Computation of the Effective Intensity Of Flashing Lights

By CHARLES A. DOUGLAS

It is generally recognized that when a light signal consists of separate flashes, the instantaneous intensity during the flashes must be greater than the intensity of a steady light in order to obtain threshold visibility. Blondel and Rey found that the threshold illuminance for an abrupt flash (a flash producing a relatively constant illuminance throughout its duration) is

$$E = E_0(a + t)/t,$$

(1)

where $E_0$ is the threshold illuminance for a steady light, $t$ is the flash duration, and $a$ is a constant. They found that $a$ was equal to 0.21 when $t$ is in seconds.

It is convenient to evaluate flashing lights in terms of their effective intensity, $I_e$, that is, the intensity of a fixed light which will appear equally bright. Then

$$I_e = I E_0/E,$$

where $I$ is the instantaneous intensity producing the illuminance, $E$,

$$I_e = \frac{It}{a + t}. \quad (2)$$

Later Toulmin-Smith and Green found that somewhat different effective intensities were obtained when the illuminance at the eye was above threshold. However, Hampton showed that their experimental results could be adequately expressed by equation (2) when $a$ is a function of the illuminance at the eye.

The flash from most lights used in aviation service, such as airway beacons and anti-collision lights, is not abrupt. The instantaneous intensity often rises and falls gradually and may vary appreciably during the flash. If the flash duration is very short or if the times of rise and fall of intensity are short in comparison to the flash duration, only small uncertainties will be introduced in the determination of flash duration and by the use of the product of the peak intensity during the flash and the flash duration for the quantity $I_e$. However, in many cases significant errors would be introduced. Some modification of equation (2) is therefore required.

Some of the specifications for flashing lights have evaluated their signals in terms of the candleseconds in the flash, integrating over a period of not more than 0.5 second, that is

$$\text{Candle-seconds} = \int_{t_1}^{t_2} Idt$$

where $I$ is the instantaneous intensity and $t_2 - t_1$ does not exceed 0.5 second. This method of evaluation provides a measure of comparison between lights of roughly the same intensity variation with time but is not suited to the comparison of lights of different flash characteristics nor to the computation of visual ranges.

Others have used the relation

$$I_e = \frac{I_{max}t}{a + t},$$

where $I_{max}$ is the maximum instantaneous intensity during the flash and $t$ is the flash duration. Often the value of $a$ is adjusted for the characteristics of the flash so that the computed value of $I_e$ is in reasonable agreement with the observed value.

When the specification for aircraft anti-collision lights was being drafted, it was suggested that a modified form of equation (2) be used for the computation of effective intensity, so that

$$I_e = \frac{\int_{t_1}^{t_2} Idt}{t_2 - t_1}, \quad (3)$$

An equation of this form was originally suggested by Blondel and Rey, but has rarely been used.

The question of choice of limits was immediately raised. Rather than use an arbitrary set of limits, such as choosing for $t_1$ and $t_2$ the times when $I$ was 10 per cent of the peak of the flash, a choice of limits which would make $I_e$ a maximum was suggested. This immediately poses the problem of developing a method, other than trial and error, of obtaining the maximum value of $I_e$. The development of such a method is the purpose of this paper.
Fundamental Theorems

The method of obtaining the maximum value of \( I_e \) will be developed by means of one theorem and two corollaries. The proofs follow.

Theorem. \( I_e \) is a maximum when the limits \( t_1 \) and \( t_2 \) are the times when the instantaneous intensity is equal to \( I_e \).

This theorem may be readily proved by application of the calculus of variations.* The proof given below is included because of the application of the method and equations used to later sections of the paper.

Consider instantaneous intensity, \( I \), in a flash as any continuous, non-negative, single-valued function of time such that \( I \) is less than \( I_e \) in the intervals \( t''_1 \) to \( t_1 \) and \( t_2 \) to \( t''_2 \), and \( I \) is greater than \( I_e \) in the intervals \( t_1 \) to \( t'_1 \) and \( t'_2 \) to \( t_2 \), where \( t''_1 < t_1 < t'_1 < t'_2 < t_2 < t''_2 \), and \( I_e \) is defined in equation (4):

\[
I_e = \frac{\int_{t_1}^{t_2} I dt}{a + t_2 - t_1}.
\] (4)

Fig. 1 shows \( I \) as a function of \( t \) for a simple one-peak flash meeting these requirements.

Case I.

Consider the case where the integration is performed over the time interval \( t'_1 \) to \( t'_2 \) which lies within the interval \( t_1 \) to \( t_2 \).

Then the intensity \( I' \) at the times \( t'_1 \) and \( t'_2 \) is greater than \( I_e \).

\[
I' > I_e.
\]

Let

\[
I'_e = \frac{\int_{t_1}^{t_2} I dt}{a + t_2 - t_1}.
\]

Then

\[
\int_{t_1}^{t_2} I dt = \int_{t_1}^{t'_1} I dt + \int_{t'_1}^{t'_2} I dt + \int_{t'_2}^{t_2} I dt,
\]

so that

\[
I_e(a + t_2 - t_1) = \int_{t_1}^{t'_1} I dt + I'_e(a + t'_2 - t'_1) + \int_{t'_2}^{t_2} I dt.
\]

But

\[
\int_{t_1}^{t'_1} I dt > I_e(t'_1 - t_1),
\] (5a)

and

\[
\int_{t'_2}^{t_2} I dt > I_e(t_2 - t'_2),
\] (5b)

Substituting and combining terms, we have

\[
I_e(a + t'_2 - t'_1) > I_e(a + t'_2 - t'_1).
\]

Therefore

\[
I_e > I'_e.
\] (6)

*The author is indebted to Dr. H. H. Seliger of the National Bureau of Standards for an elegant proof using the calculus of variations.

Figure 1. Intensity-time distribution of simple flash. (See text.)

Case II.

Consider now the case where the integration is performed over the time interval \( t''_1 \) to \( t''_2 \), which includes the interval \( t_1 \) to \( t_2 \).

Then the intensity \( I' \) at the times \( t''_1 \) and \( t''_2 \) is less than \( I_e \).

\[
I' < I_e.
\]

Let

\[
I'_e = \frac{\int_{t_1}^{t_2} I dt}{a + t_2 - t_1}.
\]

Then

\[
\int_{t_1}^{t''_1} I dt = \int_{t_1}^{t_1} I dt + \int_{t_1}^{t''_2} I dt + \int_{t''_2}^{t_2} I dt,
\]

and

\[
I'_e(a + t''_2 - t''_1) = \int_{t_1}^{t'_1} I dt + I_e(a + t_2 - t_1) + \int_{t'_2}^{t_2} I dt.
\] (7)

But

\[
\int_{t_1}^{t_1} I dt < I_e(t_1 - t''_1),
\] (8a)

and

\[
\int_{t_2}^{t_2} I dt < I_e(t_2 - t''_2).
\] (8b)

Substituting and combining terms, we have

\[
I_e > I'_e.
\] (9)

Thus \( I_e \) is greater than both \( I'_e \) and \( I''_e \). Therefore, the maximum value which can be obtained from the Bloudeel-Rey relation, equation (4), is that obtained when the intensity at the beginning and end of the interval of integration is equal to the effective intensity.

Corollary 1. If the instantaneous intensity is integrated over a period of time \( t'_1 \) to \( t'_2 \) shorter than \( t_1 \) to \( t_2 \), and \( I' \) is the instantaneous intensity at these times, a value \( I_e \) is obtained for the effective intensity that is always less than \( I' \).

From equation (6) we have

\[
I_e > I'_e.
\]
But

\[ I' > I_e. \]

Therefore

\[ I' > I_e. \]  \hspace{1cm} (10)

**Corollary 2.** If the instantaneous intensity is integrated over a period of time \( t_1' \) to \( t_2' \) longer than \( t_1 \) to \( t_2 \), and \( I' \) is the instantaneous intensity at the times \( t_1' \) and \( t_2' \), a value \( I'_e \) is obtained for the effective intensity that is always greater than \( I' \).

From equation (7) we have

\[ I_e'(a+t_2-a-t_1) = \int_{t_1'}^{t_1} I \, dt + I_e(a+t_2-a-t_1) + \int_{t_2}^{t_2'} I \, dt. \]

But

\[ \int_{t_1'}^{t_1} I \, dt > I'(t_1-t_1'), \]

and

\[ \int_{t_2}^{t_2'} I \, dt > I'(t_2'-t_2). \]

Also

\[ I_e'(a+t_2-a-t_1) > I_e'(a+t_2-a-t_1). \]

Substituting these into equation (7) and simplifying, we have

\[ I'_e > I'. \]  \hspace{1cm} (11)

**Computations of Effective Intensity**

Guides for the computation of the effective intensity from an intensity-time distribution curve may be obtained from the theorem and corollaries.

**a. Computation of \( I'_e \)**

1. Make an estimate \( I' \) of the value of the effective intensity and solve equation (3) using the values of \( t \) corresponding to this intensity, obtaining \( I'_e \).

2. Repeat step 1 above, using as limits the values of \( t \) corresponding to the \( I'_e \) obtained in step 1, obtaining \( I''_e \). Repeat as often as necessary to obtain the desired accuracy.

Note that if the estimated effective intensity is too high (\( I' \) in Fig. 1), the effective intensity, \( I'_e \), computed in step 1 will be below \( I'_e \) (\( I'' \) of Fig. 1) and thus \( I'_e \) lies between \( I' \) and \( I''_e \). If the initial estimate is lower than \( I'_e \) (\( I'' \) of Fig. 1), \( I'_e \) will be greater than both \( I' \) and \( I''_e \) and a “straddle” is not obtained but \( I'_e \) is approached continuously from the low side.

**b. Determination of Conformance of a Flashing Light to Specification Requirements**

1. Compute \( I'_e \) using the time limits corresponding to the specified effective intensity \( I_e \). If \( I'_e \) is greater than \( I_e \), the unit obviously complies, for the conditions are those of Fig. 2a (corollary 2).

2. If \( I'_e \) is equal to \( I_e \), the unit just complies, for then \( I'_e = I_e = I'_e \) (theorem).

3. If \( I_e \) is less than \( I_e \), the unit fails for then the conditions are those of Fig. 2b (corollary 1).

Note that the degree by which the unit exceeds or fails to meet the specification requirements is **not** given by the single computation described here. The method outlined in section a must be used for this purpose.

**c. Visual Range Computations**

If the visual range of the light, under specified conditions of transmittance and threshold, is desired, compute the effective intensity by using the method outlined in section a and compute the visual range by using Allard’s Law.

If the problem is only the determination of whether the light can be seen at a given distance under specified conditions of transmittance and threshold, use Allard’s Law to compute the fixed intensity required to make the source visible at this distance. Then, by using the method outlined in section b, determine if the effective intensity of the unit exceeds this intensity.

**Application to Complex Intensity-Time Curves**

Not all units have single-peak intensity-time distribution curves similar to the curve shown in Fig. 1. Consider an intensity-time distribution curve of the type whose rise is shown in Fig. 3 where \( I_b \)
is the average intensity in the time interval \( t_b \) to \( t_e \).

(The time interval \( t_e - t_b \) is sufficiently short so that the momentary decrease in intensity is not visible.) If \( I_e \) is less than \( I_a \) or is greater than \( I_e \), then the restrictions on the shape of the curve stated in theorem 1 are met and there is no problem in the determination of \( I_e \).

Consider the case where \( I_e \) lies between \( I_a \) and \( I_e \). It may be easily shown by means of equations (5) and (8) that as the shape of other parts of the intensity-time distribution curve changes, the lower limits of time to be used to obtain the maximum value of equation (4) will lie between \( t_b \) and \( t_b \), or between \( t_e \) and \( t_e \), and will never lie between \( t_e \) and \( t_e \). If \( I_e \) is equal to \( I_b \), then either \( t_b \) or \( t_e \) can be used as the lower limit.

**Application to Groups of Short Flashes**

In general a signal from a flashing light consists of regularly spaced single flashes of light and the interval between flashes is so great that each flash has little influence on the effective intensity of the adjoining flashes.

Consider first a flash with the intensity-time distribution shown in Fig. 4. This flash is similar to that of the "split-beam" beacon used at military airfields. If the threshold intensity required to make a steady light visible is much less than \( I_e \) (\( I_{T_v} \)), the flash will be seen as a continuous flash with two peaks. However, if the threshold intensity is about equal to \( I_e \) (\( I_{T_0} \)), two separate flashes will be seen. The maximum distance at which the light can be seen will be determined by the effective intensity of a single flash computed over the time interval \( t_1 \) to \( t_2 \).

There are lights that produce a number of very short flashes in rapid succession so that this group of flashes is seen as a single flash. An example of a light of this type is a unit using a number of condenser-discharge lamps to produce a single flash.

There appear to be no published data reporting studies of the effects of groups of flashes where the interval between flashes is short. Behavior of the eye under somewhat similar conditions suggests that if in a group of flashes the periods during which the instantaneous intensity of the light is below the effective intensity of the flash are of the order of 0.01 second or less, the eye will perceive this group as a single flash. The effective intensity of the group should then be computed by equation (12), choosing as times \( t_1 \) and \( t_2 \) the first and the last times the instantaneous intensity is \( I_e \).

$$I_e = \frac{\int_{t_1}^{t_2} I \, dt + \int_{t_2}^{t_3} I \, dt + \int_{t_3}^{t_4} I \, dt + \int_{t_4}^{t_5} I \, dt}{a + t_2 - t_1}.$$  

(12)

Note that \( I_e \) is the effective intensity of the group and not that of a single flash.

If the periods during which the effective intensity is less than \( I_e \) are of the order of 0.1 second or more, it is believed that the individual flashes will be seen. Therefore, the effective intensity should then be computed on the basis of a single flash.

When the dark period is between 0.01 and 0.1 second, the effective intensity will lie between that of a single flash and that of the group. The behavior during the transition is not known.

**Numerical Examples**

Although the precise determination of the maximum value of \( I_e \) may appear laborious, it is relatively easy. Since a change in the times chosen as the limits in equation (4) changes the denominator and the numerator in the same direction, it is not necessary to determine the correct limits, \( t_1 \) and \( t_2 \),
with great precision in order to obtain a satisfactorily precise determination of $I_e$. This is illustrated in Fig. 6 which shows two representative intensity-time distributions, one for a flashing light with a flash duration of about one-quarter second, and one with a flash duration of about one-twentieth second. The values of $I_e$ are computed for seven sets of time limits. The values obtained are indicated at the abscissa of the time limits. The middle value of each group is the maximum $I_e$ computed according to the method outlined above. Note that this value is equal to the instantaneous intensity at the corresponding time limits. Typically, the maximum effective intensity occurs lower on the curve for the short duration flash than for the long one and the variation of computed values of effective intensity with changes in time limits is smaller.

Experience indicates that if the times chosen for the initial integration are the times when the instantaneous intensity is about 20 per cent of the peak intensity, only one additional step is required to obtain a value for the effective intensity which is within one or two per cent of the maximum value. This is within the limits of accuracy with which the integral is evaluated by means of a planimeter. Often a single computation is sufficient if, instead of using as limits for the initial integration the times when $I_e$ is 20 per cent of the peak intensity, the times used are the times when the instantaneous intensity is equal to the product of the peak intensity and the number of seconds between the times when the instantaneous intensity is roughly five per cent of the peak intensity.

The maximum value of $I_e$ for the curves of Fig. 6 were computed by using as limits for the first integration the times corresponding to an intensity equal to about 20 per cent of the peak intensity and as limits for each succeeding integration the times corresponding to the instantaneous intensity obtained from the preceding step. (The curves have the shape of probability curves so the values of integrals may be computed with the desired accuracy. The accuracy is not limited by the accuracy of planimetric measurements.) Successive values for $I_e$ of 1.66, 1.75, and 1.75 kilocandles were obtained for the longer flash and of 3.41, 3.42, and 3.42 kilocandles for the shorter flash.

**Discussion**

As noted above, concern has frequently been expressed about the choice of the limits for the integral of the Blondel-Rey relation for computing the visual range of a flashing light. It seems illogical to extend the limits of the integral beyond the

![Figure 6. Examples of the effects of time limits on the computed value of effective intensity. The effective intensities are indicated at the abscissa of the time limits used in the computations.](Image)
times when the instantaneous intensity is below the threshold intensity for steady burning lights so that intensities which are below threshold, even for a steady burning light, are included, or to exclude intensities which are above threshold for steady burning lights. Using this reasoning, Blondel and Rey suggested that the limits of the integral of equation (4) be the times when the instantaneous intensity is equal to the threshold intensity. As shown above, these are also the limits which make the computed visual range of the light a maximum. Therefore, the use of these limits in evaluating the performance of a lighting unit appears to be a logical choice.

The use of the maximum value of $I_o$ as the effective intensity of a flashing light is probably not valid except when the light is at or near threshold. When the light is well above threshold, not only will the value of $\alpha$ in equation (4) be decreased, thus tending to increase the value of $I_o$, but also the limits of the integral should probably be extended to include the entire portion of the flash which is above threshold, thereby tending to decrease the value of $I_o$. In many cases this latter effect will be predominant. This is consistent with the decrease in effective intensity of airway beacons with increase in illuminance at the eye found by Neeland, Laufer, and Schaub.

This analysis should be considered only as a mathematical treatment of equation (4). The analysis neither proves nor disproves the validity of this equation in determining the effective intensity of flashing lights nor the validity of the principle of choosing the limits of integration so that the effective intensity is a maximum.

References


G. A. Horton: Messrs. Projector and Douglas are to be highly commended for presenting two papers on a subject which has been long neglected. The papers are particularly timely because of recent application of condenser discharge lights as aviation runway approach lights.

In the condenser light, the length of the flash may be of the order of 100 microseconds, while the peak of the flash may reach several million candlepower. Since the time required for the eye to reach its maximum response is of the order of one-tenth of a second, it is clear that the effective candlepower as seen by the eye is not the same as the measurable average candlepower developed by the flash.

However, by application of the formulas developed by the authors of these two papers to the measured data can the effective candlepower of the condenser lights be expressed.

I should like to ask the authors if visual field checks of the condenser lights have been made to substantiate the theory developed in the papers and, if so, what degree of agreement was obtained.

T. H. Projector and C. A. Douglas: It is a pleasure to acknowledge the comments of Messrs. Carlson and Horton and to reply to their questions.

Mr. Carlson raises the question of the relation between effective intensity and the conditions of observation. The term "effective intensity" is used for convenience, and implies narrowly restricted conditions, usually threshold illuminance, dark adaptation, etc. Strictly, the term should be "effective illuminance" for greater generality, thus automatically taking into account the effect of atmospheric attenuation on illuminance. The effect of other sources of light in the field of view and, in general, of the background condition, state of adaptation, etc., has not been sufficiently explored and is a fertile field for further investigation.

In reply to Mr. Horton, there have been a number of visual field checks of the applicability of the Blondel-Rey law to condenser discharge lights data in the United States and in Europe. All of the results of these checks show no significant deviations from the Blondel-Rey law. To our knowledge none of these results has as yet been published formally. Publication of the results of work at the National Bureau of Standards is expected in the future. It should be noted that these remarks apply only to the direct light from the source when the illuminance from the light is near threshold. Frequently, in a foggy or hazy atmosphere, the visual range of the glow of a flashing light will be considerably greater than the visual range of the direct light.

Committee Personnel

A listing of I.E.S. Committees—Standing, General, Task and Technical—with their personnel begins on page 18A.