Computation of the Effective Intensity Of Flashing Lights

By CHARLES A. DOUGLAS

It is generally recognized that when a light signal consists of separate flashes, the instantaneous intensity during the flashes must be greater than the intensity of a steady light in order to obtain threshold visibility. Blondel and Rey[1] found that the threshold illuminance for an abrupt flash (a flash producing a relatively constant illuminance throughout its duration) is

$$E = E_o(a+t)/t, \qquad (1)$$

where E_o is the threshold illuminance for a steady light, t is the flash duration, and a is a constant. They found that a was equal to 0.21 when t is in seconds.

It is convenient to evaluate flashing lights in terms of their effective intensity, I_e, that is, the intensity of a fixed light which will appear equally bright. Then

$$I_e = IE_o/E$$

where I is the instantaneous intensity producing the illuminance, E,

$$I_e = \frac{It}{a+t}. \qquad (2)$$

Later Toulmin-Smith and Green[2] found that somewhat different effective intensities were obtained when the illuminance at the eye was above threshold. However, Hampton[3] showed that their experimental results could be adequately expressed by equation (2) when a is a function of the illuminance at the eye.

The flash from most lights used in aviation service, such as airway beacons and anti-collision lights, is not abrupt. The instantaneous intensity often rises and falls gradually and may vary appreciably during the flash. If the flash duration is very short or if the times of rise and fall of intensity are short in comparison to the flash duration, only small uncertainties would be introduced in the determination of flash duration and by the use of the product of the peak intensity during the flash and the flash duration for the quantity It. However, in many cases significant errors would be introduced. Some modification of equation (2) is therefore required.

Some of the specifications for flashing lights have evaluated their signals in terms of the candle-seconds in the flash, integrating over a period of not more than 0.5 second, that is

$$\text{Candle-seconds} = \int_{t_1}^{t_2} I\,dt$$

where I is the instantaneous intensity and $t_2 - t_1$ does not exceed 0.5 second. This method of evaluation provides a measure of comparison between lights of roughly the same intensity variation with time but is not suited to the comparison of lights of different flash characteristics nor to the computation of visual ranges.

Others have used the relation

$$I_e = \frac{I_{max}\,t}{a+t},$$

where I_{max} is the maximum instantaneous intensity during the flash and t is the flash duration. Often the value of a is adjusted for the characteristics of the flash so that the computed value of I_e is in reasonable agreement with the observed value.

When the specification for aircraft anti-collision lights was being drafted, it was suggested that a modified form of equation (2) be used for the computation of effective intensity, so that

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{.2+t_2-t_1}. \qquad (3)$$

An equation of this form was originally suggested by Blondel and Rey,[4] but has rarely been used.

The question of choice of limits was immediately raised. Rather than use an arbitrary set of limits, such as choosing for t_1 and t_2 the times when I was 10 per cent of the peak of the flash, a choice of limits which would make I_e a maximum was suggested. This immediately poses the problem of developing a method, other than trial and error, of obtaining the maximum value of I_e. The development of such a method is the purpose of this paper.

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Fundamental Theorems

The method of obtaining the maximum value of I_e will be developed by means of one theorem and two corollaries. The proofs follow.

Theorem. *I_e is a maximum when the limits t_1 and t_2 are the times when the instantaneous intensity is equal to I_e.*

This theorem may be readily proved by application of the calculus of variations.* The proof given below is included because of the application of the method and equations used to later sections of the paper.

Consider instantaneous intensity, I, in a flash as any continuous, non-negative, single-valued function of time such that I is less than I_e in the intervals t''_1 to t_1 and t_2 to t''_2, and I is greater than I_e in the intervals t_1 to t'_1 and t'_2 to t_2, where $t''_1<t_1<t'_1<t'_2<t_2<t''_2$, and I_e is defined in equation (4):

$$I_e=\frac{\int_{t_1}^{t_2} I\,dt}{a+t_2-t_1}. \tag{4}$$

Fig. 1 shows I as a function of t for a simple one-peak flash meeting these requirements.

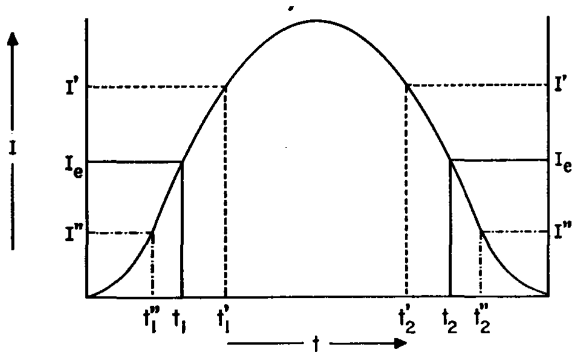

Figure 1. Intensity-time distribution of simple flash. (See text.)

Case I.

Consider the case where the integration is performed over the time interval t'_1 to t'_2 which lies within the interval t_1 to t_2.

Then the intensity I' at the times t'_1 and t'_2 is greater than I_e.

$$I'>I_e.$$

Let

$$I'_e=\frac{\int_{t'_1}^{t'_2} I\,dt}{a+t'_2-t'_1}.$$

Then

$$\int_{t_1}^{t_2} I\,dt=\int_{t_1}^{t'_1} I\,dt+\int_{t'_1}^{t'_2} I\,dt+\int_{t'_2}^{t_2} I\,dt,$$

so that

$$I_e(a+t_2-t_1)=\int_{t_1}^{t'_1} I\,dt+I'_e(a+t'_2-t'_1)+\int_{t'_2}^{t_2} I\,dt.$$

But

$$\int_{t_1}^{t'_1} I\,dt>I_e(t'_1-t_1), \tag{5a}$$

and

$$\int_{t'_2}^{t_2} I\,dt>I_e(t_2-t'_2). \tag{5b}$$

Substituting and combining terms, we have

$$I_e(a+t'_2-t'_1)>I'_e(a+t'_2-t'_1).$$

Therefore

$$I_e>I'_e. \tag{6}$$

Case II.

Consider now the case where the integration is performed over the time interval t''_1 to t''_2, which includes the interval t_1 to t_2.

Then the intensity I'' at the times t''_1 and t''_2 is less than I_e.

$$I''<I_e.$$

Let

$$I''_e=\frac{\int_{t''_1}^{t''_2} I\,dt}{a+t''_2-t''_1}.$$

Then

$$\int_{t''_1}^{t''_2} I\,dt=\int_{t''_1}^{t_1} I\,dt+\int_{t_1}^{t_2} I\,dt+\int_{t_2}^{t''_2} I\,dt,$$

and

$$I''_e(a+t''_2-t''_1)=\int_{t''_1}^{t_1} I\,dt+I_e(a+t_2-t_1)+\int_{t_2}^{t''_2} I\,dt. \tag{7}$$

But

$$\int_{t''_1}^{t_1} I\,dt<I_e(t_1-t''_1), \tag{8a}$$

and

$$\int_{t_2}^{t''_2} I\,dt<I_e(t''_2-t_2). \tag{8b}$$

Substituting and combining terms, we have

$$I_e>I''_e. \tag{9}$$

Thus I_e is greater than both I'_e and I''_e. Therefore, the maximum value which can be obtained from the Blondel-Rey relation, equation (4), is that obtained when the intensity at the beginning and end of the interval of integration is equal to the effective intensity.

Corollary 1. *If the instantaneous intensity is integrated over a period of time t'_1 to t'_2 shorter than t_1 to t_2, and I' is the instantaneous intensity at these times, a value I'_e is obtained for the effective intensity that is always less than I'.*

From equation (6) we have

$$I_e>I'_e.$$

*The author is indebted to Dr. H. H. Seliger of the National Bureau of Standards for an elegant proof using the calculus of variations.

But

$$I' > I_e.$$

Therefore

$$I' > I'_e. \tag{10}$$

Corollary 2. *If the instantaneous intensity is integrated over a period of time t''_1 to t''_2 longer than t_1 to t_2, and I'' is the instantaneous intensity at the times t''_1 and t''_2, a value I''_e is obtained for the effective intensity that is always greater than I''.*

From equation (7) we have

$$I''_e(a+t''_2-t''_1)=\int_{t''_1}^{t_1} I\,dt+I_e(a+t_2-t_1)+\int_{t_2}^{t''_2} I\,dt.$$

But

$$\int_{t''_1}^{t_1} I\,dt > I''(t_1-t''_1),$$

and

$$\int_{t_2}^{t''_2} I\,dt > I''(t''_2-t_2).$$

Also

$$I_e(a+t_2-t_1) > I''_e(a+t_2-t_1).$$

Substituting these into equation (7) and simplifying, we have

$$I''_e > I''. \tag{11}$$

Computations of Effective Intensity

Guides for the computation of the effective intensity from an intensity-time distribution curve may be obtained from the theorem and corollaries.

a. Computation of I_e

1. Make an estimate I' of the value of the effective intensity and solve equation (3) using the values of t corresponding to this intensity, obtaining I_{e_1}.

2. Repeat step 1 above, using as limits the values of t corresponding to the I_{e_1} obtained in step 1, obtaining I_{e_2}. Repeat as often as necessary to obtain the desired accuracy.

Note that if the estimated effective intensity is too high (I' in Fig. 1), the effective intensity, I_{e_1}, computed in step 1 will be below I_e (I'' of Fig. 1) and thus I_e lies between I' and I_{e_1}. If the initial estimate is lower than I_e (I'' of Fig. 1), I_e will be greater than both I'' and I_{e_1} and a "straddle" is not obtained but I_e is approached continuously from the low side.

b. Determination of Conformance of a Flashing Light to Specification Requirements

1. Compute I_{e_1} using the time limits corresponding to the specified effective intensity I_s. If I_{e_1} is greater than I_s, the unit obviously complies, for the conditions are those of Fig. 2a (corollary 2).

2. If I_{e_1} is equal to I_s, the unit just complies, for then $I_e = I_s = I_{e_1}$ (theorem).

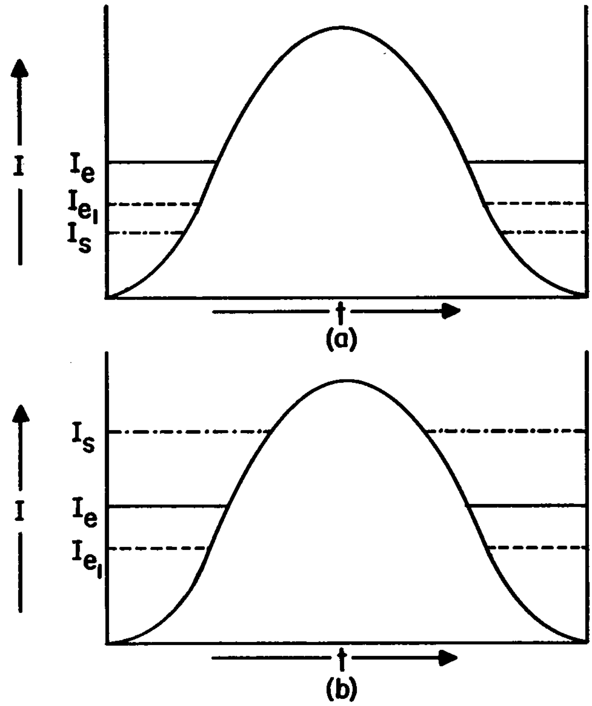

Figure 2. Illustrating the method of determining conformance to specification requirements of effective intensity. (See text.)

3. If I_{e_1} is less than I_s, the unit fails for then the conditions are those of Fig. 2b (corollary 1).

Note that the degree by which the unit exceeds or fails to meet the specification requirements is *not* given by the single computation described here. The method outlined in section *a* must be used for this purpose.

c. Visual Range Computations

If the visual range of the light, under specified conditions of transmittance and threshold, is desired, compute the effective intensity by using the method outlined in section *a* and compute the visual range by using Allard's Law.

If the problem is only the determination of whether the light can be seen at a given distance under specified conditions of transmittance and threshold, use Allard's Law to compute the fixed intensity required to make the source visible at this distance. Then, by using the method outlined in section *b*, determine if the effective intensity of the unit exceeds this intensity.

Application to Complex Intensity-Time Curves

Not all units have single-peak intensity-time distribution curves similar to the curve shown in Fig. 1. Consider an intensity-time distribution curve of the type whose rise is shown in Fig. 3 where I_b

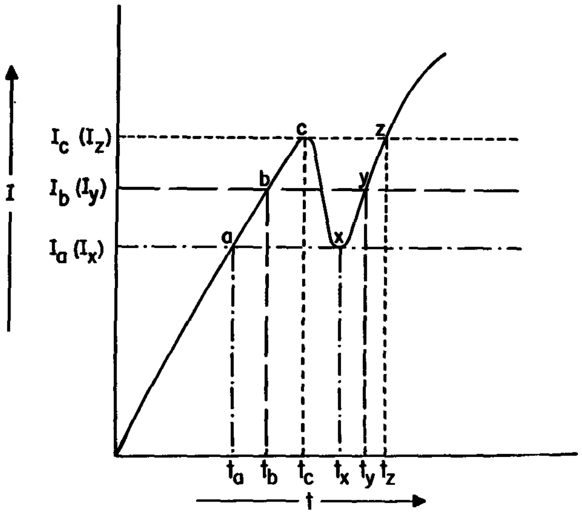

Figure 3. Illustrating application of the method to multi-peak intensity-time distributions. (See text.)

is the average intensity in the time interval t_b to t_y. (The time interval $t_z - t_c$ is sufficiently short so that the momentary decrease in intensity is not visible.) If I_e is less than I_a or is greater than I_z, then the restrictions on the shape of the curve stated in theorem 1 are met and there is no problem in the determination of I_e.

Consider the case where I_e lies between I_a and I_z. It may be easily shown by means of equations (5) and (8) that as the shape of other parts of the intensity-time distribution curve changes, the lower limits of time to be used to obtain the maximum value of equation (4) will lie between t_a and t_b or between t_y and t_z and will never lie between t_b and t_y. If I_e is equal to I_b, then either t_b or t_y can be used as the lower limit.

Application to Groups of Short Flashes

In general a signal from a flashing light consists of regularly spaced single flashes of light and the

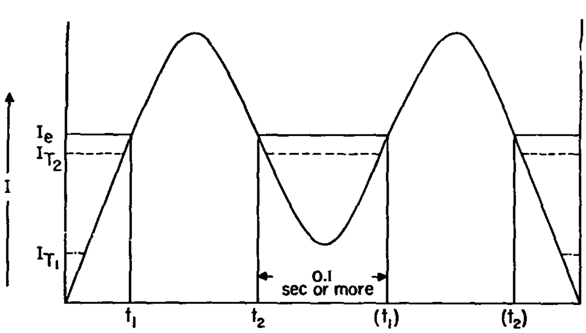

Figure 4. Illustrating application of the method to two-peak intensity-time curves. (See text.)

interval between flashes is so great that each flash has little influence on the effective intensity of the adjoining flashes.

Consider first a flash with the intensity-time distribution shown in Fig. 4. This flash is similar to that of the "split-beam" beacon used at military airfields. If the threshold intensity required to make a steady light visible is much less than I_e (I_{T_1}), the flash will be seen as a continuous flash with two peaks. However, if the threshold intensity is about equal to I_e (I_{T_2}), two separate flashes will be seen. The maximum distance at which the light can be seen will be determined by the effective intensity of a single flash computed over the time interval t_1 to t_2.

There are lights that produce a number of very short flashes in rapid succession so that this group of flashes is seen as a single flash. An example of a light of this type is a unit using a number of condenser-discharge lamps to produce a single flash.

There appear to be no published data reporting studies of the effects of groups of flashes where the interval between flashes is short. Behavior of the eye under somewhat similar conditions suggests that if in a group of flashes the periods during which the instantaneous intensity of the light is below the effective intensity of the flash are of the order of 0.01 second or less, the eye will perceive this group as a single flash. The effective intensity of the group should then be computed by equation (12), choosing as times t_1 and t_2 the first and the last times the instantaneous intensity is I_e.

$$I_e = \frac{\int_{t_1}^{t_a} I\,dt + \int_{t_b}^{t_c} I\,dt + \int_{t_d}^{t_e} I\,dt + \int_{t_f}^{t_2} I\,dt}{a + t_2 - t_1}. \qquad (12)$$

Note that I_e is the effective intensity of the group *and not that of a single flash.*

If the periods during which the effective intensity is less than I_e are of the order of 0.1 second or more, it is believed that the individual flashes will be seen. Therefore, the effective intensity should then be computed on the basis of a single flash.

When the dark period is between 0.01 and 0.1 second, the effective intensity will lie between that of a single flash and that of the group. The behavior during the transition is not known.

Numerical Examples

Although the precise determination of the maximum value of I_e may appear laborious, it is relatively easy. Since a change in the times chosen as the limits in equation (4) changes the denominator and the numerator in the same direction, it is not necessary to determine the correct limits, t_1 and t_2,

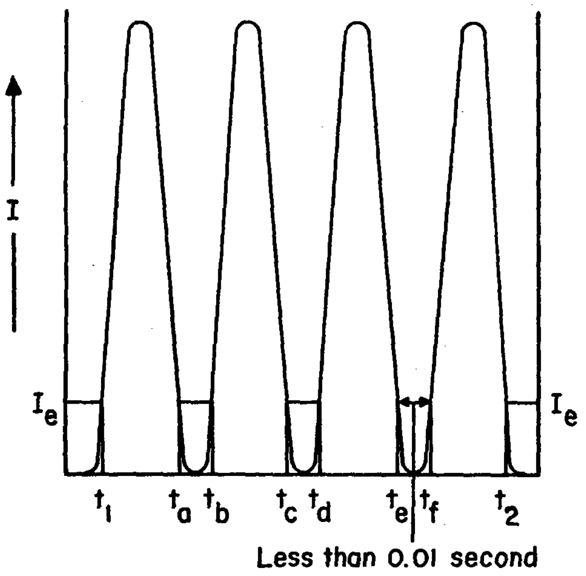

Figure 5. Illustrating application of the method to multiple flashes.

with great precision in order to obtain a satisfactorily precise determination of I_e. This is illustrated in Fig. 6 which shows two representative intensity-time distributions, one for a flashing light with a flash duration of about one-quarter second, and one with a flash duration of about one-twentieth second. The values of I_e are computed for seven sets of time limits. The values obtained are indicated at the abscissa of the time limits. The middle value of each group is the maximum I_e computed according to the method outlined above. Note that this value is equal to the instantaneous intensity at the corresponding time limits. Typically, the maximum effective intensity occurs lower on the curve for the short duration flash than for the long one and the variation of computed values of effective intensity with changes in time limits is smaller.

Experience indicates that if the times chosen for the initial integration are the times when the instantaneous intensity is about 20 per cent of the peak intensity, only one additional step is required to obtain a value for the effective intensity which is within one or two per cent of the maximum value. This is within the limits of accuracy with which the integral is evaluated by means of a planimeter. Often a single computation is sufficient if, instead of using as limits for the initial integration the times when I_e is 20 per cent of the peak intensity, the times used are the times when the instantaneous intensity is equal to the product of the peak intensity and the number of seconds between the times when the instantaneous intensity is roughly five per cent of the peak intensity.

The maximum value of I_e for the curves of Fig. 6 were computed by using as limits for the first integration the times corresponding to an intensity equal to about 20 per cent of the peak intensity and as limits for each succeeding integration the times corresponding to the instantaneous intensity obtained from the preceding step. (The curves have the shape of probability curves so the values of integrals may be computed with the desired accuracy. The accuracy is not limited by the accuracy of planimetric measurements.) Successive values for I_e of 1.66, 1.75, and 1.75 kilocandles were obtained for the longer flash and of 3.41, 3.42, and 3.42 kilocandles for the shorter flash.

Discussion

As noted above, concern has frequently been expressed about the choice of the limits for the integral of the Blondel-Rey relation for computing the visual range of a flashing light. It seems illogical to extend the limits of the integral beyond the

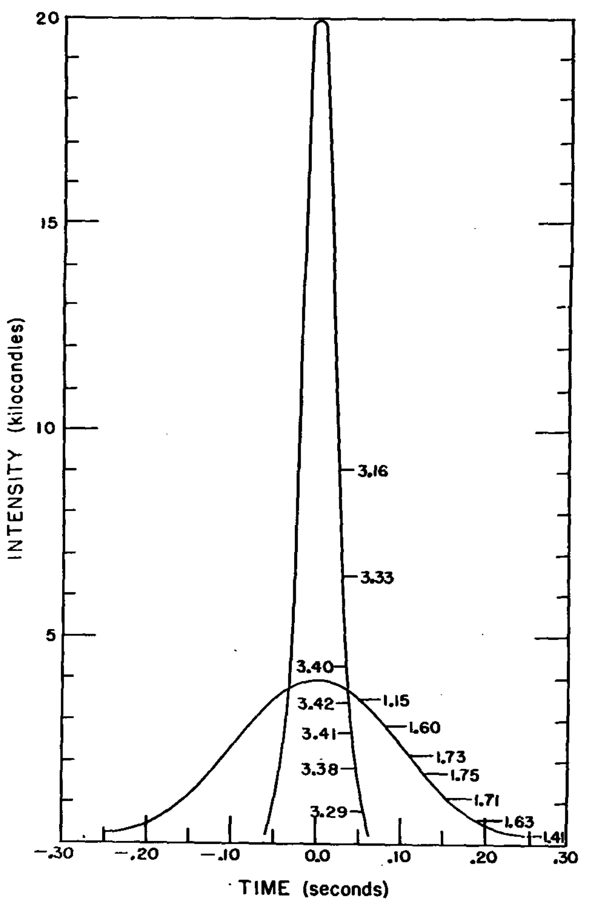

Figure 6. Examples of the effects of time limits on the computed value of effective intensity. The effective intensities are indicated at the abscissa of the time limits used in the computations.

times when the instantaneous intensity is below the threshold intensity for steady burning lights so that intensities which are below threshold, even for a steady burning light, are included, or to exclude intensities which are above threshold for steady burning lights. Using this reasoning, Blondel and Rey[4] suggested that the limits of the integral of equation (4) be the times when the instantaneous intensity is equal to the threshold intensity. As shown above, these are also the limits which make the computed visual range of the light a maximum. Therefore, the use of these limits in evaluating the performance of a lighting unit appears to be a logical choice.

The use of the maximum value of I_e as the effective intensity of a flashing light is probably not valid except when the light is at or near threshold. When the light is well above threshold, not only will the value of a in equation (4) be decreased, thus tending to increase the value of I_e, but also the limits of the integral should probably be extended to include the entire portion of the flash which is above threshold, thereby tending to decrease the value of I_e. In many cases this latter effect will be predominant. This is consistent with the decrease in effective intensity of airway beacons with increase in illuminance at the eye found by Neeland, Laufer, and Schaub.[5]

This analysis should be considered only as a mathematical treatment of equation (4). The analysis neither proves nor disproves the validity of this equation in determining the effective intensity of flashing lights nor the validity of the principle of choosing the limits of integration so that the effective intensity is a maximum.

References

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DISCUSSION

F. E. Carlson:* These interesting papers should prove very helpful in evaluating the many lighting problems involving flashing lights. It is assumed that the calculations described hold for viewing conditions approximating the ideal. I wonder if the authors could tell us something about the influence of such factors as atmospheric transmission, other sources of light in the field of view, etc.

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G. A. Horton:* Messrs. Projector and Douglas are to be highly commended for presenting two papers on a subject which has been long neglected. The papers are particularly timely because of recent application of condenser discharge lights as aviation runway approach lights.

In the condenser light, the length of the flash may be of the order of 100 microseconds, while the peak of the flash may reach several million candlepower. Since the time required for the eye to reach its maximum response is of the order of one-tenth of a second, it is clear that the effective candlepower as seen by the eye is not the same as the measurable average candlepower developed by the flash.

However, by application of the formulas developed by the authors of these two papers to the measured data can the effective candlepower of the condenser lights be expressed.

I should like to ask the authors if visual field checks of the condenser lights have been made to substantiate the theory developed in the papers and, if so, what degree of agreement was obtained.

T. H. Projector and C. A. Douglas:** It is a pleasure to acknowledge the comments of Messrs. Carlson and Horton and to reply to their questions.

Mr. Carlson raises the question of the relation between effective intensity and the conditions of observation. The term "effective intensity" is used for convenience, and implies narrowly restricted conditions, usually threshold illuminance, dark adaptation, etc. Strictly, the term should be "effective illuminance" for greater generality, thus automatically taking into account the effect of atmospheric attenuation on illuminance. The effect of other sources of light in the field of view and, in general, of the background condition, state of adaptation, etc., has not been sufficiently explored and is a fertile field for further investigation.

In reply to Mr. Horton, there have been a number of visual field checks of the applicability of the Blondel-Rey law to condenser discharge lights data in the United States and in Europe. All of the results of these checks show no significant deviations from the Blondel-Rey law. To our knowledge none of these results has as yet been published formally. Publication of the results of work at the National Bureau of Standards is expected in the future. It should be noted that these remarks apply only to the direct light from the source when the illuminance from the light is near threshold. Frequently, in a foggy or hazy atmosphere, the visual range of the glow of a flashing light will be considerably greater than the visual range of the direct light.

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Committee Personnel

A listing of I.E.S. Committees — Standing, General, Task and Technical — with their personnel begins on page 18A.