1.10	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50
Solar altitude	70°0	65°0	42°7	30°0	23°5	19°3	16°4	14°3	12°6	11°3	10°2
Foot-candles .	7,040	7,020	6,880	6,740	6,650	6,580	6,520	6,460	6,410	6,370	6,320

In the measurements made at Washington in 1921-22 a *compensated* test plate was used, and the certificate furnished by the Electrical Testing Laboratories, New York, shows no appreciable error due to an obliquity in the angle of incidence of the sun's rays. Illumination intensities were measured with the test plate horizontal and also normal to the incident solar rays, but the latter measurements were given twice the weight of the former. Comparison of these measurements with simultaneous pyrheliometric measurements give the illumination equivalents of Table VI. These are considerably higher than the equivalents determined at Mount Weather, and particularly with low sun, as one would expect.

[5]Dorno, C. Himmelshelligkeit, Himmelspolarisation und Sonnenintensität in Davos 1911 bis 1918. Veröffentlichungen des Preusischen Meteorlogischen Instituts., Nr. 303. Abhandlungen Bd. VI, Tabellen 4A und 6.

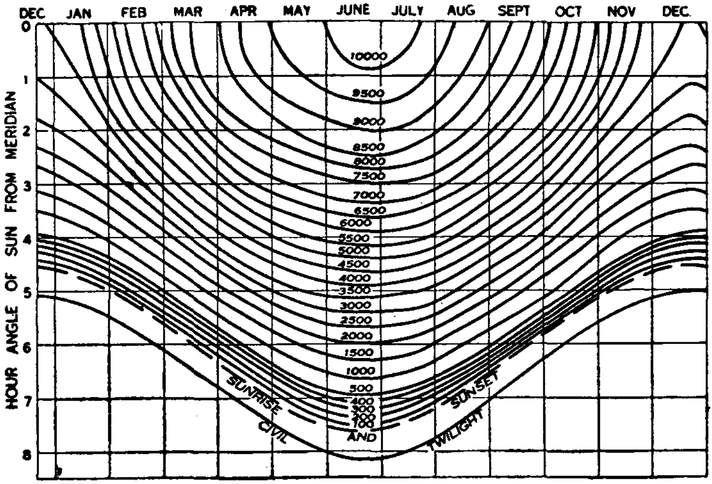

Fig. 19.—Total daylight illumination on a horizontal surface with a cloudless sky at latitude 42° north. Foot-candles.

By means of the equivalents of Table VI and the solar radiation intensity at normal incidence for latitude 42° N., given in the number of the *Review* above quoted (p. 773, Table Va), the solar illumination intensities of Table VII have been obtained.

TABLE VII.—SOLAR ILLUMINATION INTENSITY AT NORMAL INCIDENCE AT LATITUDE 42° NORTH, WITH A CLOUDLESS SKY (EAST OF THE MISSISSIPPI RIVER)

Day	Hour angle of sun from meridian							
	0	1	2	3	4	5	6	7
Dec. 21	7600	7300	6640	5190	2460	......	......	......
Jan. 21	8120	7890	7290	6040	3760	......	......	......
Feb. 21	9140	9040	8440	7450	6140	2460	......	......
Mar. 21	9270	9110	8710	7910	6700	4650	720	......
Apr. 21	9230	9060	8800	8300	7350	5860	3600	......
May 21	9070	8990	8630	8140	7480	6260	4700	1200
June 21	9080	9000	8740	8220	7430	6420	4880	2160
July 21	9070	8990	8670	8140	7550	6330	4830	1200
Aug. 21	8810	8710	8390	7830	6880	5460	2990	......
Sept. 21	8910	8760	8510	7710	6500	4590	720	......
Oct. 21	8510	8420	7960	6910	5220	2100	......	......
Nov. 21	8120	7890	7290	5960	3390	......	......	......

Representing the illumination intensities of Table VII by I_n, the illumination on a horizontal surface, I_h, and on a vertical surface, I_v, may be obtained by the equations

$$I_h = I_n \sin a \qquad (1)$$
$$I_v = I_n \cos a \cos \alpha \qquad (2)$$

where a is the altitude of the sun, and α is the difference between the sun's azimuth and the azimuth of a line normal to the vertical surface. The surface will be illuminated by the sun only when the value of α is less than 90°.

Adding the values of I_h to the skylight-illumination values for corresponding days and hours given on Figure 10, we obtain the total daylight illumination on a horizontal surface for a cloudless sky of average brightness at latitude 42° N., which is charted on Figure 19.

Similarly, by adding the values of I_v for vertical surfaces facing the eight principal points of the compass to the skylight illumination for corresponding days and hours given on Figures 11 to 16, inclusive, we obtain the total daylight illumination on vertical surfaces facing south; southeast A. M., or southwest P. M.; southwest A. M., or southeast P. M.; east A. M., or west P. M.; northeast A. M., or northwest P. M.; and north; as given on Figures 20 to 25, inclusive.

It is to be noted that with north solar declination all vertical surfaces receive direct solar radiation during only a part of the day. During the remainder of the day the total daylight illumination is the same as the skylight illumination on Figures 11 to 18, inclusive.

The data of Figures 1 to 25, inclusive, assume that the surface under consideration has an unobstructed exposure to the sky. Where a part of the sky is cut off by adjacent buildings or other obstructions, the shading effect of such obstructions may be determined by the method given in the previous report.[6]

This shading effect, and also the reflection of daylight from surrounding objects, will receive more detailed consideration in a later report.

[6] TRANS., Illum. Eng. Soc., Vol. XVI, p. 270; *Mo. Weather Rev.*, Sept., 1921, 49, p. 486.

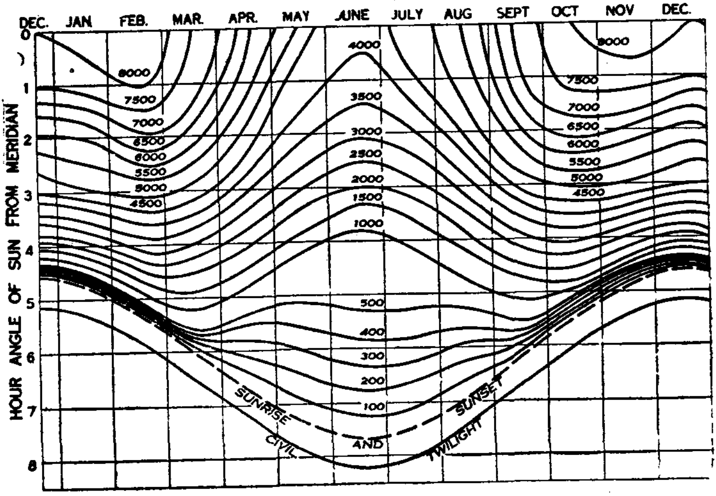

Fig. 20.—Total daylight illumination on a vertical surface facing south with a cloudless sky at latitude 42° north. Foot-candles.

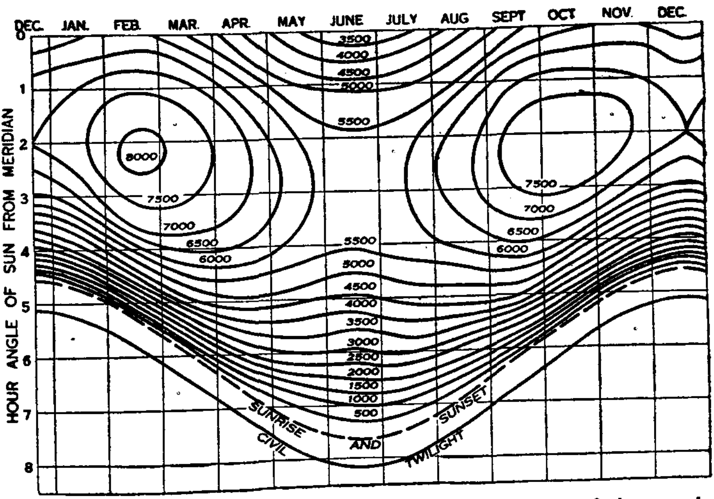

Fig. 21.—Total daylight illumination on a vertical surface facing southeast, A. M., or southwest, P. M., with a cloudless sky at latitude 42° north. Foot-candles.

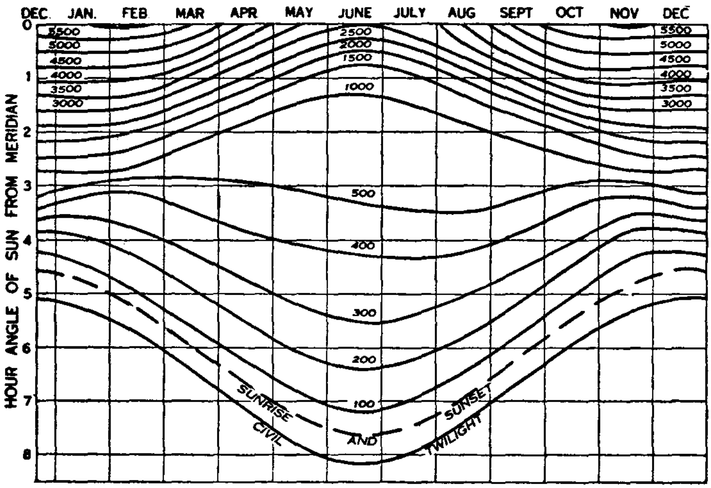

Fig. 22.—Total daylight illumination on a vertical surface facing southwest, A. M., or southeast P. M., with a cloudless sky at latitude 42° north. Foot-candles.

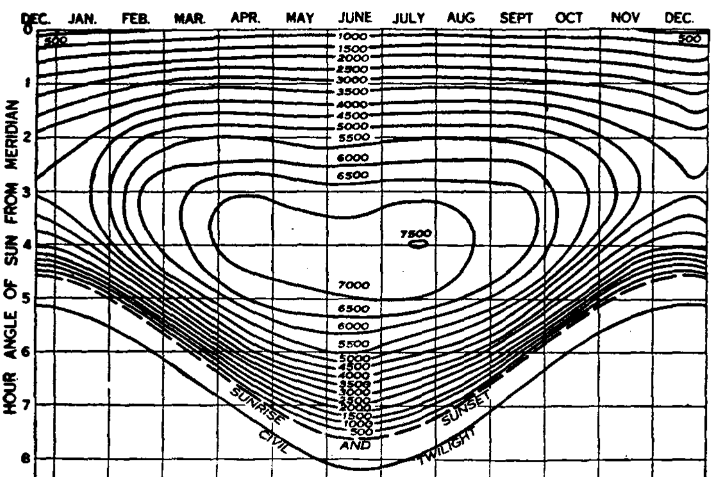

Fig. 23.—Total daylight illumination on a vertical surface facing east, A. M., or west P. M., with a cloudless sky at latitude 42° north. Foot-candles.

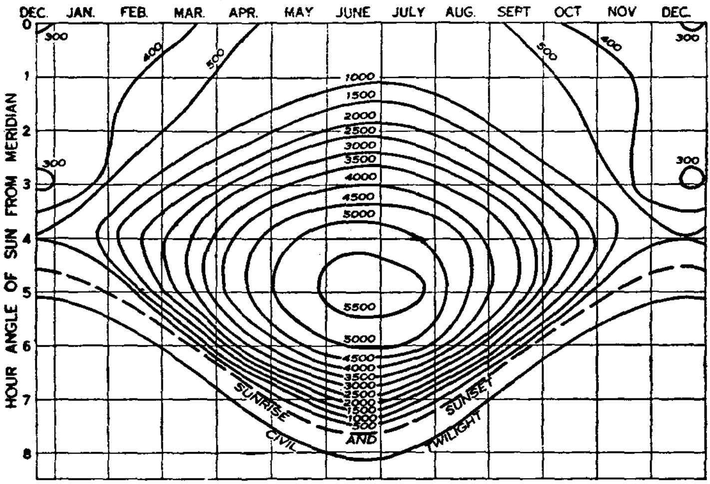

Fig. 24.—Total daylight illumination on a vertical surface facing northeast, A. M., or northwest P. M., with a cloudless sky at latitude 42° north. Foot-candles.

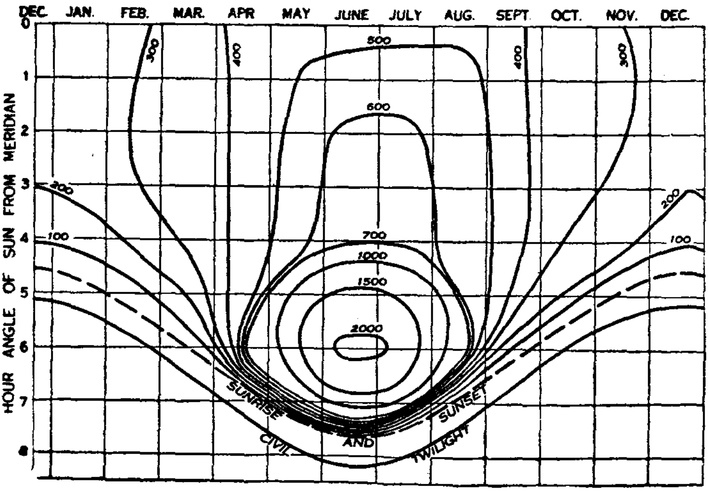

Fig. 25.—Total daylight illumination on a vertical surface facing north with a cloudless sky at latitude 42° north. Foot-candles.

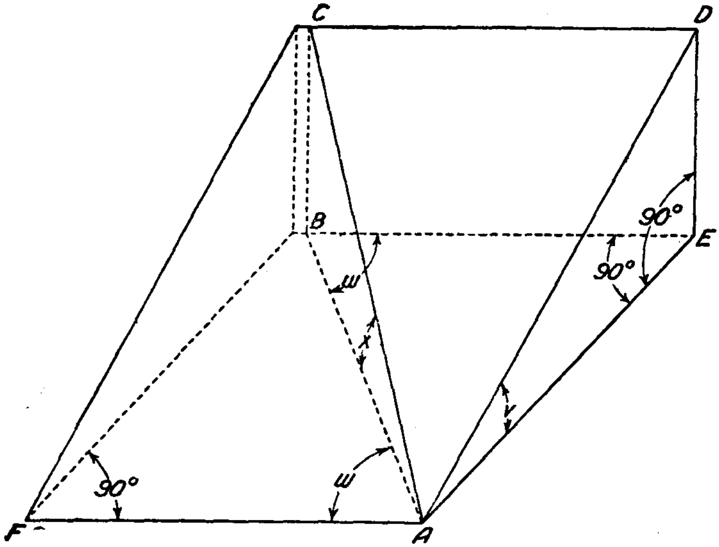

Fig. 26.—Determination of the angle of incidence of solar rays with a sloping surface.

DAYLIGHT ILLUMINATION ON SLOPING SURFACES

The intensity of solar radiation on surfaces sloping in different directions should be of importance to agriculturalists and engineers.[7] Illumination intensities on such surfaces are of especial interest in connection with the lighting of industrial plants by means of the so-called saw-tooth-roof construction.

The computation of direct solar illumination on sloping surfaces presents no special difficulties. Let v, Figure 26, represent the angle between the sloping surface and a horizontal surface; w, the difference between the azimuth bearing of AF, the intersection of these two surfaces, and AB, representing the sun's azimuth; and x, the angle between the intersections of a plane in the sun's vertical with the sloping surface and with a horizontal surface. Then

$$\tan x = \sin w \tan v, \text{ and } a' = a + x \qquad (3)^{8}$$

[7]*Mo. Weather Rev.*, Nov., 1919, 47, p. 781.

[8]In Daylight *vs.* Sunlight in Sawtooth-Roof Construction., *Transactions American Society of Mechanical Engineers*, 40, pp. 603-625, W. S. Brown derives this equation as follows:

$$\frac{AE}{ED} = \frac{\cos v}{\sin v} \quad \therefore \quad AE = ED \cot v.$$

Similarly, $AB = ED \cot x$; and $\sin w = \dfrac{AE}{AB} = \dfrac{\cot v}{\cot x}$, from which $\tan x = \sin w \tan v$.

where a is the altitude of the sun and a' is the angle between the incident solar rays and the sloping surface.

To obtain the intensity of solar illumination on a sloping surface we have only to substitute a' for a in equation (1).

We may also obtain a' by first determining the latitude and longitude of a point at which a horizontal surface is parallel to the sloping surface, by the method given in the *Monthly Weather Review,* November, 1919, **47**, p. 781.[9] Making allowance for the difference in time represented by the difference in longitude of the sloping surface and its parallel horizontal surface, we may obtain directly from an altitude table the altitude of the sun at the latitude of the horizontal surface, and therefore the angle a' which the incident solar rays make with the sloping surface at any hour of any day of the year.

The computation of the *skylight* illumination on sloping surfaces requires the replotting of the sky-brightness measurements for each surface considered.

Figure 27 shows the data for a clear sky with the sun at altitude 40°, projected on a surface for which $90° - w = 45°$ and $v = 10°$ (surface 80° out of the vertical and facing 45° in azimuth from the sun). In this case the zenith of the sky falls 10° from the zenith of the sloping surface. The line of the horizon from azimuth $+ 45°$ to $- 135°$ with reference to the sun, and the lines on which the sky-brightness measurements are to be plotted, have been determined by means of the methods given under Solution of Problems in Stereographic Projections" (pp. 52-58), in General Theory of Polyconic Projections, by Oscar S. Adams, United States Coast and Geodetic Survey, Special Publication No. 57, Serial No. 110.

[9]NOTE.—In lines 15 and 16 from the bottom of the second column of the page referred to, the words "longitude" and "latitude" should be interchanged. The difference in latitude between the sloping surface and its parallel horizontal surface is given by the equation

$$\tan \Delta \phi = \frac{\cos a'}{\cot v},$$

and the difference in longitude by the equation

$$\sin \Delta \lambda = \sin a' \sin v$$

where a' is the azimuth in which the sloping surface faces, and v its angle of slope. When $a' = 0°$ or $180°$, $\sin \Delta \lambda = 0$, and $\tan \quad = \tan v$. That is, the sloping surface and the parallel horizontal surface have the same longitude, and the difference in latitude equals the angle of slope, v.

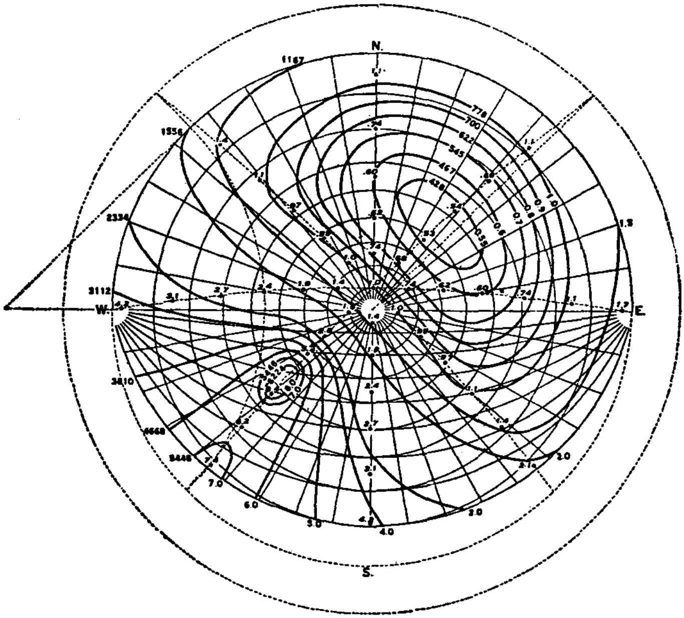

Fig. 27.—Stereographic projection of sky-brightness measurements on a sloping surface.

It is to be noted that the location of the line of the horizon consists in passing a circle through two given points, when the center of the circle falls on a given line. Or, it may also be determined by passing a circle through three given points, as is the location of the lines on which the sky-brightness measurements are to be plotted.

On Figure 27 the brightness of the entire sky is shown with the exception of a spherical lune which falls below the plane of projection on the side WNE., and for which the maximum width is 10° at N. The sky-brightness values that have been obtained by measurement on the half of the sky on one side of the sun's vertical have been plotted on both sides of this vertical.

In Tables VIII and IX is given the total (solar + sky) illumination on surfaces sloping in southerly directions, as indicated. In Tables X and XI is given the skylight illumination on surfaces sloping in northerly directions as indicated. In Table XII is given the ratio of the total illumination to the sky illumination on surfaces facing opposite each other in azimuth.

TABLE VIII.—TOTAL ILLUMINATION ON SURFACES SLOPING SOUTH

Date	Hour angle of the sun from meridan							
	0	1	2	3	4	5	6	7
	Foot-candles							
	Surface sloping 10° from horizontal							
Dec. 21	5,220	4,810	3,800	2,280	656	......	......	......
Jan. 21	6,000	5,590	4,560	2,910	1,140	......	......	......
Feb. 21	7,890	7,520	6,280	4,480	2,240	486	......	......
Mar. 21	9,250	8,800	7,660	5,880	3,660	1,560	82	......
Apr. 21	10,230	9,780	8,700	7,040	4,910	2,640	720	......
May 21	10,690	10,280	9,110	7,530	5,570	3,340	1,420	154
June 21	10,980	10,530	9,460	7,830	5,820	3,730	1,720	280
July 21	10,820	10,390	9,270	7,610	5,700	3,460	1,500	195
Aug. 21	10,050	9,660	8,580	6,920	4,790	2,620	710	......
Sept. 21	9,150	8,730	7,730	5,910	3,720	1,680	98	......
Oct. 21	7,550	7,220	6,090	4,280	2,250	440	......	......
Nov. 21	6,060	5,640	4,590	2,890	1,070	......	......	......
	Surface sloping 20° from horizontal							
Dec. 21	6,300	5,820	4,690	2,890	900	......	......	......
Jan. 21	7,070	6,600	5,460	3,600	1,490	......	......	......
Feb. 21	8,870	8,500	7,140	5,120	2,910	600	......	......
Mar. 21	10,050	9,550	8,250	6,310	3,940	1,630	47	......
Apr. 21	10,770	10,310	9,050	7,250	4,950	2,540	630	......
May 21	11,060	10,490	9,230	7,510	5,400	3,100	1,180	141
June 21	11,260	10,640	9,460	7,750	5,610	3,360	1,380	232
July 21	11,180	10,590	9,320	7,580	5,570	3,220	1,190	156
Aug. 21	10,590	10,090	8,890	7,120	4,830	2,540	620	......
Sept. 21	9,930	9,450	8,310	6,360	3,980	1,680	61	......
Oct. 21	8,510	8,130	6,890	4,920	2,630	550	......	......
Nov. 21	7,130	6,660	5,500	3,580	1,380	......	......	......
	Surface sloping 30° from horizontal							
Dec. 21	7,220	6,680	5,410	3,440	1,140	......	......	......
Jan. 21	7,790	7,360	6,190	4,150	1,820	......	......	......
Feb. 21	9,690	9,220	7,720	5,680	3,250	720	......	......
Mar. 21	10,640	9,990	8,590	6,550	4,110	1,700	50	......
Apr. 21	11,100	10,340	9,040	7,120	4,870	2,390	470	......
May 21	10,960	10,320	8,990	7,260	5,100	2,740	875	134
June 21	10,970	10,320	9,150	7,420	5,200	2,960	1,040	230
July 21	11,060	10,400	9,120	7,300	5,220	2,870	880	156
Aug. 21	10,910	10,210	8,900	7,020	4,650	2,650	500	......
Sept. 21	10,520	9,860	8,660	6,650	4,180	1,800	62	......
Oct. 21	9,250	8,790	7,420	5,380	2,940	650	......	......
Nov. 21	3,050	7,410	6,230	4,160	1,680	......	......	......

TABLE IX.—Total Illumination on Surfaces Sloping Toward Southeast or Southwest.

Date	Hour angle of sun from meridian														
	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
	A. M. for surface sloping SE.; P. M., SW.								P. M. for surface sloping SE.; A. M., SW.						
	Foot-candles														
	Surface-sloping 10° from horizontal														
Dec. 21	....	....	....	816	2,530	3,920	4,680	4,830	4,120	3,080	1,620	302	....	....	....
Jan. 21	....	....	....	1,420	3,260	4,680	5,490	5,580	4,920	3,740	2,130	623	....	....	....
Feb. 21	....	....	756	2,980	4,950	6,500	7,420	7,370	6,780	5,330	3,500	1,610	180	....	....
Mar. 21	....	171	2,120	4,340	6,390	7,960	8,800	8,890	8,180	6,760	4,840	2,790	907	48	....
Apr. 21	....	1,240	3,330	5,630	7,600	8,990	9,730	9,970	8,140	7,930	6,010	3,890	1,850	305	....
May 21	366	2,090	4,160	6,310	8,060	9,400	10,360	10,480	9,790	8,320	6,600	4,580	2,530	850	180
June 21	589	2,380	4,520	6,620	8,470	9,880	10,690	10,770	10,050	8,900	6,970	4,900	2,920	1,180	235
July 21	336	2,210	4,240	6,520	8,260	9,640	10,520	10,600	9,920	8,570	6,730	4,760	2,640	902	160
Aug. 21	....	1,130	3,330	5,540	7,520	8,960	9,710	9,800	9,150	7,850	6,000	3,870	1,890	330	....
Sept. 21	....	190	2,170	4,380	6,430	8,010	8,730	8,820	8,130	6,890	4,920	2,850	839	63	....
Oct. 21	....	....	676	2,710	4,740	6,310	7,110	7,180	6,520	5,040	3,380	1,520	184	....	....
Nov. 21	....	....	....	1,320	3,230	4,700	5,540	5,640	4,970	3,760	2,110	594	....	....	....
	Surface sloping 20° from horizontal														
Dec. 21	....	....	....	1,260	3,360	4,880	5,560	5,550	4,650	3,310	1,580	232	....	....	....
Jan. 21	....	....	....	2,040	4,190	5,740	6,420	6,310	5,340	3,920	2,060	467	....	....	....
Feb. 21	....	....	1,130	3,990	6,020	7,560	8,340	8,120	7,150	5,380	3,250	1,270	157	....	....
Mar. 21	....	253	2,790	5,290	7,370	8,820	9,470	9,160	8,260	6,580	4,400	2,190	485	38	....
Apr. 21	....	1,610	3,960	6,590	8,400	9,720	10,290	10,170	9,030	7,500	5,330	3,100	1,060	270	....
May 21	505	2,440	4,670	6,910	8,660	10,000	10,800	10,470	9,610	7,770	5,790	3,640	1,640	418	140
June 21	713	2,700	4,990	7,180	9,000	10,340	11,040	10,720	9,730	8,130	6,100	3,930	1,920	540	190
July 21	412	2,550	4,790	7,160	8,850	10,220	10,920	10,610	9,650	8,020	5,910	3,820	1,700	450	120
Aug. 21	....	1,430	3,900	6,270	8,270	9,650	10,270	9,980	9,070	7,450	5,350	3,110	1,110	250	....
Sept. 21	....	276	2,770	5,220	7,370	8,910	9,430	9,260	8,250	6,700	4,530	2,300	500	52	....
Oct. 21	....	....	1,010	3,510	5,720	7,310	7,970	7,840	6,870	4,900	3,160	1,200	163	....	....
Nov. 21	....	....	....	1,880	4,160	5,730	6,480	6,360	5,350	3,950	2,050	451	....	....	....
	Surface sloping 30° from horizontal														
Dec. 21	....	....	....	1,670	4,180	5,740	6,280	6,080	4,960	3,380	1,530	175	....	....	....
Jan. 21	....	....	....	2,660	5,090	6,580	7,140	6,800	5,580	3,970	1,940	325	....	....	....
Feb. 21	....	....	1,490	4,850	6,940	8,410	8,950	8,480	7,290	5,220	2,900	859	174	..	....
Mar. 21	....	334	3,230	5,910	8,090	9,500	9,930	9,460	8,140	6,240	3,880	1,600	450	46	....
Apr. 21	....	1,990	4,600	7,150	9,040	10,340	10,650	10,010	8,700	6,850	4,560	2,240	540	250	....
May 21	467	2,830	5,150	7,380	9,120	10,360	10,860	10,140	8,900	7,040	4,860	2,610	651	284	120
June 21	821	2,970	5,380	7,490	9,280	10,570	10,960	10,270	9,070	7,240	5,020	2,770	820	410	180
July 21	470	2,910	5,240	7,550	9,180	10,550	10,970	10,240	9,050	7,180	4,850	2,700	703	380	110
Aug. 21	....	1,790	4,350	6,790	8,770	10,160	10,530	9,830	8,630	6,780	4,540	2,180	580	270	....
Sept. 21	....	350	3,340	6,060	8,110	9,650	9,650	9,370	8,040	6,280	3,950	1,660	455	58	....
Oct. 21	....	....	1,320	4,280	6,540	8,120	8,530	8,140	6,890	4,740	2,840	859	143	....	....
Nov. 21	....	....	....	2,450	5,060	6,640	7,210	6,880	5,640	4,000	1,950	319	....	....	....

TABLE X.—SKYLIGHT ILLUMINATION ON SURFACES SLOPING NORTH

Date	Hour angle of sun from meridian							
	0	1	2	3	4	5	6	7
	Foot-candles							
	Surface sloping 10° from vertical							
Dec. 21	310	310	285	221	115	......	......	......
Jan. 21	320	330	310	259	160	......	......	......
Feb. 21	340	352	348	340	293	112	......	......
Mar. 21	421	428	406	429	436	327	48	......
Apr. 21	520	540	562	564	550	485	346	......
May 21	554	600	660	685	641	650	590	228
June 21	580	640	758	770	740	790	719	365
July 21	580	635	733	730	712	700	630	288
Aug. 21	570	605	610	635	629	560	371	......
Sept. 21	468	480	462	494	487	354	72	......
Oct. 21	368	386	372	374	331	147	......	......
Nov. 21	333	343	328	270	167	......	......	......
	Surface sloping 20° from vertical							
Dec. 21	355	350	320	236	112	......	......	......
Jan. 21	362	365	355	276	172	......	......	......
Feb. 21	390	400	410	395	294	120	......	......
Mar. 21	489	498	471	516	486	416	46	......
Apr. 21	619	635	638	640	642	589	412	......
May 21	660	720	782	805	759	764	625	241
June 21	698	777	888	901	956	879	750	365
July 21	691	760	874	860	834	817	720	312
Aug. 21	671	710	715	730	730	660	418	......
Sept. 21	542	560	538	558	555	411	60	......
Oct. 21	430	446	460	420	338	122	......	......
Nov. 21	378	394	366	278	182	......	......	......
	Surface sloping 30° from vertical							
Dec. 21	377	380	345	255	120	......	......	......
Jan. 21	398	407	384	302	172	......	......	......
Feb. 21	431	450	441	424	341	147	......	......
Mar. 21	569	575	524	572	582	451	47	......
Apr. 21	754	760	742	753	745	651	409	......
May 21	801	873	913	882	898	871	643	200
June 21	830	920	1,035	1,028	1,035	1,004	770	337
July 21	837	917	998	990	980	962	655	217
Aug. 21	828	870	840	837	850	735	442	......
Sept. 21	631	645	596	630	626	465	58	......
Oct. 21	473	497	502	472	378	189	......	......
Nov. 21	413	423	417	323	200	......	......	......

TABLE XI.—SKYLIGHT ILLUMINATION ON SURFACES SLOPING NORTHEAST OR NORTHWEST

Date	Hour angle of sun from meridian														
	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
	A. M., sloping NE.; P. M., sloping NW.								P. M., sloping NE.; A. M., sloping NW.						
	Foot-candles														
	Surface sloping 10° from vertical														
Dec. 21 . .	. .	. .	. .	122	292	380	380	336	317	298	186	108	. .	. .	. .
Jan. 21 . .	. .	. .	. .	220	355	418	418	351	330	307	245	142	. .	. .	. .
Feb. 21 . .	. .	. .	152	457	503	478	445	375	360	338	308	236	97	. .	. .
Mar. 21 . .	. .	80	630	780	660	614	544	465	425	373	370	325	245	42	. .
Apr. 21 . .	. .	650	985	990	920	790	700	569	527	465	410	375	320	220	. .
May 21 . .	450	1,150	1,300	1,240	1,100	1.010	815	624	575	540	485	425	430	330	150
June 21 . .	600	1.350	1,500	1,325	1,383	1,070	900	661	600	590	533	478	475	400	180
July 21 . .	450	1,220	1,500	1,300	1,130	1,200	870	660	600	585	510	440	450	380	120
Aug. 21 . .	. .	900	1,110	1,080	1,080	1.000	830	640	588	520	465	430	355	275	. .
Sept. 21 . .	. .	108	700	860	760	670	655	529	470	420	400	360	260	47	. .
Oct. 21 . .	. .	. .	185	496	597	562	529	402	358	335	310	230	97	. .	. .
Nov. 21 . .	. .	. .	. .	268	376	444	488	372	345	308	242	148	. .	. .	. .
	Surface sloping 20° from vertical														
Dec. 21 . .	. .	. .	. .	170	337	405	440	391	370	308	225	105	. .	. .	. .
Jan. 21 . .	. .	. .	. .	230	402	480	525	418	385	340	257	145	. .	. .	. .
Feb. 21 . .	. .	. .	202	502	580	580	565	440	420	380	345	250	98	. .	. .
Mar. 21 . .	. .	78	640	755	785	705	660	547	495	425	370	360	245	38	. .
Apr. 21 . .	. .	720	980	1,050	1,010	935	810	676	625	550	485	440	355	225	. .
May 21 . .	450	1,050	1,360	1,280	1,190	1,125	935	747	690	655	560	490	500	360	130
June 21 . .	600	1,350	1,500	1,420	1,300	1,210	1,070	800	730	685	630	530	570	450	165
July 21 . .	450	1,220	1,500	1,320	1,300	1,195	1,020	792	715	675	605	508	530	420	115
Aug. 21 . .	. .	900	1,080	1,140	1,220	1,080	920	760	685	610	535	480	390	240	. .
Sept. 21 . .	. .	105	680	935	1,010	803	750	624	560	492	450	375	262	44	. .
Oct. 21 . .	. .	. .	185	500	684	680	638	488	460	425	365	260	91	. .	. .
Nov. 21 . .	. .	. .	. .	247	424	518	556	440	394	340	275	125	. .	. .	. .
	Surface sloping 30° from vertical														
Dec. 21 . .	. .	. .	. .	220	320	430	480	420	387	340	255	135	. .	. .	. .
Jan. 21 . .	. .	. .	. .	285	455	550	580	448	422	375	294	164	. .	. .	. .
Feb. 21 . .	. .	. .	202	530	660	645	592	491	450	428	400	277	107	. .	. .
Mar. 21 . .	. .	69	625	840	975	980	790	640	570	486	425	385	290	42	. .
Apr. 21 . .	. .	755	910	950	1,200	1,050	920	824	785	650	525	475	385	240	. .
May 21 . .	450	1,050	1,320	1,350	1,380	1,300	1,100	888	830	800	665	530	535	390	120
June 21 . .	600	1,350	1,500	1,450	1,360	1,350	1,180	941	855	840	738	595	600	445	220
July 21 . .	450	1,220	1,500	1,400	1,400	1,320	1,160	949	870	835	695	575	535	420	140
Aug. 21 . .	. .	900	1,085	1,270	1,320	1,200	1,070	928	850	720	620	510	425	270	. .
Sept. 21 . .	. .	100	740	960	810	857	910	730	627	535	485	420	310	48	. .
Oct. 21 . .	. .	. .	215	580	784	750	847	524	505	470	415	275	107	. .	. .
Nov. 21 . .	. .	. .	. .	326	510	620	590	484	440	380	285	167	. .	. .	. .

TABLE XII.—Ratio of Total Illumination, T, to Sky Illumination, S

CLOUDLESS SKY, LATITUDE 42° N.

Date	Hour angle of sun from meridian 0	1	2	3	4	5	6	7
	T, vertical surface facing south; S, vertical surface facing north							
Dec. 21	29	27	22	22	16	......	......	......
Feb. 21	28	26	22	17	12	8	......	......
Apr. 21	13	12	10	7	4	1.4	0.3	......
June 21	9	7	5	3	1.1	0.3	0.2	0.1
Aug. 21	12	11	9	6	3	1	0.5	......
Oct. 21	25	23	20	15	11	5	......	......
	T, surface sloping south 10° from horizontal; S, sloping north 10° from vertical							
Dec. 21	18	16	13	10	6	......	......	......
Feb. 21	23	21	18	13	8	4	......	......
Apr. 21	20	18	16	12	9	5	2	......
June 21	19	16	12	10	8	5	2	0.8
Aug. 21	18	16	14	11	8	5	2	......
Oct. 21	20	19	16	11	7	3	......	......
	T, surface sloping south 30° from horizontal; S, sloping north 30° from vertical							
Dec. 21	19	18	16	14	10	......	......	......
Feb. 21	22	20	18	13	10	5	......	......
Apr. 21	15	14	12	10	6	4	1.2	......
June 21	13	11	9	7	5	3	1.4	0.7
Aug. 21	13	12	11	8	6	4	1.1	......
Oct. 21	20	18	15	11	8	3	......	......

HOUR ANGLE OF SUN FROM MERIDIAN

Date	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
	T, surface facing SE., A. M., or SW., P. M. S, surface facing NW., A. M., or NE., P. M.								T, surface facing SW., A. M., or SE., P. M. S, surface facing NE., A. M., or NW., P. M.						
	Surfaces vertical														
Dec. 21	...	...	...	28	29	28	24	20	12	7	2	0.7	...	...	...
Feb. 21	...	...	30	30	28	28	24	19	10	4	0.9	0.6	0.5	...	...
Apr. 21	...	14	17	22	21	17	13	9	4	0.9	0.6	0.4	0.3	0.3	...
June 21	6	8	11	16	14	11	9	6	1.6	0.7	0.5	0.3	0.3	0.3	0.3
Aug. 21	...	11	15	18	18	15	12	8	3	0.9	0.6	0.4	0.3	0.3	...
Oct. 21	...	...	24	25	25	24	20	16	9	4	0.9	0.6	0.5	...	...

TABLE XII.—(*Continued*)—RATIO OF TOTAL ILLUMINATION, T, TO SKY ILLUMINATION, S

Date	HOUR ANGLE OF SUN FROM MERIDIAN														
	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
	T, surface sloping 10° from horizontal; S, 10° from vertical														
Dec 21	...	...	...	8	14	13	15	14	11	8	6	2	...	...	...
Feb. 21	...	...	8	13	16	19	21	20	15	11	7	4	1.2	...	...
Apr. 21	...	6	10	15	18	19	18	17	13	10	6	4	2	0.5	...
June 21	3	6	10	14	16	17	17	16	11	8	5	4	2	0.9	0.4
Aug. 21	...	4	9	13	16	17	18	15	11	8	6	4	2	0.4	...
Oct: 21	...	...	7	12	15	19	20	18	12	9	6	3	1.0	...	...
	T, surface sloping 30° from horizontal; S, 30° from vertical														
Dec. 21	...	...	...	12	16	17	16	14	10	8	5	0.8	...	...	...
Feb. 21	...	...	14	18	18	20	20	17	12	8	4	1.6	0.9	...	...
Apr. 21	...	8	12	15	17	16	14	12	9	7	4	2.4	0.6	0.3	...
June 21	4	7	9	13	13	13	13	11	8	5	4	1.9	0.7	0.3	0.3
Aug. 21	...	7	10	13	14	14	12	11	8	6	3	1.7	0.5	0.3	...
Oct. 21	...	...	12	16	16	17	17	16	8	6	4	1.4	0.7	...	...

Table VIII shows that in general on surfaces sloping south and with south solar declination the total illumination increases with v. With north solar declination the illumination reaches a maximum in the middle of the day when v equals about 20°, and decreases as v increases with the sun near the horizon.

Table IX shows that in the morning on surfaces facing southeast, and in the afternoon on surfaces facing southwest, there is an increase in the total illumination with increase in v, except near midday in midsummer with v greater than about 20°. Also in the morning, on surfaces facing southwest, and in the afternoon, on surfaces facing southeast, the illumination generally decreases with increase in v, except near midday with south solar declination.

Tables X and XI show an increase with v in skylight illumination on vertical surfaces sloping northward, as one would expect. It must be remembered, however, that with saw-tooth construction a very considerable part of the skylight is cut off by shading.

This is unimportant when considering the total illumination on surfaces sloping in a southerly direction, but becomes important in connection with the skylight illumination on surfaces facing towards the north, since it is the brightest part of the sky that is cut off.

Let it be assumed that the ridges of the saw-teeth of the roof are horizontal, and of infinite length, and let $\theta =$ the maximum angular width of the spherical lune of the sky cut off. Then Table XIII gives the percentages of decrease in the skylight illu-

TABLE XIII.—SHADING EFFECT IN SAW-TOOTH ROOF CONSTRUCTION

$90° - w$.	Solar altitude				θ
	20°	40°	60°	70°	
	PERCENTAGE OF SKYLIGHT CUT OFF				
°	Surface 10 degrees out of vertical				°
180	32	24	20	17	10
135	26	25	18	17	10
90	25	19	17	15	10
	Surface 20 degrees out of vertical				
180	43	37	31	25	
	Surface 30 degrees out of vertical				
180	39	33	24	21	20
135	38	28	23	21	20
90	34	26	24	18	20
180	53	44	33	31	30
135	50	41	33	28	30
90	46	37	33	30	30
	Surface 60 degrees out of vertical				
0	19	11	6	6	10
45	14	9	6	5	10
90	9	7	5	4	10
0	53	39	23	21	30
45	44	32	22	21	30
90	28	22	18	16	30
	Surface 80 degrees out of vertical				
0	11	6	3	3	10
45	9	5	3	3	10
90	4	3	2	2	10
0	35	29	18	15	30
45	32	21	13	12	30
90	16	12	10	9	30
0	64	54	36	30	50
45	55	42	28	27	50
90	30	23	20	19	50

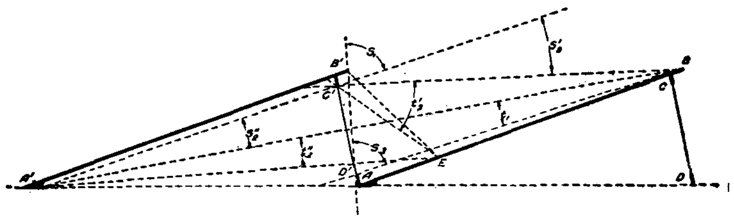

Fig. 28.—Cross section of a saw-tooth roof.

mination,[10] due to shading by the adjacent saw tooth.

Table XII shows that during most of the working hours of the day (except in midsummer) the total daylight illumination on surfaces sloping southward exceeds by more than tenfold the skylight illumination on surfaces sloping northward.

Computations from the sky-brightness data given in TRANSACTIONS, Illuminating Engineering Society, Vol. XVI, p. 260, Figures 6 and 7, show that skylight illumination on vertical or sloping surfaces facing away from the sun is about twice as intense when the sky is covered with thin clouds or haze or partly covered with white clouds, which is its usual condition, as when clear. The total (solar + sky) illumination on surfaces facing the sun is usually diminished by the presence of haze or clouds of the above character. In consequence, when the angle w lies between about 45° and 135°, the ratios of Table XII will be diminished, on the average, by at least one-half, and will vary in value from their maxima with clear-sky conditions, given in Table XII, to about 2 for a sky completely covered with dense clouds. This will be made clear from a comparison of the sky-brightness data of Figures 4 (opposite p. 259) and 13 (p. 263) with Figures 6, 7 and 8 (p. 260) and 11 (p. 262), TRANSACTIONS, Illuminating Engineering Society, Vol. XVI, October, 1921, and Figure 3 (p. 442) of this paper; and by reference to the illumination intensities of Table IV (p. 443) of this paper.

These ratios are of use in computing the daylight that can be made available for illuminating working space in a building through saw-tooth-roof construction. In general, there are two sources from which the light may be obtained as follows:

[10]The percentages of Table XIII have been computed from Figure 27 and other similar figures. See also TRANS., Illum. Eng. Soc., Vol. XVI, p. 270 and *Mo. Weather Review*, Sept., 1921, 49, p. 486.

(1) Light from the northern sky incident at the working space, or reflected thereto from the ceiling of the saw-tooth roof (sky angles S_1 to S_3, and S'_2 to S''_2, respectively, Figure 28).

(2) Solar and skylight reflected from the outside surface of the saw-tooth directly to the working space, or through a secondary reflection from the ceiling of the saw-tooth roof (roof angles t_1 to o, and t'_2 to t''_2, Figure 28).

It is to be understood that the angles here shown are cross-sections of spherical wedges.

Assuming the ratio of the intensity of the total light reaching the southerly-sloping roof surface of a saw-tooth window, AB (Figure 28) to the light received from the sky on the northerly-sloping window surface C'D' (Figure 28) to be 4, Brown[11] computed the relative values of (1) and (2) to be 14.6 and 6.1, respectively.

Let us consider a saw-tooth construction that gives a window surface facing north and sloping 20° from the vertical, and a roof surface sloping south 20° from the horizontal. Let the latitude be 42° north, the sky clear, the date March 21, and the hour 10 A. M., or 2 P. M., apparent time. The solar altitude will be 40° and its azimuth 41°. Disregarding shading, Tables VIII and X give 8,250 and 471 foot-candles for the illumination intensity on the roof surface and the window surface, respectively, the ratio of the former to the latter being 18.

The ridge of the adjacent saw tooth would cut off from the window a spherical wedge near the horizon for which the average value of θ would be about 10°. The window is facing 139° from the sun, and from Table XIII it is estimated that the shading by the roof diminishes the skylight illumination at the window surface 20 per cent. The roof between B and E will be in sunlight, and between E and A it will be illuminated by skylight only. We may therefore disregard the small quantity of light this latter can reflect to the under side of A'B'. The skylight from a spherical lune near the southern horizon for which θ averages about 30° will be cut off from BE by the adjacent saw tooth. From Table XIII we estimate that the illumination from skylight will be decreased by about 27 per cent. Therefore, the available sky illumination on the north-sloping window surface is 471 × 0.80

[11]Loc. cit., p. 620.

= 377, and on the south-sloping roof surface it is 1,100 × 0.73 = 800 foot-candles. The total illumination on the south-sloping roof surface is 7,950 foot-candles, and its ratio to the illumination on the window surface is 7,950/377 = 21. Substituting this value for 4, we obtain for the relative values of (1) and (2) 14.6 and 32, respectively. Or, if we suppose the sky to be covered with thin clouds, or partly covered with white clouds the values become 11.6 and 16.

Apparently, therefore, for clear-sky conditions, or even for the most usual sky conditions, when thin clouds or haze, or scattered white clouds are present, most of the daylight received through a saw-tooth-roof window will be from the reflection of skylight and sunlight from the roof of an adjacent saw tooth. In cloudy weather, however, nearly all the light will be received from the northern sky.

No attempt has been made to express the illumination intensity at the working space in absolute units. In order to do so, it is necessary to know the average of the solid sky angles S_1 and S_3, and of S'_2 and S''_2 (Figure 28); the brightness of the sky included in each of these angles, from which the sky illumination may be computed; the solar illumination intensity on the roof surface BE; the solid roof angle t_1 and the average of the solid roof angles t'_2 to t''_2; the coefficients of reflection of the surface of the ceiling A′B′, and the roof, BE; the solid angle subtended by the ceiling at the working space, and the angle at which light is incident at the roof or the ceiling, and is received at the working space either directly or by reflection from the outside roof or the inside ceiling.

Of the above factors the brightness of the sky and the intensity of the solar illumination are given with reasonable accuracy in this paper for latitude 42° N. The remaining factors depend upon the design of the saw-tooth roof and its window openings, and must be determined for each individual case.

During the winter months, in a smoky city like Chicago, disregarding the probable decrease in the reflecting power of the ceiling of A′B′, and the roof surface BE, the absolute values of (1) and (2) can not exceed two-thirds and one-half, respectively of their values in a comparatively smokeless region.