Interim Study of Procedures for Investigating the Effect of Light on Impression and Behavior

John E. Flynn Terry J. Spencer Osyp Martyniuk Clyde Hendrick

THIS PAPER reports some preliminary findings concerning the effect of environmental lighting as a medium that affects user impressions and behavior. It begins with recognition of the fact that visual consciousness does not seem to be completely explainable with the simple notion of an optical image imposed on the retina of the eye and "photographically" interpreted by the brain. Instead, we find indications that there is considerable selectivity in the process of visual experience—a search for meaningful information. This suggests that light can be discussed as a vehicle that facilitates the selective process and alters the information content of the visual field. It further suggests that lighting design should be evaluated, in part, for its role in adequately establishing cues that facilitate or alter the user's understanding of his environment and the activities around him.

This initial study involved a search for evidence that variations in environmental lighting do (or do not) affect human behavior in some noticeable way. In this, we were looking specifically for evidence that some patterns of light might serve as environmental cues or signals, and that the occupants might tend to respond or act upon these cues in some consistent way. If so, we should find some consistent and shared patterns of impression among the occupants of a room—and we should find some consistent changes in impression as we vary the lighting conditions.

Parameters of the Study

As we began this study, we were aware of a number of recently developed scientific techniques for evaluating the subjective quality of a space—most notably (1) Semantic Differential rating scales for Factor Analysis;^{1,2} (2)

A paper presented at the Annual IES Conference, Philadelphia, Pa., July 9-12, 1973. AUTHORS: Kent State University, Kent, Ohio. Respectively, Professor of Architecture, Assistant Professor of Psychology, Associate Professor of Architecture, and Associate Professor of Psychology. Multidimensional Scaling;^{3,4,5} and (3) various observation and mapping methods.^{7,8} However, we found little work that explained how these methods might be used to anticipate the quality of various lighting decisions that are made during the design stage. So this became a major focus in our study to date—an effort to test the usefulness of these new study methods as techniques for lighting research.

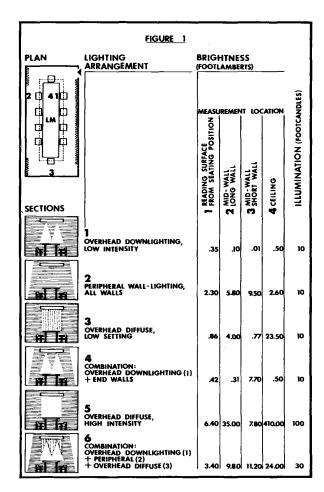
For these initial tests, a lighting demonstration room at the General Electric Lighting Institute at Nela Park in Cleveland was used as a laboratory. This room has several important advantages: (1) it has a number of lighting arrangements that permit significant variation in the visual character of the space without changing any of the other physical conditions; and (2) the Lighting Institute provides access to subjects with widely diverging backgrounds, thus allowing for study of individual and group differences. The room was made available at our request, and all tests were designed, supervised, and evaluated by the Kent State University research team.

The procedures used to assess judgments of lighting arrangements in the room are described below.

Rating Scales: Procedures

Judgments on "Osgood-type"^{1,2} Semantic Differential (SD) rating scales were obtained for each of six different lighting arrangements in a medium-sized conference room (see Fig. 1 for a description of the room and lighting arrangements). Ratings were analysed from 12 groups with a total of 96 subjects who were distributed in groups of eight. These adult subjects were well distributed in age and educational background.

For each group, *initial* ratings of the room were obtained for the lighting arrangement that was in effect when the subjects first entered the room. Each of the six lighting arrangements was in effect for two of the 12 groups as they first entered. Subjects were not initially informed that variations in lighting arrangement were the primary



focus of this experiment and presumably judged the room with respect to all of its characteristics. Thus any significant differences in initial ratings between the groups should represent the differential effect of lighting variations on the overall impression of the physical space. (This may not have been strictly true here, since the location of the room at G.E.'s Lighting Institute would probably cue the subjects to the likelihood that lighting manipulations were of interest. Nevertheless, most subjects would still be judging the room with reference to an absolute standard, since they would not be exposed to other lighting arrangements in the room prior to the initial ratings.)

Following the initial ratings of the room, comparative ratings were obtained by informing each group that we wanted judgments of the room under a variety of lighting arrangements. Prior to these comparative judgments, the experimenter showed the six arrangements in a fairly rapid manner to provide the subjects with a general frame of reference. Each of the six lighting arrangements was then presented one at a time (but in a different order for each group); and after a short period of time for adaptation, the subjects rated each arrangement on the rating scales. The initial lighting arrangement was repeated as one of the six comparative arrangements.

Rating Scales: Results

An analysis of data from the *initial* ratings of the room indicated that 17 (of 34) rating scales showed a significant

FIGURE 2								
FACTOR 1 - EVALUATIVE								
FRIENDLY		6 4	121	3	5		HOSTILE	.89
PLEASANT	6	4 2	1		35		UNPLEASANT	.89
LIKE		6	4 2	1	5 3		DISLIKE	.86
HARMONY		6 42	1	35			DISCORD	.83
SATISFYING		6	24 1	5	3		FRUSTRATING	.83
BEAUTIFUL		6	42	1 5	3		UGLY	.83
SOCIABLE		64	2 1		53		UNSOCIABLE	.83
RELAXED		4 :	261	3	5		TENSE	.83
INTERESTING		6 4	2	1	53		MONOTONOUS	.81
FACTOR 2 - PERCEPTUAL CLARITY								
FACTOR 2 - PERCEPTUAL CLARITY CLEAR 5 6 24 1 3 HAZY .86								
CLEAR			Γ.		Γ]	
BRIGHT	1	6		4	2 1	3	DIM	.84
FACES CLEAR	5 6	-	2			-	FACES OBSCURE	
DISTINCT		6	42		3		VAGUE	.77
FOCUSED	-	56		2	-		UNFOCUSED	.71
RADIANT	L	6	5 4	2 1	L	[3	DULL	61
FACTOR 3 - SPATIAL COMPLEXITY								
SIMPLE			426				COMPLEX	.65
UNCLUTTERED	L L	16	3	5	L		CLUTTERED	.56
The same of the sa								
FACTOR 4 - SPACIOUSNESS								
LARGE		625	1	3	- '		SMALL	.84
LONG			5 3	1	\vdash	 -	SHORT	.77
SPACIOUS		6 2	54	3 1	<u> </u>	11_	CRAMPED	.76
FACTOR 5 - FORMALITY								
ROUNDED		1	. 1	63425	_	1 .	ANGULAR	.60
INFORMAL	Ш	<u> </u>	42	631	5	<u> </u>	FORMAL	.58

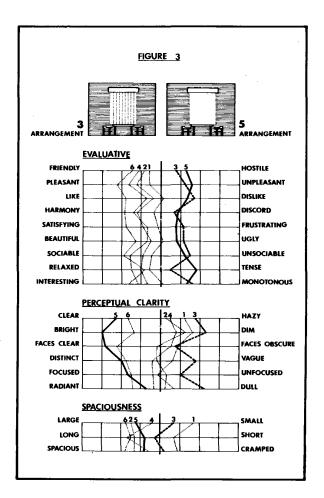
difference in response to the room under the six lighting arrangements. Apparently the groups were evaluating the room differently, according to the lighting system in effect as they entered the room.

An analysis of data from the *comparative* ratings of all six arrangements were in good general agreement with the initial ratings. Those rating scales that were significant in differentiating judgments of the initial room conditions were also significant for the subsequent or comparative ratings. In fact, all of the 34 rating scales showed a significant difference in impression between two or more of the six lighting arrangements; although on a few of the scales, the separation between mean ratings was small.

Most importantly, the pattern or order of lighting arrangements on each rating scale was often virtually identical when comparing the initial and comparative mean ratings. It appears that providing a frame of reference (as we did in the comparative ratings) served to enhance the effectiveness of rating scales but did not alter the basic nature of the judgments.

As one part of our analysis, the ratings were factor analysed to find areas of redundancy and repetition in the use of our scales. This factor analysis resulted in identification of five factors or "categories of impression"—and three of these showed a significant difference in impression between two or more of the six lighting arrangements

Factor 1 was a general evaluative impression—as shown by the character of the rating scales that load on



this factor. In Fig. 2, the mean rating of each of the six lighting arrangements is represented by the location of the corresponding arrangement number on each rating scale. As can be seen, lighting arrangements 6 and 4 were generally most preferred, and arrangements 3 and 5 were least preferred.

Factor 2 appeared to represent an impression that is tentatively named PERCEPTUAL CLARITY; although it might also have been named "spatial brightness," since it seemed to relate to this physical variation (see measured brightnesses, Fig. 1). In this category, lighting arrangements 5 and 6 were grouped as the clearest, brightest, etc. (which in fact they were) while arrangements 1, 2, 3, and 4 represented lesser photic energy.

Factor 3 seemed to be an impression of SPATIAL COMPLEXITY. The terms "visual noise" or "visual clutter" might also have been appropriate. The two rating scales that loaded on this factor did not strongly differentiate between the six lighting arrangements. Fig. 2 shows that the arrangements tend to cluster together, and changes in lighting arrangement did not significantly affect impression in this case. However, this factor may serve to differentiate between rooms, and current research is investigating this possibility.

Factor 4 was an impression of spaciousness. Lighting arrangements 1 and 3 were judged significantly less spacious than arrangements 6 and 2. Since the physical dimensions of the room and the furnishings did not vary,

this result was solely an induced psychological effect brought on by the variations in lighting arrangement. Thus, lighting all four walls (as in arrangements 6 and 2) seemed to induce a feeling of greater spaciousness, when compared with low intensity overhead arrangements 1 and 3 in which wall brightnesses were subdued.

Factor 5 consisted of two rating scales: rounded-angular and informal-formal. It was not obvious what property these two scales have in common, but the term formality seemed tentatively appropriate. This may also represent an impression of "style or fashion," and this possibility is being investigated in more current research. At any rate, this factor did not strongly differentiate between the six lighting arrangements, although it may differentiate between different rooms.

Rating Scales: Discussion

These tests suggest that lighting variables do induce some consistent and apparently shared impressions for the users. Furthermore, the tests tend to confirm that rating scales may be useful techniques for providing insight about this phenomena. As an example of this latter possibility, some tentative but representative conclusions can be made concerning the subjective implications of some lighting design decisions.

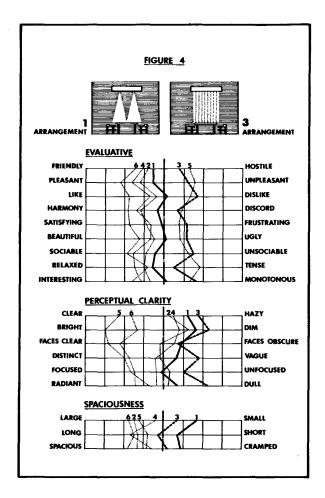
(1) What happens to subjective spatial impression when there is a significant change in the intensity of horizontal illumination from an overhead lighting system (with no change in the distribution characteristics of the system)?

Arrangements 3 and 5 were overhead diffusing systems, but at 10 and 100 fc respectively. Fig. 3 shows that changing only light intensity had a negligible effect on general "evaluative impressions." Both systems were rated (almost together) as somewhat more hostile, monotonous, etc., than the other four lighting systems. On the other hand, impressions of arrangements 3 and 5 varied widely for "perceptual clarity." Not unexpectedly, the higher intensity system (arrangement 5) was rated as most clear, bright, and distinct among the six conditions; while arrangement 3 was at the opposite extreme in these ratings.

It appears, then, that intensity of light emission from overhead diffusing systems may affect the impression of "perceptual clarity;" but that this has relatively little effect on general "evaluative impressions" like pleasantness or friendliness. On the other hand, higher brightness levels tended to produce an impression of increased "spaciousness."

(2) What happens to subjective spatial impression when there is a significant change in the distribution characteristics from an overhead lighting system (with no change in the intensity of horizontal illumination)?

Fig. 4 shows that arrangements 1 and 3 (each at 10 fc to minimize the effect of inter-reflected light) produced a roughly similar impression regarding "perceptual clarity;" although both were expectedly toward the negative end of the scale (hazy, dim, etc.). However, arrangement 1 (downlighting) consistently produced more positive "evaluative impressions" than arrangement 3. There were also marginally significant differences in the impression of "spaciousness."



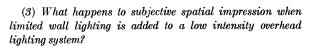
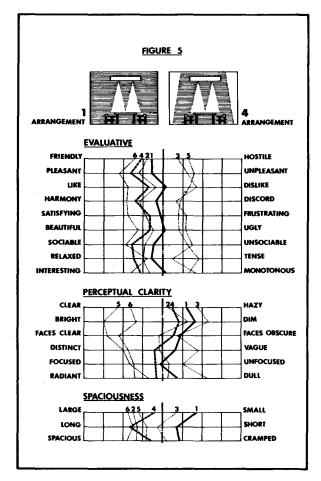


Fig. 5 suggests that this design action will enhance both "evaluative impressions" and the impressions of "spaciousness." In our tests, arrangement 4 (downlights plus wall lights) was consistently evaluated more positively than arrangement 1 (downlights only). The limited wall lighting also improved "perceptual clarity" somewhat, and significantly affected the impression of "spaciousness." In general, the wall lighting seemed to create a more favorable attitude about the space in all three of the affected categories of impression.

Recognize that these are very tentative conclusions. They are discussed here simply to demonstrate the potential value of comparative rating procedures as a research device for providing broader insight into questions of lighting quality.

Multidimensional Scaling: Procedures

It was felt that it would be useful to compare the semantic differential rating scale data with independent data obtained from other recently developed methods of evaluation. With this in mind, we initiated a second rating experiment, using a different set of subjects. In this experiment, we collected data which consisted of judgments of relative similarity or difference for 38 pairwise compari-



sons of the six lighting conditions in the same test (conference) room. These data were then subjected to a multi-dimensional scaling.

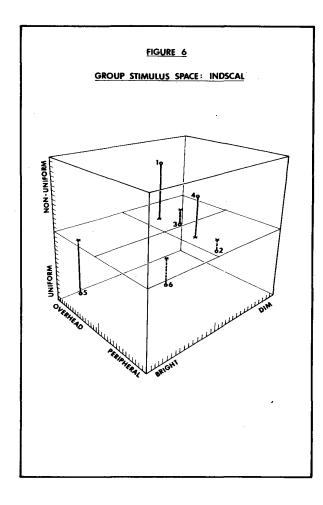
This procedure differs basically from the semantic differential rating scales used in the first phase of the study. The semantic differential methods specified areas of impression for evaluation (one at a time) by the participants—cheerfulness, spaciousness, brightness, formality, etc. In the multidimensional scaling procedure, we asked only for a judgment of overall similarity or difference, and the participants were left to establish their own criteria for making this judgment.

Specifically, in this phase of the study, subjects were asked to judge the degree of change in going from one lighting condition to the next. To facilitate these judgments, the experimenter asked the subjects to choose a number from 0 to 10, where 0 represented "no change" and 10 represented a "very large change." Thus a medium change would be assigned a number such as 4, 5, or 6.

The various judgments of perceived change were then processed by a computer program (INDSCAL⁵), which helps determine the criteria that each participant used for his judgments.

Multidimensional Scaling: Results

In evaluating the 46 subjects involved in this phase of our study, an INDSCAL computer analysis indicated that a three-dimensional perceptual solution best accounted



for the judgments of similarity and difference. When fewer dimensions were tested, the model produced a much poorer correlation with the obtained data; while models with additional dimensions contributed little additional explanatory power.

The group stimulus model for the three-dimensional solution is shown in Fig. 6. Dimension 1 was tentatively labeled a "peripheral/overhead" mode of lighting (arrangements 2,4,6, vs 1,3,5). Dimension 2 was tentatively labeled a "uniform/non-uniform" mode (arrangements 2,3,5,6 vs 1,4). (However, dimension 2 may also represent a "warm-cool" variation in light tone, because this physical manipulation was confounded with the "uniform/non-uniform" variation.) Dimension 3 was tentatively labeled a "bright-dim" mode (5,6 vs 1,2,3,4).

In addition to the group response pattern, the INDSCAL program also provides a measure of the relative weight that each individual subject placed on each dimension. Although three dimensions were used by the group as a whole, it was not necessarily true that each individual subject used all three dimensions. An inspection of the dimension weights for individuals indicates that subjects could be classified into three general categories:

(1) One-Dimensional Perceivers

There were only three subjects in this category—two who judged similarities and differences along the "bright/dim" dimension; and one who judged along the "peri-

pheral/overhead" dimension.

(2) Two-Dimensional Perceivers

There were a total of 20 subjects in this category—arranging themselves into three sub-categories:

- a. Sub-category A represents 13 who judged along "bright/dim" and "uniform/non-uniform" dimensions.
- b. Sub-category B represents four subjects who judged along "peripheral/overhead" and "uniform/non-uniform" dimensions.
- c. Sub-category C represents three subjects who judged along "bright/dim" and "overhead/peripheral" dimensions.

(3) Three-Dimensional Perceivers

There were a total of 23 subjects in this category, all of whom utilized each of the three-dimensions of perceptual judgment to a significant extent. However, there were individual differences with respect to which dimension was given the greatest weight. Eight subjects gave greater weight to the "bright/dim" dimension; five subjects to the "uniform/non-uniform" dimension; and three subjects to the "peripheral/overhead" dimension. Seven subjects weighted two or more of the dimensions about equally.

In summary, it appears that there were three primary dimensions of perceptual experience to which the subjects could or did respond: (1) bright/dim, (2) uniform/non-uniform, and (3) peripheral/overhead. However, some individual differences in the response modes of the participants led the subjects to respond to either one, two, or three of these dimensions.

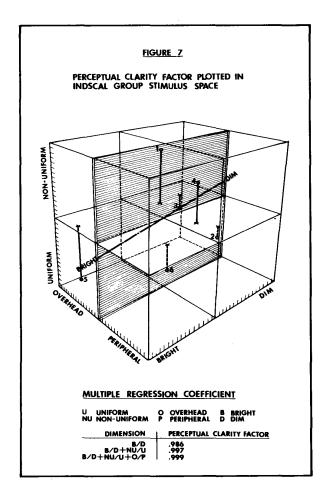
These differences in response modes may reflect real differences in visual perception of the lighted space. Or they may be explained by a suggestion made by Arnold, that observers have a limited capacity for information processing, and that this capacity may vary between individuals. He suggests that when subjects are asked to make comparisons of complex stimuli (such as lighting arrangements) in a multidimensional scaling situation where possible dimensions of experience are unspecified, some subjects may abstract out or isolate one or a few dimensions and submerge the others. If this is true, tests such as our MDS-plots might give some significant insight into the internal priorities established by various groups of individuals in regard to lighting quality.

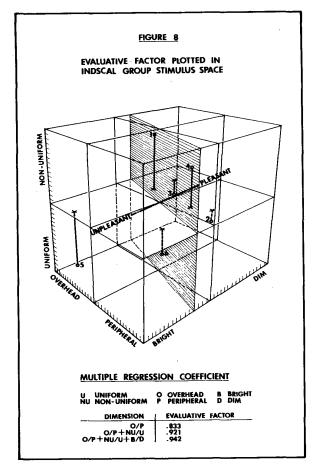
Comparison of Factor Analysis and Multidimensional Scaling Results

It is interesting to note at this point that three strongly discriminating factors (or categories of impression) were obtained from the semantic differential rating scale procedure, and three dimensions were obtained from the multidimensional scaling procedure. To what extent, if any, do they represent compatible characteristics of visual experience?

(1) Perceptual Clarity

The mean ratings on the bright/dim rating scale (chosen as representative of the perceptual clarity factor) were subjected to a stepwise multiple regression with the three dimensions of the INDSCAL group stimulus space. This procedure determines which dimension and combination





of dimensions have the greatest predictive power.

A correlation of 0.99 between the perceptual clarity SD ratings and the "bright/dim" INDSCAL dimension was achieved in the first step (see Fig. 7). Thus both procedures seem to be yielding essentially equivalent results concerning this aspect of visual experience.

(2) Evaluative Impressions

The mean ratings on the pleasant/unpleasant rating scale (chosen as representative of the evaluative factor) were also subjected to a stepwise multiple regression with the indexal space. A correlation of 0.83 with the "overhead/peripheral" dimension was obtained in the first step; and a correlation of 0.92 was achieved in the second step, using both the "overhead/peripheral" and "uniform/non-uniform" dimensions as predictors (see Fig. 8). The addition of the "bright/dim" dimension contributed little additional explanatory power—only increasing the correlation to 0.94.

Thus, evaluative impressions appear to be adequately represented as a linear combination of the "overhead/peripheral" and "uniform/non-uniform" dimensions. A pleasant lighting arrangement (such as arrangement 6) would tend to be one that contains non-uniform and peripheral lighting modes, as represented by the projection of arrangement 6 on the evaluative impression line. A less pleasant system is one that contains both overhead and uniform modes of lighting. Notice that the "bright/dim"

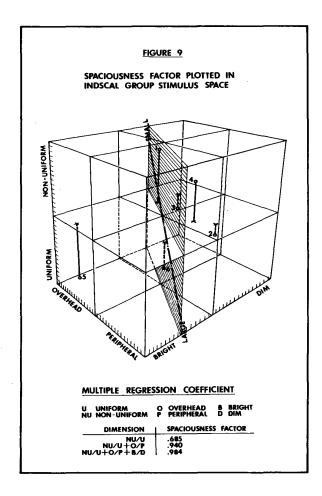
dimension plays little role in this respect, as suggested previously in the semantic differential rating scale discussion.

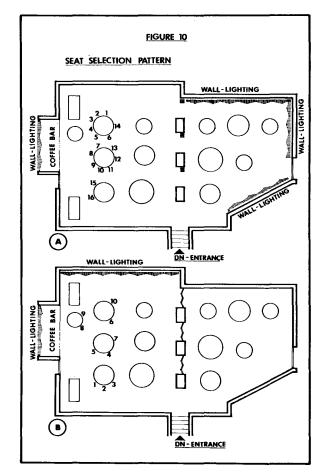
(3) Spaciousness

The mean ratings on the large-small rating scale (chosen as representative of the spaciousness factor) were also subjected to a stepwise multiple regression with the INDSCAL space. A correlation of 0.69 with the "uniform/non-uniform" dimension was obtained in the first step; and a correlation of 0.94 was obtained in the second step, using both the "uniform/non-uniform" and "overhead/peripheral" dimensions as predictors (see Fig. 9). The addition of the "bright-dim" dimension as a predictor increased the multiple regression coefficient to 0.98.

Thus, the impressions of spaciousness identified by the semantic differential ratings in Fig. 2 seem to be represented quite well as a linear combination of the "uniform/ non-uniform" and "overhead/peripheral" dimensions, but an even better fit is obtained using all three dimensions.

Of course these interpretations are again very tentative and must await additional data collection with a larger variety of lighting arrangements for confirmation. Our purpose in noting such conclusions at this point is to suggest that semantic differential rating scales and multidimensional scaling may, in combination, become useful research devices for providing greater insight into the





types of impressions that can be fostered or reinforced by various lighting systems. And perhaps most important, these methods seem to offer promise for providing specific insight into the design decisions that actually trigger or signal such impressions.

Informal Observation of Overt Behavior

To this point, our discussion has been limited to the use of rating techniques for evaluating lighting systems. In this sense, we are suggesting that various rating scale techniques may be useful for generating a better understanding of the way a user *thinks* about a given lighting system. But we also have some evidence that lighting variations influence overt behavior.

To explore this question, some individual patterns of overt behavior were recorded informally during the rating scale studies (usually during the initial room ratings, when participants were first entering the room). Although incomplete, these observations tend to support our previous conclusions. Several examples will be noted.

1. Diffuse overhead lighting arrangements 3 and 5 elicited generally negative evaluative judgments on the rating scales. To reinforce this conclusion, arrangement 3 induced several "thumbs down" gestures from subjects seated at the conference table awaiting the beginning of the experiment. Such comments as "very unflattering" and "reminds me of an elevator" were heard. Subjects first exposed to arrangement 5 exhibited such behavior

as squinting, eye rubbing, and sudden removal and cleaning of glasses. Comments were "get out your typewriter" and "we have ways of making you talk, Ivan."

2. As another example, rating responses for the low intensity down-lighting showed some variability (even inconsistency in a few cases). Perhaps to help explain this, subject comments suggested that this lighting arrangement cued two very different types of associations. A night club association seemed to be suggested for some subjects (drinking-type gestures, comments about psychedelic lights, etc.). Other subjects had church-like or spiritual associations. Sample comments were "is this a seance"? "let us pray," "do we hold hands?" etc.

Mapping of Overt Behavior

Observations of subjects' actions during the rating study suggest that lighting variations also influence other forms of overt behavior—such as circulation patterns, seat selection patterns, posture, comments, gestures, facial expressions, etc.

To explore this in a preliminary way, data were collected on the effect of high contrast lighting arrangements on seating patterns and preferences among uninstructed subjects in a restaurant-type setting at G.E.'s Lighting Institute. This limited study involved various groups of uninstructed subjects who were offered coffee from a lighted coffee-bar. They were then free to select seats at nearby tables (these tables located in an unlighted area,

except for stray light from the adjacent areas); or they could select seats in a more remote part of the room (an area that was lighted in an interesting and pleasant way).

Interestingly, most people selected seats in the nearby darker part of the room. But our mapping of seat selection patterns showed that most of those first approaching an empty table (singly or in small groups) would tend to select a seat which oriented them toward the light. There was a distinct tendency for those first selecting their seats at each table to face the entrance stair, which was a center of activity located in the corner of the lighted part of the room. Fig. 10-A shows a typical pattern of seat selection under this lighting condition.

It was possible to change this seat selection pattern rather dramatically with major changes in the light distribution. In the experiment, the lighting was changed as follows: (1) the coffee-bar was illuminated the same as before; (2) all of the tables and most of the room was basically unlighted, in that they were illuminated only by stray light that was adequate for simple circulation; (3) a drapery was drawn to reduce the visual size of the room; and (4) wall-lighting was provided along a major interior wall that was opposite from the entrance. When these changes were made, there was a strong tendency for subjects approaching an empty table to select seats in a different manner—now facing the interior of the room (again, generally toward the light). With this second lighting condition, most users who had a choice in their seat selection seemed to turn their backs on the entrance area that had been an attraction before, tending to confirm the theory that some lighting conditions may induce instinctive or learned tendencies in overt behavior. Fig. 10-B shows a typical pattern of seat selection under the modified lighting arrangement.

Conclusion

This paper is an interim report. Its findings are tentative in the sense that they represent limited samples of lighting variables. Nevertheless, the findings definitely seem to encourage further work in this direction. They seem to support the theory that the experience of lighted space is, to some extent, a shared experience in that groups of room occupants tend to have somewhat similar impressions. Furthermore, the findings tend to sustain the

idea that lighting can be discussed as a vehicle that alters the information content of the visual field, and that this intervention has some effect on behavior and on sensations of well-being. And perhaps the item to be most stressed here, the findings suggest that recently evolving psychological procedures for rating and mapping of behavior may be useful methods for gaining improved insight and broader perspective on the functions of light in spaces for human beings. In this sense, our experiments suggest that realistic studies of lighting quality and lighting value may depend, in part, on the function of the system as a device for communicating ideas or reinforcing appropriate impressions for the user.

Acknowledgments

We wish to acknowledge the assistance and encouragement we have received from Mr. Robert Dorsey, Mr. Fred Dickey, Mr. James Jensen, Dr. Sylvester Guth, and their associates at Nela Park in Cleveland. The efforts of Mr. Gordon Guthrie and Mr. John Suter of the General Electric Co. staff were particularly helpful in the data collection phase.

Special thanks are due to Professor Roy Lilly, Psychology Department, Kent State University, for his expert assistance in computer programming for the multidimensional scaling.

References

- Sanoff, Henry, Techniques of Evaluation for Designers, Design Research Laboratory, North Carolina State University, Raleigh, N.C., 1968.
- 2. Osgood, C. E., Suci, G. J., and Tannenbaum, P. H., *The Measurement of Meaning*, University of Illinois Press, Urbana, Illinois, 1957.
- 3. Kruskal, J. B., "Multi-dimensional Scaling by Optimizing Goodness of Fit to a Nonmetric Hypothesis," *Psychometrika*, Vol. 29, 1964, p. 1.
- 4. Kruskal, J. B., "Nonmetric Multi-dimensional Scaling: A Numerical Method," *Psychometrika*, Vol. 29, 1964, p. 115. 5. Chang, J. J., Carroll, J. D., "How to Use indscal: A Computer Program for Canonical Decomposition of N-way Tables and Individual Differences in Multi-dimensional Scaling," Bell Telephone Laboratories, Murray Hill, N. J., 1972 (mimeographed).
- Arnold, T. B., "A Multi-dimensional Scaling of Semantic Distance," *Journal of Experimental Psychology*, Vol. 90, 1971, p. 349.
- 7. Proshansky, H. M., Ittelson, W. H., and Rivlin, L. G., "Freedom of Choice and Behavior in a Physical Setting," Environmental Psychology, Holt, Rinehart and Winston, Inc., New York, N. Y. 1970, pp. 173–183.
- 8. Ittelson, W. H., Rivlin, L. G., and Proshansky, H. M., "The Use of Behavioral Maps in Environmental Psychology," *Environmental Psychology* Holt, Rinehart and Winston, Inc., New York, 1970, pp. 658-668.