Brightnesses in Visual Field at Borderline Between Comfort and Discomfort (BCD)

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UALITY of lighting has been defined as a function of the brightness characteristics of certain unique portions of the visual field.1 The brightnesses of these areas and the brightnessratios between them contribute favorably or unfavorably to the seeing conditions. They may influence the visibility of a visual task, or their effects may be more subtle, insidious or deep-seated and result in decreased ease of seeing. Obviously, all the effects may be produced simultaneously. In general, it is an easy matter to provide the footcandle-levels within the restricted area of the visual task for the attainment of definite or generally adequate visibility-levels. However, the proper adjustment of all of the brightnesses in the entire visual field is of great importance to the worker who must perform tasks of critical seeing for prolonged periods in the resulting visual environment. Thus the ultimate goal of lighting practice is to provide brightnesses in the entire visual environment which produce the most satisfactory seeing conditions. This can be accomplished only by proper attention to, and application of, what has been termed brightness engineering.2

It may be considered axiomatic that the brightness relationships within the central field determine directly the visibility of any specific visual object or task. The brightness relationships in the surrounding field and between the central and surrounding fields determine directly the comfort of the visual environment and usually have an indirect or less obvious effect upon visibility. In other words, the quality of lighting may be such as to have little or no significant effect upon visibility but may result in extremely uncomfortable or distracting seeing conditions.

Many aspects of quality of lighting or the distribution of brightness in a visual environment have been investigated. The majority of these investigations* have dealt with the effects of brightnesses in the central and surrounding fields upon visual acuity,³ contrast sensitivity,⁴ rate of working,⁵ and precision. ⁶ More recently a few investigations have

A paper presented at the National Technical Conference of the Illuminating Engineering Society held September 20-23, 1949, French Lick, Indiana. Authors: General Electric Co., Nela Park, Cleveland, Ohio. been extended into a less obvious realm and have evaluated ease of seeing in terms of a number of psycho-physiological responses. The available data permit the establishment of rather definite relationships concerning the more or less physical or visibility aspect of the problem. However, the relatively meager data of the evaluation of comfort and ease of seeing result only in qualitative conclusions. Therefore, the gap in our knowledge of the overall problem of providing ideal seeing conditions needs to be filled by additional comprehensive and quantitative investigations which can be correlated with lighting practice.

Since comfort and discomfort are sensations, their appraisal can be made only by those who experience such sensations. Different individuals may vary considerably in their appraisals, but the trends are decisive. Many years ago, Luckiesh and Holladay,8 in their comprehensive investigation of the effects of bright light-sources upon visibility, included an attempt at appraising the psycho-physiological effects of glare. They developed a scale of comfort-discomfort or "degrees of sensation" from a scarcely noticeable sensation to intolerable and painful sensations. Holladay continued and extended this investigation. This early work involved glare-sources on the line of vision which, while useful in lighting practice, always assumed direct fixation of the glare-source. More recently we presented the results of a limited exploratory investigation of the effects upon comfort of glare-sources at various angular distances from the line of vision.9 These data were somewhat useful for extending the general concepts and results of the previous work. However, all these investigations were limited in scope, and the results were difficult to apply directly to the evaluation of specific lighting installations. Therefore, a more extensive study of brightnesses in the visual field was planned which would include the wide range of the various factors involved with sufficient corollary investigations to make possible the practicalization of the basic and fundamental data.

Any comprehensive study of quality of lighting¹

^{*}These references are not intended to be a complete bibliography of the literature but rather an indicator of the trends of researches during several periods.

or the environmental brightness relationships must include:

- 1. The brightness-level to which the eyes are adapted.
- 2. The brightnesses of various areas in the visual field.
- 3. The area and position of sources of brightness.
- 4. The criticalness of the visual tasks to be performed.

The present paper presents the results of an extensive series of researches of the effects of sources of brightness in the visual field upon comfort and discomfort. Because of the scope of these researches it is necessary to limit this paper to a presentation chiefly of the basic relationships. Numerous additional results and analyses which make possible the correlation of the basic data with lighting practice will be the subject of a future paper.

Experimental Criterion, Environment and Procedures

Criterion. As has been stated previously, only those who experience and pay the penalties of glare can appraise discomfort and determine a scale of comfort and discomfort. When only two or three subjects are involved, it is possible to agree upon a series of definitions of various degrees of comfort and discomfort. However, any particular sensation would be a matter of considerable debate among a large group of individuals. Therefore, it was decided to confine the criterion of the present series of researches to a single sensation which could be defined by the experimenter and which could be interpreted by the subjects as a relatively definite sensation. Furthermore, it was desirable to establish a criterion which is meaningful from a practical viewpoint. It was decided that the sensation at the borderline between comfort and discomfort would fulfill these primary requisites. We have termed this the BCD sensation.

Experimental Environment. An important consideration of any investigation which involves a subjective evaluation of luminous areas in the visual field is the attainment of as complete control as possible of the environmental conditions. Without such control, extraneous factors may exert undue influence upon the subject whose appraisal may be of these factors rather than of the variable being studied. The experimental situation used in the present investigation involved (1) an extended visual field of uniform brightness termed the field brightness F and (2) a series of limited luminous areas of various brightnesses termed sources. For most of the present researches these conditions were obtained in an 80-inch photometric sphere, one-third of which was cut away as is illustrated in Fig. 1. The subject's head was positioned in a headrest so as to locate his eyes at the center of the sphere, or

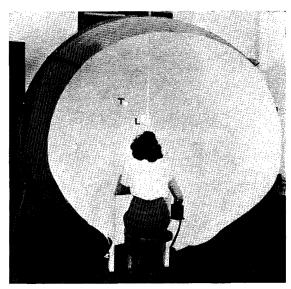


Figure 1. The experimental environment used throughout most of the present investigation was the illuminated white inner surface of an 80-inch sphere at the center of which the subject's eyes were located. This unrestricted surrounding visual field was illuminated by the lamp L located near the center of the sphere and concealed from the subject. The brightness of the test-sources T, which consist of circular apertures in the spherical surface, is adjusted by the control under the subject's right hand. The photograph illustrates the brief period during which a test-source T located 20 degrees above the line of vision was exposed.

at a distance of 40 inches from the inner surface which occupied the *entire* unrestricted visual field. The brightness of this inner surface was the field brightness.

The illumination of the entire visual field was obtained by placing a lamp immediately above the head of the subject. Since this position was approximately at the center of the sphere, the brightness F of the entire field was uniform throughout. A small shield was interposed between the lamp and head in order to eliminate any sensation of heat or brightness from above.

Light-sources mounted behind circular openings in the surface of the sphere provided the sources of brightness or luminous areas which the subjects were required to evaluate at the borderline between comfort and discomfort. By using appropriate filters in front of the light-sources, it was possible to obtain apparent constancy of color of the sources for a relatively large change in voltage. Therefore, by the selection of a lamp of proper wattage and a corresponding color-correcting filter, a reasonably constant spectral quality was maintained over the range of brightnesses from zero to approximately 30,000 footlamberts. This minimized the possibility of the color of the source influencing the judgment

of the subjects, since the spectral character of a 100-footlambert source was approximately the same as that of a 10,000-footlambert source. Each lamp and filter combination covered a limited range of brightnesses, an appropriate one being selected for each subject depending upon the variable being studied. In every case, the subject had available a sufficient brightness-range so that his evaluation would not be affected by the available limiting brightnesses.

Those sources which were not used during any particular part of the investigation were adjusted to equal the field brightness F and became indistinguishable from it. When two sources were being compared by alternate exposures, the brightness of either source during its "off" period was identical to the brightness of the field. Thus, the experimental environment always consisted of a uniform field brightness F with a circular luminous area periodically superimposed upon it.

Technique. The technique used for appraising the brightnesses of the sources was similar to that originally developed by Luckiesh and Holladay⁸ but with refinements of procedure which were dictated by later experience and knowledge. The subject was required to evaluate the initial sensation of brightness when a source was momentarily exposed to view amid the surrounding field of uniform brightness F. In other words, the condition approximated that of a worker looking up from the adaptation brightness of a large area, such as his desk, to a source of higher brightness and evaluating the sensation received during the brief period that the source was viewed. Short exposures were used instead of prolonged exposures owing to the desire to maintain the adaptation brightness-level as close to that of the surrounding field as possible. However, the exposure-time was sufficiently long for the observer to receive and to experience the full building up of the sensation. The authors believe it is desirable and even essential to control the duration of exposure to the source. After considerable experimentation we have standardized upon 1-second exposures separated by 1-second intervals during which the subjects were exposed only to the field brightness. A 10-second cycle was devised during which the sources were presented for a subjective evaluation three times and the remaining short period was allowed for the subject to alter the brightness of the test-source.

An absolute method and also a comparison method were used. It was found that both types were reliable and consistent. The absolute method was most suitable when a single test-source was used or when the variables being investigated included more than one field brightness. The comparison method was quite suitable for those phases of the investiga-

tion in which the test-source being evaluated was displaced from the line of vision, which permitted locating a standard or comparison-source on the line of vision, or when the angular displacement between sources was sufficiently small to permit alternate direct fixation with minimal movement of the eyes.

The exposure-cycle of the absolute method involved three 1-second "on" periods of the test-source separated by 1-second "off" periods and followed by a 5-second "off" period. The 5-second period was provided so that the subject could resolve the sensations received during the "on" periods and alter the brightness of the test-source for the next appraisal if he wished. The criterion used throughout the present investigation was the sensation at the borderline between comfort and discomfort, or the BCD sensation. The subject was free to establish his own criterion of BCD brightness and was permitted as many 10-second cycles as he deemed necessary to make a complete appraisal for each condition.

When the comparison method was used, the comparison-source C was set at a predetermined brightness by the experimenter. The brightness of the test-source T was then adjusted by the subject so that the initial sensation received from it was the same as the initial sensation received from the comparison-source. The sources were exposed alternately for 1-second periods with 1-second intervals between exposures and a 5-second "off" period for evaluating the sensation of brightness and for altering the brightness of the test-source. During two successive cycles the order of presentation of the two sources was reversed. For example, the order during even-numbered cycles was C-T-C, each exposure being for one second with 1-second "off" intervals between exposures. This was followed by a 5-second interval during which the subject saw only the uniform field brightness. During the oddnumbered cycles the order was T-C-T so as to minimize any influence of the order of presentation. As with the absolute method, the subject was permitted as many cycles as necessary for making an appraisal. The field brightness F was maintained constant and continuous for each specific condition and series of observations.

In considering controlled conditions, it is also necessary to minimize the influence of the experimenter upon the subjects. After adequate explanation of the procedure, the subject was left alone to arrive at his conclusion at each step in the investigation. The handling of subjects is very important.

Basic Relationships

In this section of this paper the basic factors which affect the visual sensation produced by a

source or luminous area are restricted to the size and brightness of the source and to the uniform brightness of the entire adapting field. The position of the source in the visual field, the configuration of the source and the number of sources that are visible are modifying factors which are considered separately in later sections.

Standard BCD Brightness. It was first necessary to establish a meaningful brightness value for standardized conditions and one that could be used as a common denominator for relating all the variables. This was accomplished by selecting as the criterion a brightness sensation which was at the borderline between comfort and discomfort. We have termed this the BCD brightness for the specified condition. This datum should be the average from a large number of subjects in order for it to be a representative value. It then becomes the standard BCD brightness for the standard test-conditions as viewed by a standard subject. The standard test-conditions included (1) a circular source having a diameter of 1.48 inches, (2) located on the line of vision, (3) at a distance of 40 inches from the eyes and (4) a surrounding field brightness of 10 footlamberts. At this viewing distance, the circular source subtended a solid angle of 0.0011 steradian. The absolute method was used for determining the basic relationships. The source brightness was always viewed directly by the subjects who adjusted it until it was judged to be at the BCD brightness for the specific conditions.

It will be noted that the background brightness, against which the source is viewed, has not been included among the basic or modifying factors. In order that the results of these researches be useful, the background brightnesses are considered to be less than the brightness of the source and of the same order as the adaptation brightness as is often encountered in practice. In this investigation the background and adaptation brightnesses were identical. Obviously, if the background is of the same brightness as the source, it becomes a part of the source and thus contributes to the visual sensation received from the source.

A random group of 50 subjects varying in age from 20 to 40 years was used for determining the standard BCD brightness of the standard source viewed under the standard test-conditions. The average BCD brightness for each of the subjects is presented in Table I. It is seen that the individual BCD brightnesses range from 315 to 1600 footlamberts and that the geometric mean BCD brightness of the standard source is 830 footlamberts. An analysis of the data indicates that an approximately normal distribution was obtained from the 50 subjects. The variation between individuals is not un-

TABLE I.—The BCD brightness in footlamberts of the standard circular source (subtending 0.0011 steradian) located on the line of vision which produced an initial momentary sensation as judged by each of 50 different subjects to be at the borderline between comfort and discomfort. The field brightness was 10 footlamberts.

Subject	Brightness	Subject	Brightness
1	410	26	1040
2	820	27	910
3	1040	28	1530
4	880	29	1300
5	910	30	670
6	1170	31	990
7	580	32	850
8	910	33	1380
9	1300	34	625
10	885	35	405
11	1600	36	630
12	450	37	950
13	1060	38	810
14	1530	39	1440
15	930	40	810
16	370	41	985
17	445	42	640
18	400	43	650
19	750	44	470
20	990	45	1230
21	860	46	820
22	1170	47	980
23	955	48	1080
24	1000	49	840
25	315	50	770
		Arithmetic Mean	891
		Geometric Mean	830

expected nor extraordinary because of the many physiological and psychological factors which may influence the subjective appraisal of bright areas. Contrast sensitivity, for example, has been used as a criterion of retinal sensitivity and has been demonstrated to vary widely for a group of subjects who may be otherwise rated equal, and are socalled normal, on a visual-acuity basis.10 Furthermore, standards of comfort or discomfort vary greatly among individuals. However, the normalcy of the distribution is an indication that the mean value is representative of a much larger group. The geometric mean and the arithmetic mean are of the same order of magnitude, differing by approximately 7 per cent. The geometric mean is particularly useful for these types of data since it is less affected by extreme values than the arithmetic mean, and hence it is a more typical average especially when there is a relatively wide spread of the data. In actual practice "factors of safety" should be provided so that no sources in the visual field are in the discomfort realm beyond the borderline BCD for any one of a representative group of persons. Thus the average BCD brightness is suitable for establishing general relationships, but it may be desirable to use the lowest BCD brightness as a conservative value for determining the brightness relationships in a visual environment when comfort is of primary importance.

It is of interest to compare the average BCD brightness obtained in the present investigation

with the value obtained by applying the socalled Holladay formula,8

$$K = \log B + 0.25 \log Q - 0.3 \log F$$
 (1)

where K expresses the magnitude of the initial sensation of a "glare-source" having a brightness of B millilamberts and subtending a solid angle of Q steradians when the adapting field brightness is F millilamberts. The value of K was originally determined by Luckiesh and Holladay to be 1.9 when the sensation was at the boundary between comfort and discomfort. Substituting Q = 0.0011 steradian, F = 10 footlamberts and K = 1.9 in the formula, the calculated BCD brightness is 827 footlamberts as compared with the mean value of 830 footlamberts found in the present study. In other words, the two groups of subjects, under similar test conditions, appraised the same brightness as being at the borderline between comfort and discomfort. However, in the present investigation the group of subjects was much larger and, therefore, may be considered to be more representative of the average.

Influence of Field Brightness. The effect of the field or adaption brightness upon the BCD brightness is illustrated in Fig. 2. These data were determined by a group of ten subjects so selected as to be representative of the 50 subjects used previously for the determination of the standard average BCD brightness. All factors, excepting the field brightness, were maintained at the standard values. That is, the size of the source was 0.0011 steradian and was located on the line of vision. For field brightnesses of 1, 10 and 100 footlamberts the average BCD brightnesses were found to be 302, 830 and 2325 footlamberts, respectively. These values, which

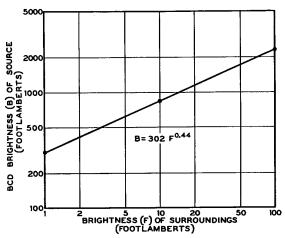


Figure 2. The relationship between BCD brightness B of a source, located on the line of vision and subtending a solid angle of 0.0011 steradian, and the surrounding field brightness F.

plot as a straight line on logarithmic coordinates, may be represented by the equation

$$B = 302F^{0.44} \tag{2}$$

where B is the BCD brightness of the source and F is the field brightness. From this relationship it is seen that doubling the field brightness F permits a 36 per cent increase in the brightness B of the source. Conversely, if the brightness B of the source is doubled, the field brightness F must be increased approximately five times in order to maintain the BCD sensation.

The straight-line relationship agrees with that found by Luckiesh and Holladay⁸ and by Nutting.¹¹ The coefficient of F as determined by these earlier investigators was higher primarily because the criterion used by them involved the sensation of glare rather than the BCD brightness which is at the borderline between comfort and discomfort and is defined as being less than the sensation of glare in The difference between the its usual meaning. values of the exponent may be due to slightly different experimental situations and techniques and to the differences between the subjects used in the various investigations. We believe that the group of subjects used in the present investigation is more representative of the average than in the earlier work. Holladay, for example, found that the exponent was somewhere between 0.15 and 0.4 with a probable average value of 0.3. The value found in the present work is 0.44 which is approximately the same as Holladay's upper limit.

Influence of Size of Source. A series of five circular sources was used for determining the effect of size upon the BCD brightness. The relative sizes of these sources, which subtended solid angles from 0.0001 to 0.126 steradian, are illustrated in Fig. 3. The actual sizes, which were viewed at a distance of 40 inches, varied from 0.46 inch to 16 inches in diameter. In table II are summarized the physical data pertaining to the circular sources and the average BCD brightnesses as determined by the representative group of subjects for a field brightness of 10 footlamberts. These BCD brightnesses are indicated in footlamberts on Fig. 3 for each of the circular sources and are plotted as the solid line of Fig. 4 for a field brightness of 10 footlamberts.

The relationship between the size of the source Q in steradians and the BCD brightness B in footlamberts approximates a linear logarithmic one for a range of Q from 0.0001 to 0.01 steradian. However, the BCD brightness decreases more rapidly for larger sources, from 0.01 to 0.13 steradian, than for smaller sources. The straight-line portion of the curve includes the range of sizes used by Holladay who also found a straight-line relationship. The

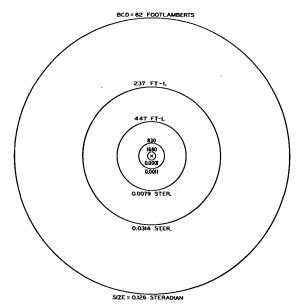


Figure 3. The relative sizes of the five circular sources used for determining the effect of size upon BCD brightness for a field brightness of 10 footlamberts. The point of fixation "X" was at the center of each of the sources.

empirical equation for the straight-line portion of the curve is

$$B = 102Q^{-0.30} \tag{3}$$

For the entire range of Q used in the present researches, the empirical equation is

$$B = 296 Q^{-0.21} - 377 \tag{4}$$

The exponents of Q are in general agreement with Holladay who found that a value of 0.25 gave a good approximation for the range he studied.

The inverse relationship between BCD brightness and size of source is obvious. However, it is a variable relationship and the change in one factor required as a result of an increase or decrease in the other factor is dependent upon the absolute values of size and brightness that are involved. For ex-

TABLE II.—Dimensions of the circular sources and the average BCD brightnesses in footlamberts of these sources, located on the line of vision which produced an initial momentary sensation judged to be at the border-line between comfort and discomfort. The field brightness was 10 footlamberts.

Size of source, steradians Visual angle, degrees			1 0.0079 5.72		4 0.120 22,62
When viewing distance is	4 0 inc	hes or 3	1/3 feet		
Diameter, inches		$1.48 \\ 1.72$	4.0 12.6	8.0 50.3	16.0 201
When viewing distance is	120 i	nches or	10 feet		
Diameter, inches Area. square inches		4.44 15.5	12.0 113	24.0 453	48.0 1810
BCD Brightness, footlamberts 16	50	830	447	237	82

ample, when the size Q of the source is less than 0.01 steradian, doubling its brightness requires that it be reduced to one-tenth of its original size. As the size of the source is increased progressively above 0.01 steradian, a doubling of its brightness requires progressively less of a reduction in size. The reasonableness of these results is obvious when it is considered that large sources cover a substantial part of the visual field and become a greater factor in the brightness to which the eyes are adapted. Furthermore, the BCD brightnesses of large sources are relatively low in comparison with the BCD brightnesses of small sources and therefore, the effects of changes in size are relatively diminished.

It is emphasized that the relationship given in equation (4) holds only for the range of the experimental data and cannot be extrapolated for larger sizes of sources without obtaining BCD values which are without meaning. For example, if equation (4) were extended in range to a source subtending a solid angle of about 0.32 steradian, the BCD brightness would become zero, and for larger sources would be a negative quantity. Obviously this is an empirical equation of limited range. Nevertheless, the results do indicate that large expanses of luminous areas become uncomfortable when their brightnesses are relatively low. For many years we have emphasized12 this fault of indirect lighting in large interiors and have advocated general illumination plus supplementary lighting.1,2 However, a large source may include a considerable portion of the visual field and part of it may be in view at all times. Thus the large source may contribute directly to the adaptation brightness which will be higher. Therefore, the sensation of brightness when the source is viewed directly may be appreciably mitigated.

It is interesting to compare the diameters and areas of sources of the same solid angle when they are viewed at 40 inches, the distance used in this investigation, and, for example, when they are viewed at 10 feet. The sizes of the sources for the two distances are presented in Table II. At the viewing-distance of 10 feet a circular source subtending a solid angle of 0.0079 steradian has a diameter of 12 inches, and an area of 113 square inches and approximates an enclosing globe. The largest source used in the investigation was 16 inches in diameter at a viewing-distance of 40 inches. A source of the same visual size would have a diameter of 48 inches when viewed at 10 feet. The area at this distance is 1810 square inches or approximately 12.5 square feet. When a source of this size is fixated centrally, the visual angle subtended by its diameter is 22.6 degrees. In other

words, the central visual field is covered by this source. Furthermore, a source of this size begins to approach a large segment of an indirectly illuminated or luminous ceiling. Thus these data can be used to indicate the BCD brightness of either small or relatively large areas in a visual environment.

Basic Relationships. When the relationships between the BCD brightness B of a source in the visual field, the size of source Q, and the field brightness F for the entire range of the experimental data are combined the following empirical formula is obtained from equations (2) and (4):

$$B = 108F^{0.44}(Q^{-0.21}-1.28) \tag{5}$$

This is plotted in Fig. 4 for three values of F. The points for which experimental data were obtained are indicated by the solid dots. Dotted lines have been drawn parallel to the 10-footlambert curve for field brightnesses of 1 and 100 footlamberts. This parallel relationship has been adequately established by Holladay and was spot checked in the present investigation.

From Fig. 4 it is seen that, for a constant BCD brightness, if the field brightness is increased, the size of the source may also be increased, but not in a constant ratio. For a given BCD brightness of a source, the permissible increase in the size of the source depends upon its absolute size. For example, for a given BCD brightness of a source when the size of the source is only 0.001 steradian, the area of the source may be increased about 2.6 times, or to 0.0026 steradian, if the field brightness is doubled. However, when the size of the source is as large as 0.1 steradian, its area may be increased by a factor of only 1.3, or to 0.13 steradian, when the field brightness is doubled. In other words, when the

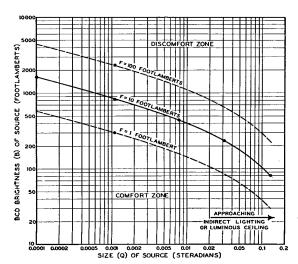


Figure 4. The relationship between BCD brightness B and size Q of sources located on the line of vision for three surrounding field brightnesses F.

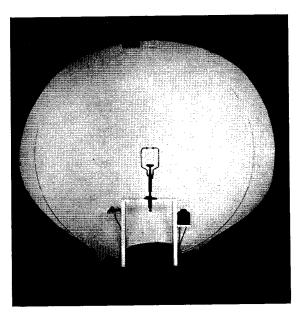


Figure 5. Locations of the circular test-sources along the vertical, diagonal and horizontal meridians. The diameter of the sources was 1.48 inches or 0.0011 steradian at a viewing distance of 40 inches.

brightnesses of sources are increased, considerable care must be exercised to make the necessary compensating increase in the field brightness or decrease in the size of the source or both.

Position of Source in Visual Field

In a previous paper,⁹ the authors presented the results of a limited investigation of the relative brightnesses above and to one side of the line of vision which produced the same initial visual sensation. The present researches were designed to extend the scope of the previous work and to determine more explicitly the effect of displacing a source from the line of vision.

Circular sources of brightness were located at various angular distances from the line of vision along three meridians-vertically, diagonally and horizontally—as is illustrated in Fig. 5. The diameter of the sources was 1.48 inches which, at a viewing distance of 40 inches, subtended solid angles of 0.0011 steradian. These circular sources were located on the inner surface of the sphere at a constant distance from the eyes which were at the center of the sphere. Therefore, these sources were constant in visual size and shape. The brightness of the surrounding field, which extended over the entire visual field, was maintained at 10 footlam-The comparison method as previously described was used for determining the BCD brightness of the sources as they were displaced at various angular distances from the line of vision. central source which was viewed directly by the

TABLE III.—BCD brightnesses in footlamberts of circular sources (subtending 0.0011 steradian) located at various angles (θ) from the horizontal line of vision. The field brightness, which extended over the entire visual field, was 10 footlamberts. (See Fig. 5).

Angle θ from line of vision	BCD Br	BCD Brightness in Footlamberts					
in degrees	V ertical	Diagonal	Horizontal				
0	830	830	830				
5	1165	1020	886				
10	1344	1051	928				
20	1764	1338	1037				
30	2764	1967	1254				
40	4452	2937	1463				
50	7615	4500 .	2015				
60	14000*	7658	2566				
70		13500*	3423				
80			5212				
90		*******	8396				
100		*******	15000*				

^{*}Outside of visual field for some subjects.

subjects was set at the standard BCD brightness of 830 footlamberts determined in the basic phase of the investigation. The eyes of the subject remained fixed on the position of the central source and the off-axis sources were seen by averted vision. Since the comparison source was at the BCD brightness, and the off-axis sources were adjusted by the subjects for the same initial momentary sensation of brightness, the latter are also BCD brightness. The comparison-source and the off-axis source were exposed alternately for 1-second intervals.

In Table III are summarized the average BCD brightnesses as determined by the representative group of 10 subjects. The observed values are also diagrammatically presented in Fig. 6, where the cross at zero degrees represents the point of fixation while appraising the displaced sources by averted vision. The angular displacement of the sources was extended to cover the limits of the visual fields for all subjects. For example, all of the subjects were able to see a source located 50 degrees above

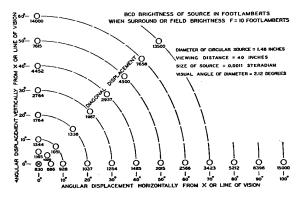


Figure 6. A diagrammatic representation of vertical, diagonal and horizontal displacement of test-sources from the point of fixation "X." The BCD brightness for each source location is indicated below the respective circles for a field brightness of 10 footlamberts.

the line of vision, but only four were able to see the source at 60 degrees. The latter datum was used for extrapolating the average curve to 60 degrees. Similarly, sources displaced 70 degrees diagonally and 100 degrees horizontally were outside the visual field for some of the subjects.

Averages obtained by the individual subjects indicate that the relative brightnesses selected for the BCD values varied approximately the same as those obtained for the standard BCD brightness. That is, some subjects appear to be more sensitive to brightness than others, and hence their BCD brightness is lower than that obtained by the less sensitive subjects. However, it was found that a subject generally fell into the same position throughout all of the tests. In other words, a subject selecting high BCD brightnesses always selected high values. It is interesting to note that when the angular displacement of the source from the line of vision is small, the variation among subjects is small, being about a 2:1 ratio. However, as the angular displacement increases, the variation among subjects becomes greater and reaches a maximum of about 10:1. In other words, central vision appears to be more uniform among a group of subjects than does peripheral vision. Such a variation is to be expected because of individual variations in judgment, retinal sensitivity and other psychological and physiological factors. Seeing by averted vision involves focusing the attention on a given object while focusing the eyes on another object. inherent difficulty apparently increases with the angular separation of the two objects. Nevertheless, the average BCD brightness is a useful value for specifying brightness relationships in the visual field provided a suitable "factor of safety" is used. In other words, the average BCD brightness for a large group of persons is always of a higher value than the lowest individual value. This point is discussed later.

In Fig. 7 is presented graphically the manner in which the BCD brightnesses increase as a source is displaced from the line of vision in different merid-The heavy lines representing the vertical, diagonal and horizontal displacement are smooth curves drawn through the average observed values which are indicated by the circles. The thinner lines for the intermediate meridians have been interpolated. From these curves it is seen that a source of a given brightness is more effective in producing a given sensation when displaced horizontally from the line of vision than it is when displaced the same angular distance vertically above the line of vision. For example, when the field brightness is 10 footlamberts, a source having a brightness of 2000 footlamberts must be displaced 50 degrees horizontally

but only 23 degrees vertically in order to be at the borderline between comfort and discomfort. Or, stated in another way, a 2000-footlambert source displaced 50 degrees horizontally and a 7600-footlambert source displaced 50 degrees vertically are both BCD brightnesses for the standard experimental conditions.

No significant change in the curve of Fig. 7 for horizontal displacement is noticeable at the transition point between binocular and monocular perception of the sources. The actual transition point varied from subject to subject, being dependent upon facial configuration, and ranged from approximately 40 to 50 degrees. An examination of individual data revealed no sudden change in BCD brightness which could be attributed to this factor.

The relationships between the vertical BCD brightnesses and the diagonal and horizontal BCD

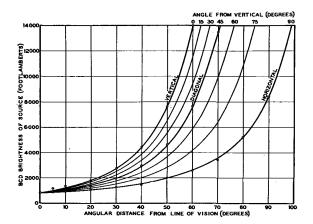


Figure 7. Illustrating how the BCD brightness increases as a circular source, subtending a solid angle of 0.0011 steradian, is displaced along various meridians from the line of vision. The heavy lines represent observed data for vertical, diagonal and horizontal displacement. The thinner lines for the intermediate meridians have been interpolated. These data were obtained when the field brightness was 10 footlamberts.

brightnesses are illustrated in Fig. 8. In this diagram the BCD brightness at each diagonal and horizontal angle is expressed relatively to the vertical BCD brightness at the same angle. For example, a source located on the diagonal meridian 50 degrees from the line of vision can be only 60 per cent of the corresponding BCD brightness of a source located 50 degrees from the line of vision on the vertical meridian. Similarly, a source displaced 50 degrees horizontally from the line of vision can be only 27 per cent of the corresponding vertical BCD brightness. The observed points are indicated by the circles and result in approximately linear rela-

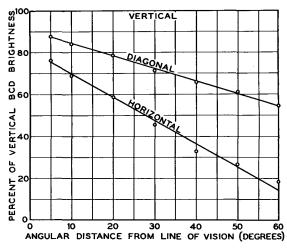


Figure 8. The relative BCD brightnesses of sources displaced along the diagonal and horizontal meridians in per cent of the BCD brightness of a source displaced the same angular distance along the vertical meridian. The size of the test-sources was 0.0011 steradian and the field brightness was 10 footlamberts.

tionships for the useful range to 60 degrees from the line of vision.

A graphical illustration of the variation of BCD brightnesses of the circular sources located in the visual field above the horizontal is presented in Fig. 9 when the point of fixation is at zero degrees. The heavy solid curved lines represent iso-BCD brightnesses and are lines of equal BCD brightnesses. A striking point is the similarity between the shape of the BCD curves and the curve representing the boundary of the visual field. The latter dotted curve is based upon data published by Duke-Elder, Sheard and others. The extent of the visual field is greatly dependent upon physiognomy or facial topography and upon the retinal characteristics of any particular individual. The slight dip in the curves along the vertical meridian is real and also

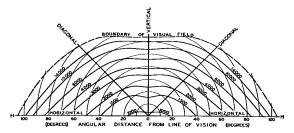
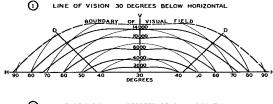
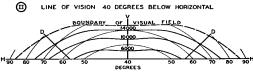


Figure 9. A graphical illustration of the variation of BCD brightnesses of circular sources (subtending 0.0011 steradian) located in the visual field above the horizontal. The point of fixation was at zero degrees and the brightness of the field was 10 footlamberts. The heavy solid curved lines represent points of equal BCD brightness. The broken curve represents the boundary of the visual field.







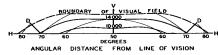


Figure 10. Illustrating the extent of the visual field above the horizontal and the permissible BCD brightnesses of sources (subtending 0.0011 steradian) when the line of vision is depressed below the horizontal. The field brightness was 10 footlamberts.

appears to follow the shape of the boundary of the visual field.

Fig. 9 is predicated upon the subject's line of vision being horizontal and the point of fixation at zero degrees. However, one often is more interested in that part of the visual field above the horizontal when the line of vision is depressed below the horizontal. Diagrammatic views of this are presented in Fig. 10. It is seen that, as the line of vision is depressed more and more below the horizontal, that part of the visual field in which bright areas ordinarily are located becomes progressively smaller. For example, as shown in I, Fig. 10, when the line of vision is 30 degrees below the horizontal, the visual field extends vertically above the horizontal only about 33 degrees and laterally on the horizontal about 60 degrees. In III, Fig. 10, when the line of vision is 50 degrees below the horizontal, which approximates looking down at a desk, the extent of the visual field is only 13 degrees vertically and 30 degrees horizontally. In other words, there is but a small area in which sources can be seen and for the latter example the minimum BCD brightness is about 8000 footlamberts when the size of the source is 0.0011 steradian and the field brightness is 10 footlamberts. Nevertheless, the line of vision at times may and often will be horizontal or above the horizontal. Therefore the complete diagram of Fig. 9 cannot be neglected or ignored.

In Table IV are presented the BCD brightnesses taken from the smooth curves of Fig. 7 for sources located at various positions in the visual field, having a size of 0.0011 steradian (1.48 inches in

diameter, viewed at 40 inches) and a surrounding field brightness of 10 footlamberts. These BCD brightnesses may be increased by 36 per cent if the field brightness is 20 footlamberts and by 85 per cent if the field brightness is 40 footlamberts. However, if the size of the source is increased by a factor of ten, the brightnesses of Table IV must be halved.

The relationship between the BCD brightness and the position of a source in the visual field is necessarily a complex one since it involves physiological and psychological factors. Either Fig. 7 or Fig. 9 may be used for determining the relative BCD brightness of a source located anywhere in the visual field. However, they involve the measurement or calculation of the angle of a meridian from the vertical and the angular displacement of the source from the line of vision along this meridian. In other words, on these diagrams, the source is considered to be on the surface of a sphere.

In establishing a more practical relationship, it is desirable to consider that the source is located in a vertical plane normal to the horizontal line of vision. Thus, the position of a source may be defined by a vertical distance V above the horizontal line of vision and a lateral distance L from the point where the horizontal line of vision intersects the vertical plane. By expressing these distances in terms of the distance R from the eye to the vertical plane, the numerical values $\frac{V}{R}$ and $\frac{L}{R}$ become simple

factors for determining a position index P. If the position index P is in terms of the relative BCD brightness, it becomes a relatively simple matter to determine the BCD brightness of a source located anywhere in the visual field above the line of vision. The position indexes P, or the relative BCD brightnesses of sources located at various positions in the visual field, are presented in Fig. 11. It is seen that Fig. 9 and Fig. 11 are similar except that linear measurements are involved in the latter.

TABLE IV.—BCD brightnesses of a circular source located at various positions in the visual field when the size of the source is 0.0011 steradian and the brightness F of the surrounding field is 10 footlamberts.

Angular BCD Brightnesses for Various Radial An Distance from the Vertical (Footlamberts) from Line									
of Visi	on	Vertica	al	1	Diagonal			Horizontal	
(Degre	es)	0	15	30	45	60	75	90	
0		830	830	830	830	830	830	830	
10		1230	1160	1120	1070	1020	980	930	
20		1790	1670	1530	1400	1290	1170	1060	
30		2770	2520	2260	1980	1730	1470	1250	
40		4400	3910	3420	2940	2480	1990	1540	
50		7640	6450	5550	4500	3730	2900	1960	
60		14000	11800	9700	7660	5860	4150	2600	
70					13500	10000	6400	3580	
80							11140	5200	
90			*******		••••••		*******	8400	
100					•••••			15000	

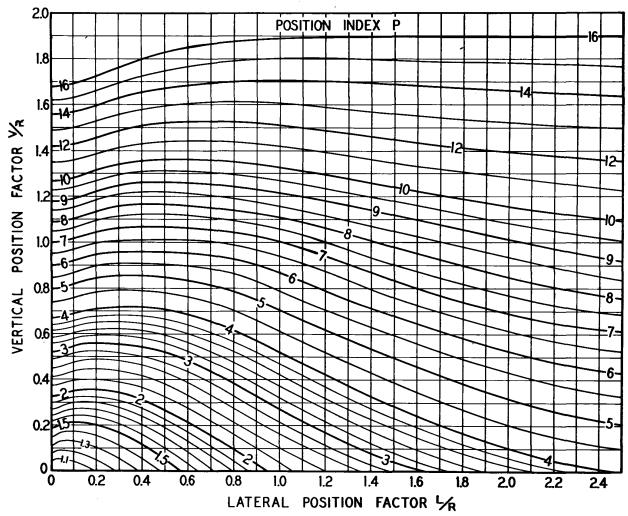


Figure 11. Chart for determining the Position Index P of sources located at various positions in the visual field. V and L are the vertical and lateral distances, respectively, from the line of vision and R is the distance from the eye to the vertical plane normal to the line of vision in which the source is located.

Equation 5, which is for a source located on the line of vision may be extended to include a source located anywhere in the visual field above the line of vision by incorporating the position index P. The position index P indicates the relative BCD brightness of a source displaced from the line of vision in terms of the BCD brightness of a source located on the line of vision. In other words, if the BCD brightness of a specific source located on the line of vision is B footlamberts, the permissible BCD brightness is BP footlamberts when the same source is displaced from the line of vision. Thus equation (5) becomes

$$B = 108PF^{0.44}(Q^{-0.21} - 1.28) \tag{6}$$

This expression may be rewritten as

$$M = \frac{B}{PF^{0.44}(Q^{-0.21} - 1.28)} \tag{7}$$

 \mathbf{or}

$$\log M = \log B - \log P - 0.44 \log F - \log (Q^{-0.21} - 1.28)$$
(8)

Thus M becomes an index of the sensation of visual comfort when a source is exposed to view. When M is equal to 108, the combination of source-brightness B, size Q and position index P and the field brightness F are such that the initial sensation of brightness received from the source is at the borderline between comfort and discomfort or BCD. Smaller and larger values of M indicate greater comfort or discomfort, respectively. Log M corresponds to the K-factor developed by Luckiesh and Holladay⁸ who found a value of K equal to 1.9 for the BCD brightness. In the present investigation log M (when M=108) is 2.03 when the sensation is at the borderline between comfort and discomi

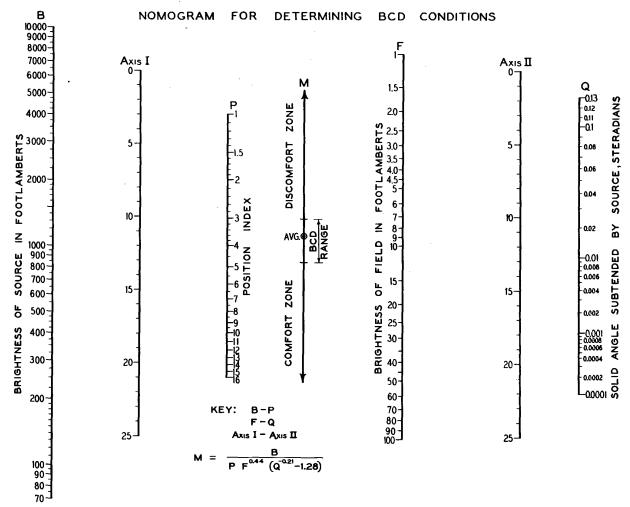


Figure 12. A nomogram for calculating the factors which combine to determine the BCD brightness of a source.

fort. The absolute values of K and $\log M$ are dependent upon exponents of F and Q, which in turn are a function of the characteristics of the specific subjects used in the respective investigations.

The nomogram of Fig. 12 and the disk calculator illustrated in Fig. 13 have been designed to facilitate the calculation of the various factors which combine to determine the BCD brightness of a source. It will be noted that the M-line has not been given scale values. It is re-emphasized that the present investigation had as its purpose the establishment of the relationships between the factors which produce a sensation at the borderline between comfort and discomfort. Therefore, any value of M other than the BCD value has no numerical significance except to indicate that a source is in the comfort zone or discomfort zone. Three relative BCD values of M have been indicated. average value has been used throughout this discussion and is indicated by the circled dot on the M-line. The BCD range has been determined from the individual maximum and minimum BCD brightnesses selected by the 50 subjects who were involved in the basic phase of this series of researches. The average BCD value merely means that half of any given group can be expected to find the sensation of brightness from a source to be at, or more comfortable than, the borderline between comfort and discomfort. If comfort is of major importance in a visual environment, the lower BCD limit may be used and the resultant sensation would be expected to be less than BCD for most of the group. Conversely, if comfort is of little moment, the upper BCD limit might be used.

The scales along Axis I and Axis II of Fig. 12 are merely for convenience and have no significance. These axes are turning points for combining the various factors and the scales should be helpful in relating the right and left sections of the nomogram.

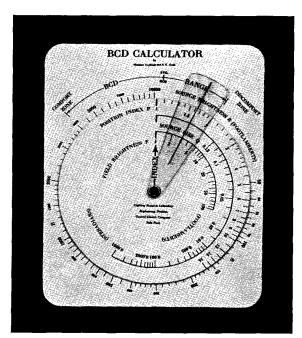


Figure 13. A disk calculator which contains in circular form the scales of the nomogram of Fig. 12.

Any one of the factors may be determined on the nomogram if the other factors are known or specific values are assumed. Several examples will serve to illustrate the method of using the nomogram.

Example I. It is desired to ascertain whether a given source produces an effect of discomfort for the following conditions:

Brightness B of source = 3000 footlamberts Size Q of source = 0.02 steradian Brightness F of field = 50 footlamberts Position index P = 5 (from Fig. 11)

Draw a line from B=3000 to P=5 and from F=50 to Q=0.02. From the intersections of these lines with Axis I and Axis II, respectively, draw a third line. The intersection of this third line with M indicates the relative degree of the sensation produced by the source, which in this case is BCD.

Example 2. It is desired to determine the BCD brightness of a source when F=40 footlamberts, Q=0.01 steradian and P=2. From the intersection on Axis II of a line between F=40 and Q=0.01, draw a line through the BCD point of line M to Axis I. The point where the line from P=2 through the turning point on Axis I intersects line B indicates the average BCD brightness which in this case is about 1500 footlamberts. In other words, if the brightness of the source is 1500 footlamberts, approximately half of a large group of individuals will find the sensation to be at or less than their own BCD value or to be in the comfort zone.

Example 3. At times it may be desirable to determine the brightness which will be at or near the BCD sensation for the most sensitive individual. Using the values of Example 2, but drawing the line from Axis II to Axis I through the lower limit of the BCD range, it is found that the source brightness must be reduced to about 550 footlamberts. In other words, if the brightness of the source is 550 footlamberts, it will be BCD or less for almost all who may view it.

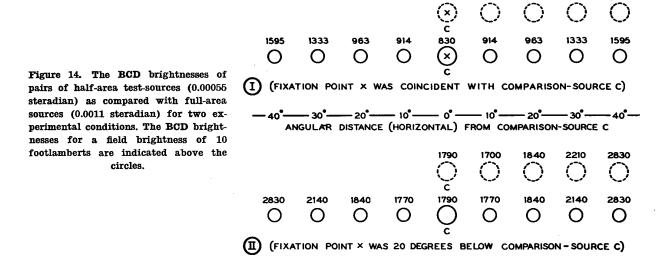
Multiple Sources

With most lighting installations, the sensation of brightness is induced by more than one source in the visual field. Therefore, it is desirable to be able to determine relationships which can be used to establish the BCD brightnesses for multiple sources. The simplest approach is to determine the characteristics of two or more sources in terms of a single source which produces the same sensation of comfort or discomfort. Since light-sources usually are spaced symmetrically in a visual environment, it is possible to consider certain basic positions of the sources as being representative of more complex arrangements.

The standard experimental conditions were used in this phase of the investigation. That is, the comparison-source subtended a solid angle of 0.0011 steradian at a viewing distance of 40 inches and the field brightness was 10 footlamberts. In the two parts of this investigation the test-sources were one-half the area and equal to the area of the comparison-source, respectively. The comparison method was used to determine the BCD brightnesses of the test-sources.

Additive Effect of Different Areas of Equal Brightness. In this part of the investigation two test-sources were located on a horizontal plane equidistant from the comparison-source which was on the line of vision. Each test-source subtended one-half the solid angle of the comparison-source, or 0.00055 steradian (1.045 inches diameter). The subject was required to adjust the brightness of the two test-sources until the sensation produced by them was equal to the sensation produced by the single comparison-source. The general arrangement is illustrated in Fig. 14 for (I) when the fixation point was coincident with the comparison-source and (II) when the fixation point was 20 degrees below the comparison-source.

When the comparison-source C was on the line of vision, it was set at the standard BCD brightness of 830 footlamberts. The test-sources T were used in pairs, both of which were simultaneously exposed. For example, test-sources were located 10 degrees to the right of and 10 degrees to the left of the com-



parison-source and were seen by averted vision. The two test-sources and the comparison-source were exposed alternately for 1-second intervals.

In I, Fig. 14, the average BCD brightness of two half-area sources displaced equal distances horizontally from the comparison-source are indicated above the solid circles. The BCD brightnesses of the single full-area source, indicated above the dotted circles, were obtained from the curve for horizontal displacements of Fig. 7. It is seen that the BCD brightness of the two half-area sources are approximately the same as the BCD brightnesses of a full-area source displaced the same distance from the line of vision, with an average variation of approximately 5 per cent.

When the fixation point was 20 degrees below the comparison-source, the predetermined brightness of the latter was 1790 footlamberts (from the curve for vertical displacement, Fig. 7). In this case all the sources were seen by averted vision. The BCD brightnesses of the half-area sources are indicated above the solid circles. The BCD brightnesses of the single full-area sources were obtained from Fig. 9. The BCD brightnesses of full- and half-area sources are approximately the same, the average variation being about two per cent.

From the data on half-area sources it may be concluded that two sources of equal brightness and area located symmetrically on either side of the line of vision are equivalent to a single source of the same brightness and total area located at the point where one of the half-area sources is located.

Additive Effect of Sources of Equal Area and Brightness. In Fig. 15 are presented the BCD brightnesses of multiple sources located at various angular distances above the line of vision. Standard experimental conditions were used. That is,

the sizes of the test- and comparison-sources were 0.0011 steradian, the brightness of the surrounding field was 10 footlamberts and the brightness of the comparison test-source was 830 footlamberts. From one to four test-sources were exposed simultaneously and alternately with the comparison-source. For example, with Condition V test-sources were located at angular distances of 10, 20 and 30 degrees above

1060

1540

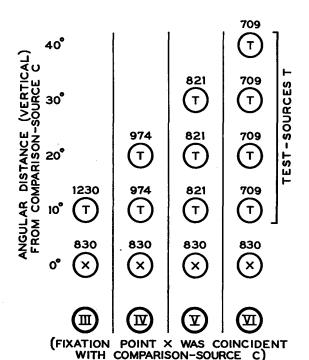


Figure 15. The BCD brightnesses of multiple test-sources T located above the line of vision. The size of the sources was 0.0011 steradian and the field brightness was 10 footlamberts. The point of fixation "X" was coincident with the comparison-source C.

the line of vision. The subjects were required to evaluate the initial momentary visual sensation induced simultaneously by the three test-sources and adjust their brightnesses so that the sensation was equal to that induced by the comparison-source which was coincident with the point of fixation. Similar evaluations were made with Conditions III, IV and VI except that one, two and four test-sources were involved, respectively. Since the comparison-source was set at the BCD brightness, the brightnesses of the multiple sources also were BCD values. The average BCD brightnesses as determined by the group of representative subjects are indicated directly above the circles which represent the sources.

As additional sources are added above the line of vision, the brightnesses of all the sources must be reduced in order to maintain the sensation at BCD. When the sources are progressively added at greater angles above the line of vision, the necessary reduction in brightness becomes progressively less. This is to be expected, since, from Fig. 7, it is seen that for the same sensation, sources displaced at greater angles from the line of vision may be considerably brighter than sources closer to the line of vision. Thus, the source displaced the smallest angular distance from the line of vision contributes more to the sensation than do the other sources.

By means of the nomogram of Fig. 12 it is possible to calculate the size or position of a single source above the line of vision having the same brightness and producing the same sensation as the multiple sources. For example, the two sources of Condition IV are located at 10 and 20 degrees and the average angle is 15 degrees above the line of vision. Thus, using a source brightness B of 974 footlamberts, a position index P of 1.79, a field brightness F of 10 footlamberts and M equal to the average BCD value, the corresponding value of the size Q of the source is 0.0044 steradian. Alternately, if the size Q of the single source is assumed to be equal to the sum of the areas of the two test-sources, or 0.0022 steradian, the position index obtained from the nomogram is 1.39. From Fig. 11 this may be converted into a vertical displacement of slightly more than 8 degrees. In other words, the two test-sources may be represented by either one of two single sources: (a) by one which is four times the area of a single source located at the average angle above the line of vision; or (b) by one which is the sum of the individual areas (0.0022 steradian) and located approximately at the position of the source that is closest to the line of vision.

Three sources above the line of vision (Fig. 15, Condition V) may be represented by (a) a single source of ten times the area (0.011 steradian) of

each of the multiple sources located at the average angle of 20 degrees above the line of vision or by (b) a single source of three times the area (0.0033 steradian) of the multiple sources located eight degrees above the line of vision. Calculations of the single source which is equivalent to four sources above the line of vision (Fig. 15, Condition VI) result in (a) a source of 0.024 steradian located 25 degrees above the line of vision or (b) a source of 0.0044 steradian (the sum of the four individual solid angles) located approximately seven degrees above the line of vision. From the foregoing it is seen that a simple and practical equivalent for multiple sources of equal brightness and area which are located above the line of vision is the total of the individual areas located at the position of the source that is closest to the line of vision. Such a conclusion appears to be a conservative one.

The results of a similar investigation of sources located to one side of the comparison-source when the fixation point was 20 degrees below the comparison source are presented in Fig. 16. It is seen that as additional test-sources T are added at greater angles from the comparison-source C, the brightnesses for the BCD sensation become progressively less. Since both vertical and horizontal components are involved in defining the positions of these sources in the visual field, the determination of a single equivalent source becomes considerably more complex. However, the test-sources T of Conditions VIII, IX and X may be represented by single

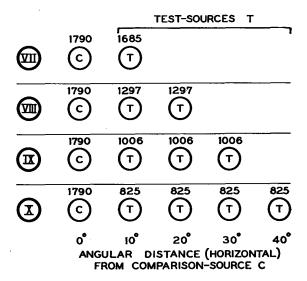


Figure 16. The BCD brightnesses of multiple test-sources located above and to the right of the line of vision. The size of the sources was 0.0011 steradian and the field brightness was 10 footlamberts. The point of fixation "X" was 20 degrees below the comparison-source C.

(FIXATION POINT X WAS 20° BELOW

COMPARISON-SOURCE C)

sources located at average angles to one side of the comparison-source C. The brightnesses of the single sources are the same as the respective multiple sources. For example, for Condition VIII a single source located midway between the two test-sources could subtend a solid angle of 0.0030 steradian, which is approximately three times the area of each test-source. The three test-sources of Condition IX may be represented by a single source located 20 degrees to the right of the comparison-source C and which subtends a solid angle of 0.0068 steradian or about six times the size of each test-source. Similarly, a single source subtending a solid angle of 0.0135 steradian located 25 degrees to the right of the comparison-source C will be equivalent to the four test-sources T of Condition X.

It has been emphasized that values of M other than the average, minimum or maximum values have no absolute significance. Therefore, calculation of M merely results in a numerical index of the sensation of comfort or discomfort of the brightnesses within a visual environment. Values of M are not directly additive. That is, if a group of multiple sources is collectively at the average BCD brightness, the sum of the individual M values does not necessarily equal the average value of 108. In general the sum will be higher than the average value. Calculations based upon the individual sources illustrated in Figs. 15 and 16 result in summations ranging from about 140 to 175, with a maximum individual M of approximately 80. A tentative conclusion, which must be substantiated by additional experimental data, is that if the sum of the M values of the individual sources of a group is not more than 150, and if the M value of any single source is less than 80, the sensation induced by the group of sources probably will be at or less than the BCD sensation.

Linear Sources

Since many lighting installations involve fluorescent lamps and lighting equipments which present areas that are relatively long and narrow, it is de-

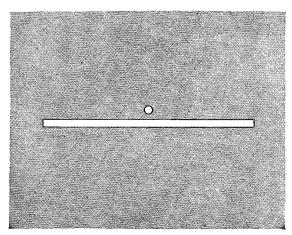


Figure 17. The arrangement for determining the BCD brightness of linear sources. Each source was alternately viewed directly by the subjects. The diameter of the circular source was 1.48 inches, approximately the same as the diameter of the tube of the 40-watt lamp which was used as the linear source. The viewing distance was 10 feet and the field brightness was 10 footlamberts.

sirable to establish the relationships between circular and linear sources. Thus the basic relationships between the factors which influence the degree of sensation induced by a circular source can be applied with reasonable accuracy to sources of certain other configurations.

In this part of the investigation the circular source was located immediately above the linear source as is illustrated in Fig. 17. The two sources were close to the line of vision and therefore each could be fixated directly with a minimum movement of the eyes. The brightness of the surrounding field amidst which the sources were viewed was 10 footlamberts. The diameter of the circular source was 1.48 inches, approximately the same as the diameter of the tube of a 40-watt fluorescent lamp which was used as the linear source. At the viewing distance of 10 feet, the circular source subtended a solid angle of 0.00012 steradian which is within the range of sizes used in the basic phase of the present investigation.

TABLE V.—BCD brightnesses in footlamberts for equal areas of circular and linear sources, for a viewing distance of 10 feet and a field brightness of 10 footlamberts. The linear source was actually a 40-watt fluorescent lamp (1.5-inch diameter).

Area of Sources Square Inches	Cir	cular Source	Lin	BCD	
	Diameter Inches	BCD Brightness Pootlamberts	Length Inches	BCD Brightness Footlamberts	Ratio Lin./Cir.
0.78	1	2000	0.52	2000	1.00
3.14	2	1340	2.10	1340	1.00
7.07	3	1040	4.71	1070	1.03
12.6	4	880	8.40	940	1.07
19.6	5	770	13.1	855	1.11
28.3	6	685	18.9	805	1.18
38.5	7	625	25.7	775	1.24
50.2	8	575	33.4	760	1.32
63.6	9	535	42.5	750	1.40
78.6	10	500	52.4*	740*	1.48
113.	12	445	75.3*	735*	1.65

Brightnesses in Visual Field—Luckiesh-Guth

*Extrapolated.

The comparison method was used, the linear source being considered the comparison-source. The brightness of the circular source was adjusted by the subject until the initial momentary sensation induced alternately by the two sources was identical. The experimental variables included the length and brightness of the linear source. Thus, with the basic data for circular sources it was possible to determine the BCD brightnesses of linear sources of various lengths.

In Table V are summarized the BCD brightnesses for equal areas of circular and linear sources. The BCD brightnesses are approximately equal for both types of sources for areas not greater than about 7 square inches when viewed at a distance of ten feet. However, as the area of the source is increased beyond 7 square inches, the permissible brightness for the BCD sensation for the circular source becomes progressively less than the BCD brightness of the linear source. In other words, as a linear source begins to deviate from a concentrated area, adding to its horizontal length does not require much of a decrease in its brightness in order to maintain a BCD sensation. Apparently if the length of a linear source is less than approximately four times its width, it may be considered equivalent to a circular source.

In the present investigation it was found that the 40-watt fluorescent lamp at the normal lamp brightness of 1610 footlamberts produced the same sensation as a circular source 1.48 inches in diameter and a brightness of 3000 footlamberts when both were viewed directly at a distance of 10 feet. From the nomogram of Fig. 12 it is determined that this circular source must be positioned above the line of vision in order for the sensation to be at the BCD level. The position index P is 1.8 which indicates, for example, that the source should be located approximately 15 degrees above the line of vision.

Summary

A complex subject such as quality of lighting cannot be resolved into a simple summary. In reality, this paper is a summary of an extensive series of researches dealing with the complex relationships among the various factors which govern whether a visual environment is comfortable or uncomfortable. These factors are divided into two groups. The first group involves those that are basic and fundamental such as:

- 1. The brightness of the source or luminous area.
- 2. The visual size of the source.
- The brightness of the surrounding field.

The relationships among these factors have been determined by evaluating them in terms of a visual

sensation which is at the borderline between comfort and discomfort. We have termed this the BCD sensation or BCD brightness.

Certain other factors which have a modifying influence upon the basic factors include:

- 1. The position of the source in the visual field.
- 2. The number of sources in the visual field.
- 3. The configuration of the sources.

By combining the effects of the modifying factors with the basic relationships, it has been possible to devise a nomogram and a disk calculator which simplify the determination of the brightness conditions which should produce an average BCD sensation or BCD brightness.

Glimpses have been given of the manner by which the basic and modifying data may be practicalized and rationalized with lighting practice. Because of the limitation of space, it is necessary to postpone to a future paper a discussion of researches involving critical seeing, continuous exposure of sources, distraction thresholds, simulated practical visual environments, etc. Considerable data already have been obtained on these, and a thorough analysis should materially assist in further practicalization of the fundamental relationships.

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DISCUSSION

Leland H. Brown:* I consider this paper an outstanding piece of engineering research in the field of brightness studies. Direct engineering application of their findings is assured for they chose as the criterion for their studies the "BCD sensation." By fixing the source diameter and brightness of the field they were able to first establish a standard BCD brightness. Then by varying one factor at a time, and keeping everything else constant, they determined the BCD brightness for: different field brightnesses, different source sizes, different source positions in the visual field, multiple sources, and linear sources.

The data they secured for all these different conditions are all quite consistent and conclusive. They consequently constitute excellent engineering material for use in lighting applications. However, after accumulating these results, the authors summarized their findings by mathematical equations and then prepared an alignment chart and a calculator to enable one to calculate the BCD brightnesses for combination conditions. Presumably such calculations are justified, but to be conclusive more data than the two meager spot checks for 1 and 100 footlamberts shown in Fig. 4 should be given if engineers are to use the chart or calculator with confidence.

In researches such as this one it is always difficult to get a group of "typical" observers. It would therefore be very interesting, and helpful to future researchers, if the authors would state just which of subjects 1 to 50 made up the 10-man group that was used for most of the tests.

The authors earefully point out that their studies, as reported in this paper, only cover the BCD brightnesses for a source, or for sources, exposed momentarily to a person's view. However, they added that they had similar studies underway covering continuous exposures of sources, etc. Illuminating engineers will await their completion of these studies and the publication of the results with keen anticipation.

WARD HARRISON:** Much appreciation is due the au-

thors who have today brought us data which are new and badly needed. In fact, the test information is so voluminous and so diversified that it is difficult to present an adequate discussion of the paper and at the same time be brief. I would like to emphasize just a few points which seem to me most significant, and of course I shall be particularly interested to compare the authors' results with appraisals according to the glare factor method.

1. It should be kept in mind that the BCD represents the point where half of the observers feel that the glare source is on the comfortable side, and the other half think it is on the uncomfortable side. The authors established BCD values for sources from 6 inches in diameter to 36 inches viewed at a distance of 10 feet. These same BCD figure out to have glare factors ranging from 22½ to 40, and represent a very good agreement with Mr. Meaker's description*** of glare factors of this magnitude, which I quote:

15-25 Some people will find such rooms slightly annoying, particularly when they are engaged in critical work for long periods. This range is usually satisfactory where the occupants are changing activities and position at intervals of an hour or so.

25-40 This range should not be exceeded where quality of lighting is a consideration, in rooms where critical work is being done. Such values are illustrated by large rooms indirectly lighted to 75 footcandles or more in which some persons are persistently conscious of the large ceiling area.

2. In Mr. Holladay's work published in 1927 he found an exponent for Q of -.25 with relation to B. In the glare factor formula an exponent of -.5 has been used for Q, which indicates an apparent discrepancy of serious magnitude. It happens, however, that the largest glare source used in the Holladay experiments had an area of .006 steradians (approximately a 10-inch globe viewed at a distance of 10 feet); Luckiesh and Guth have most commendably carried the work on up to 0.126 steradians, or an area of about 30 times the former size. They check Holladay's experiments quite well for sources of the small area which he investigated (here they find B varies as Q -.3) but their results are quite different for larger sizes. They have developed a formula:

$$B = 296Q^{-0.21} - 377$$

which covers the entire range of their work, but the "-377" has such powerful effect on the final results that it renders the fundamental relationships difficult to visualize. A simple equation which satisfied quite well the new data for the larger area is:

$$B = 27Q^{-.6}$$

As a matter of fact, this equation fits all the points from .01 up as far as 0.1 steradians, with a maximum error of not more than 7%. Likewise, the equation

$$B = 39.3Q^{-5}$$

checks the Luckiesh-Guth data very well from .006 (the largest area tested by Holladay) to .06 steradians with about the same margin of error. In the opinion of the writer, areas of these general magnitudes are the ones of real importance in lighting practice—that is, they are representative of the sum total area of light sources usually found in modern lighting installations, particularly

^{*}Stanford University, Stanford, California (Department of Electrical Engineering).

^{**}Consulting Engineer, Cleveland, Ohio.

^{***}ILLUMINATING ENGINEERING, July, 1949, p. 404.

those of the fluorescent type. For these areas it will be noted that there is a very good check between the BCD-method exponents and the exponent of -.5 used in the glare factor formula.

An exponent of —.25 as found in the range explored by Holladay would mean that doubling the brightness of a source was as bad from the discomfort standpoint as increasing its area 16 times, whereas an exponent of —.5 means that doubling the brightness is only as bad as increasing the area four times. Much of the I.E.S. recommended practice has been based on the former concept, namely, that brightness is all important and that brightness limitations or brightness contrast limitations alone are sufficient to assure comfort. The new data confirm the long-standing opinion of some of us that both brightness and area must be included in any reasonable recommended practice limitations.

- 3. It is noteworthy that throughout the important range from .006 up, light source area is of greater importance to comfort than is surround brightness—that is, the exponents for Q are higher than for F. In other words, when the number of light sources in a room is doubled, comfort will be decreased—not increased.
- 4. The data in Figure 14 for light sources placed 10°, 20° or 30° on either side of the line of vision are of particular interest, and from these data the authors draw the conclusion that the glare effect of two small sources equally removed on both sides of the line of sight is the same as for one source of the same total area at the same angle. This would seem to give corroboration to the idea that the glare effect of multiple light sources is additive directly, as is done in the glare factor formula. On the other hand, in VIII, Figure 16, where the light sources are not symmetrically arranged, their effect appears to be no longer directly additive.
- 5. Again, it is possible that the shock method of appraisal employed by Luckiesh and Guth may unduly penalize light sources at wide angles to the line of vision, for the eye is notably sensitive to moving objects and flashing lights near the periphery of vision.
- 6. The data on lineal vs circular sources are interesting indeed, and somewhat puzzling, particularly the very slight effect on glare of increasing the length of the fluorescent source beyond 10° on each side of the line of sight (21 inches at 10 feet). However, there seems to be certain corroboration of this in data which I saw last year in Holland which indicated a 4-foot fluorescent lamp viewed in its entirety was less discomforting than when the area of visible source was reduced to one-half by covering each alternate inch with a strip of opaque paper. This whole matter has far-reaching implications and raises numerous questions to be answered. For example, are continuous rows of fluorescent fixtures less glaring than interrupted rows, and how far can the space be between luminous areas of adjacent fixtures before they are considered discontinuous? Also, will the fluorescent fixture with luminous side panels four inches high have the same effect placed in continuous rows as will bare fluorescent lamps? In view of the great volume of helpful data already supplied, I hesitate to suggest it, but it would seem that these questions cannot be an-

swered fully without still more laboratory investigation.

H. L. LOGAN:* Permit me to congratulate the authors on a splendid piece of work. It is a painstaking, careful, intelligent, ingenious and logical exploration of some of the response phenomena of human beings exposed to visual stimuli. It is research in an area that has been crying for such exploration. Dependable quantitative data such as the authors offer has been too generally absent

A proper evaluation of this research will require a lot of time. It is something I would not attempt, upon such brief acquaintance. The paper offers many ramifying possibilities, both of application and of further research, and will unquestionably influence future practice for the better.

Offhand, the data would not seem to support the advocates of rigid, or very limited, so-called brightness-ratios. It also indirectly challenges—and this time with impressive evidence—the dictum first published in the Journal of the Optical Society of America in April, 1926, "that the angle of orientation of the glare source about the visual axis makes no difference in the glare effect." It would be extremely interesting to see what results the authors would get if they would carry their research into that part of the visual field that lies below the line of sight. They might find definite evidence to support the conclusion of lay people that glare is more "glaring" below the line of sight than above. Such evidence would be helpful in a rational attack on reflected glare.

The very completeness and thoroughness of the research, and its implementation with the "BCD Calculator," may cause many people to regard it as the final word. It is the *latest* word, but the authors are very careful to point out that it is a long way from being the final word. They state that they still expect to report on effects of continuous exposure to sources, to distraction thresholds, to simulated practical visual environments, and perhaps to color; all of which factors must then be integrated into a unified treatment before more than tentative practical applications can be made. This is a very ambitious, albeit necessary program, and one can only wish the authors "God-speed" in its execution.

The engineer is always under the temptation to take material of this kind and immediately use it, not only for all it is worth, but for more than it is worth. Scientists, like the authors, are the first to deprecate such "extrapolation." To guard against it they carefully state all the restrictions surrounding it, but the engineer, whose work will not wait, is inclined to disregard the limitations. This leads to delusionary conclusions: conclusions which may be true in the restricted laboratory sense, but are false-to-reality in their general application. results of activity of this nature can be seen in installations all over the country. It has found expression in pronouncements of professional and industry committees that have even been incorporated into State and other Codes, thus freezing delusionary conclusions into law. This impedes progress in those areas until the laws can be ignored or changed.

It is to be hoped that the results of this research will

^{*}Holophane Company, Inc., New York, N. Y.

not be clouded by such unwise and premature activity.

There is a long step, in this field of human response particularly, between laboratory data and practical application. Laboratory tests in this field must, by their very nature, be statistical. The use of small groups of observers (and groups of 10 and 50 are very small groups where human reactions are concerned), can only establish trends of interaction between stimulus and response. Further, the variables that must be controlled, or eliminated, in order to isolate one for test, sets up an artificial situation that cannot be translated, a priori, into the actual working and living situations of people. This may become possible when the statistical base becomes large enough, and this paper is a significant step in the right direction.

I appreciate the reasons that impelled the authors to build the paper around the term "brightness," but wish they could have found it possible to avoid this. Whenever they mention "brightness" they couple it with area and time, either explicitly or implicitly, and so they are actually talking about light flux. They know this but may believe they will only be understood if they continue the use of the ambiguous abstraction "brightness."

The fundamental drawback to the term is the unconscious belief of practically everybody that brightness has objective reality—in the words of the layman, that it is real. It may come as a shock to many to be told that brightness is a label for a creation of the mind, and does not exist outside of one's skin. Light flux exists independently of an observer, but brightness does not. It is an abstraction the mind makes as the result of evaluating one effect of a visual stimulus on the nervous system.

Another effect that the mind likewise abstracts it labels "color." Color, like brightness, has no existence outside of an observer. The "green" is not in the leaf; it is not even in the eye; it is in the mind—or, putting it into 1949 terms, it is an abstraction made in the supra-granular layer of the cortex of the brain, of nervous activity occurring at lower levels, which in turn is an abstraction of nervous activity occurring in the optical thalamus, which in its turn is an abstraction of nervous activity occurring in the retina, optic nerve, and in-between connections. It can also be triggered internally without the aid of an external visual stimulus, and so can brightness.

Other forms of life, that have a different nervous organization, will not see the "green" in the leaf. They will see something entirely different, depending upon the particular band of radiation they are equipped to handle, and the degree of simplicity or complexity of their nervous organizations. Their "brightness" responses will likewise be very different.

The partial lack of dependence of the response upon the stimulus, and the very indirect and complicated connection between the two, makes it undesirable to use a feature of the response as if it were a feature of the stimulus. Brightness is a response phenomenon. Its present use in engineering practice confuses two different levels of activity. It confuses the independent reality in the external world with the semantic symbol arising from internal nervous activity. This leads to delusionary conclusions, as before mentioned, and undesirable engineering practices. The authors are well aware of this, because, while talking brightness as if it were a feature of the stimulus they have actually operated with flux. They know what they are doing but so that everyone else can be just as clear about it, a re-statement of their work on the basis of terms that indubitably belong to the stimulus, and terms that indubitably belong to the response, would avoid infusing new life into long-standing confusions.

This is in no sense a criticism of the able research reported by the authors. We should, and do, welcome dependable quantitative data obtained htrough controlled researches such as this. There is no other way to supply a solid foundation of knowledge for the determination of optimal seeing conditions.

PHELPS MEAKER:* I am very much pleased to note that the authors have not limited their studies to just a single light source in the visual field, or to sources on the axis of vision. If one adopts the notion that fixation of the glare source should be made the basis for comfort evaluation, it follows that you can only look at one source at a time. However, in all the comparison tests, and the paper deals with many of them, the fixated source is limited to serving as a reference for comparison, by which other sources off the line of vision are given an equivalent evaluation.

Out of the 17 illustrations in the paper, nine of them deal exclusively with the concept that discomfort can result from sources which are not fixated, i.e., sources off the line of vision. Three of the remaining figures include reference to off-axis sources. Half of the tables are similarly devoted to positional data.

It is obvious that there may be any number of sources in the visual field, each of them an off-axis source, amenable to some sort of comfort or discomfort evaluation. To use the terms adopted by the authors, the BCD brightness of any group of sources depends upon the number and disposition of all of those which can affect the eyes. This conclusion is supported adequately by the results of their experiments dealing with multiple sources.

It is unfortunate that there was not room for more information on details of the additive evaluation of many sources in the visual field. Let us hope that the publication of this will not be delayed.

In the present initial phase it is right that there should be no numerical scale of answers on the nomogram and the disk calculator. The difficulty, however, is that to calculate M, in order to test out the tentative rule for multiple sources, one must wade through the cumbersome exponential formula. So far, the disk and nomogram put the spotlight on the single source, and this is perhaps regrettable. I believe it will be apparent to the careful reader of this paper that it is not enough to keep each individual source in an installation below BCD brightness. Perhaps we may tentatively suggest that each source should be well below the average BCD value in order to play safe.

MATTHEW LUCKIESH and S. K. GUTH:** We wish to thank the discussors for their complimentary remarks regarding our paper. However, several points have been

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raised which require clarification.

Mr. Brown wonders whether Fig. 4 is sufficiently conclusive to be used as the basis for the nomogram and calculator. The displacement of the curves for the three field brightnesses was obtained from the relationship illustrated in Fig. 2. The points on the curves of Fig. 4 represented by solid circles were established by all of the representative subjects. The parallelism of the curves was checked for the smallest and largest sources.

The selection of the representative subjects for the extensive series of tests was limited to those who would be readily available over an extended period of time. Those selected included Nos. 1 to 5, 8, and 10 to 13. It is interesting to note that the arithmetic and geometric mean BCD brightnesses of the standard source for this group are 896 and 839 footlamberts, respectively. Those values are almost identical to the average BCD brightnesses obtained by the 50 subjects.

Mr. Harrison has tried to simplify the formula which relates the size Q of a source and the BCD brightness B. Obviously a number of empirical equations can be made to fit the experimental data, especially if only a limited range of sizes is considered at one time. In this initial paper we have developed an equation which would include all of the experimental data and emphasize that this equation holds only for the range of data obtained. We believe that it is desirable to have a single equation for all sizes of sources rather than two or three different equations for certain ranges of sizes. The latter could result only in confusion. The complexity of the empirical formula should not be discouraging. The curves of Fig. 4 illustrate the relative and variable effectiveness of size of source and surrounding field brightness. Furthermore, the nomogram and disk calculator obviate the necessity for using the equation and simplify the calculations.

The use of the initial momentary sensation, controlled as to duration, should give a very real evaluation of lightsources at wide angles to the line of vision. It must be remembered that the eyes are seldom fixated upon a point for an extended period. The complete act of seeing involves continual shifting of the line of vision. As a result, peripheral sources of brightness are being "seen" by different retinal areas as the eyes fixate various objects. Thus, various retinal areas are receiving momentary stimuli of brightness. The dynamic technique used in our investigation actually may be resulting in a more accurate and practical evaluation of these sources than a static technique. Certainly our technique has the virture of control of exposure, and controls are essential to any researches of this character. However, a preliminary analysis of data using prolonged rather than momentary exposure of sources indicate BCD brightnesses of the same order as those presented in this paper.

The effect of increasing the length of linear sources should be evident from the data on multiple circular sources horizontally displaced from the line of vision which is illustrated in Fig. 16. The necessary reduction in the brightnesses of the sources to maintain the BCD sensation become progressively less as more sources are added at greater angles from the line of vision. A linear

source is merely an extreme or limiting condition of the multiple sources where the sources are adjacent to each other. That portion of a source closest to the line of vision produces the greatest part of the sensation. The addition of equal incremental areas requires correspondingly less of a reduction in brightness to produce the BCD sensation.

In our paper we stated that space limitations made it necessary to postpone to a future paper a discussion of numerous additional results and analysis which make possible the correlation of the basic data with lighting practice. Included among these is a series of investigations of extended linear sources mentioned in the last paragraph of our paper. When we publish the results, they should materially fill the gap indicated by Mr. Harrison.

We have speculated upon the effect of sources of brightness located below the line of vision. It should be pointed out that the work of Holladay referred to by Mr. Logan involved the obscuring power of a dazzle-source and not the sensation of discomfort. In other words, Mr. Logan refers to that part of Holladay's investigations which pertained to what has been termed "disability glare" which does not necessarily correlate with "discomfort glare."

Fig. 9 of our paper should present a glimpse of the expected relative BCD brightness of sources below the line of vision. There is marked parallelism between the curve of equal BCD brightness and the curve representing the boundary of the upper half of the average visual field of subjects with normal vision. If this parallelism continues to hold in the lower portion of the visual field, the boundary curve in this region would indicate that a source below the line of vision would be more uncomfortable than a similar source located in the corresponding position above the line of vision.

Mr. Logan's brief discussion of stimulus versus response is interesting. However, it should be obvious that our results are presented in terms of the physical characteristics of a stimulus which produces a specific sensation or response. He may have preferred us to use "luminance" when referring to the photometric characteristic source or stimulus and "brightness" when referring to the sensation. The terminology used in the paper should be clear to everyone and we do not believe that there should be any confusion as to the meaning of any terms used. To this end, for example, we have avoided the use of the word "glare" which as commonly used is so generally not sufficiently clear or specific in meaning.

Further discussion of Dr. Luckiesh's and Mr. Guth's Conference paper, received too late for inclusion in this issue, will be published together with rebuttal in subsequent issues of ILLUMINATING ENGINEERING.

Discussion of other Conference papers, to be published with the papers, should be forwarded promptly to Hoyt P. Steele, Chairman of the Committee on Papers, c/o Benjamin Electric Mfg. Co., Des Plaines, Ill. A copy of the discussion should also be sent to the author.